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# Procedural mathematical knowledge and use of technology by senior high school students

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**Abstract**
The article at hand deals with students' procedural knowledge, the frequency of technology use (CAS, graphics calculators) during mathematics education in upper secondary level and their self-assessed technology knowledge. In this study, the participating students (representative sample of Austrian high school students in the final year, n=455) had to solve procedural, curriculum-related tasks without any aids (neither technology nor formula booklets). We examined how the frequency of technology use in the classroom affects the students' success rate on procedural tasks. On average, GeoGebra or graphic calculators with CAS are used once a week by the teacher and the students in class, respectively, and unexpectedly, there is no significant correlation between the frequency of technology use during mathematics education in upper secondary level and the procedural knowledge acquired. Regardless of the success in solving the procedural tasks, the students rate their technology knowledge for solving the procedural tasks as rather high.

**Keywords:** procedural knowledge, technology use, technology knowledge, mathematics education

## INTRODUCTION

Even if the present study's results are not limited to German-speaking countries in terms of its informative value, it draws its motivation from an interesting educational policy situation and discussion that has been occurred in Austria in recent years. There were (again) complaints, even in daily newspapers, that graduates of higher schools or first-year students in mathematics-related courses do not have sufficient arithmetic and algebraic skills (Die Presse, 2015; Kurier, 2018). More recent is the criticism that mathematics in school is primarily reduced to finding a suitable command using more advanced technology like computer algebra systems (CASs) or graphic calculators. For example, lecturers at the Technical University of Vienna observe a "constant decline in mathematical knowledge and skills" among first-year students.

These deficits (e.g., in the case of term transformations) are attributed, among other things, to the "currently unrestricted possibility of using CASs" in the standardized written school leaving exam (Matyas & Drmota, 2018). The Austrian Mathematical Society (ÖMG) also notes in a statement on the future of the standardized school leaving exam that operational skills suffered from the approval of technology for all tasks (ÖMG, 2019). From their point of view, it is not required that technology-free calculating should be in the foreground in the future, but that it should at least "appear" in the school leaving exam, since it can make important contributions to understanding "to a modest extent" (ÖMG, 2019, p. 1). In line with these findings, the concept of the school leaving exam at Austrian high schools (AHS, German abbreviation for "Allgemeinbildende höhere Schule"; translates to "high school for general education") is currently being revised. It is envisaged that from the 2027/28 school year on, some calculations will have to be carried out without calculators and more advanced technologies (i.e., a "technology-free" part in the school leaving exam is planned, BMBWF, 2022, p. 2). "Basic calculations" should again be mastered with paper and pencil, such as simple term transformations, solving equations or deriving a function using the product rule (BMBWF, 2022, p. 2).


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**Contribution to the literature**

- This study closes a gap in the literature, where studies on procedural knowledge and frequency of technology use were usually only examined separately.
- This research uses a validated test instrument to show that, different than expected, there is no statistically significant connection between the frequency of technology use in mathematics classes and procedural mathematical knowledge.
- This work surveys students' intention to use more advanced technology in procedural tasks and how they self-assess their technology knowledge.

A look at Germany shows that there, too, a large number of university lecturers find deficiencies among first-year students in areas such as fractions, binomial formulas, and term transformations, which cannot be made up for either in preliminary courses or in bridging courses (Open Letter, 2017). After there are technology-free parts in the school leaving exams (Abitur) in almost all German federal states, the reasons for the deficits described are located in the introduction of educational standards, the competence orientation, and the use of modeling tasks in the school leaving exam (Open Letter, 2017). But there, too, it is required to ensure that the use of calculators and CASs "does not impair the important phase of practicing elementary and symbolic calculation techniques" (Open Letter, 2017).

Internationally, procedural knowledge seems to play a central role in STEM (science, technology, engineering, and mathematics) courses, despite the fundamental commitment to strengthening conceptual understanding (Altieri, 2016; Bosse & Bahr, 2008; Hallett, 2006; Qetrani et al., 2021) and the ongoing digitalization of our society. The literature confirms that mathematics is mainly taught, learned, and tested procedurally in tertiary STEM courses (e.g., Bergqvist, 2007; Bergsten et al., 2017; Engelbrecht et al., 2009; Zerr, 2009). This makes the desire from this direction for a strengthening of procedural skills in school education understandable, although we do not see it uncritically ourselves. Empirical evidence is needed at this point before appropriate educational policy decisions can be made.

In fact, there are no empirical studies that investigated the procedural knowledge of high school graduates over the last few decades, especially in Austria. There is a lack of systematic, representative surveys that would scientifically and comprehensibly confirm the above-mentioned deficits and the causes listed or at least suggested. What is needed here is an inventory of the procedural knowledge of high school students in their final classes, their technology knowledge, and the frequency of technology use in the classroom and for homework. Furthermore, we are interested in finding out to what extent the frequency of technology use in the classroom and at homework influences students' technology-free procedural skills. The aim of this paper is to close these research gaps.

The results are of course also of interest beyond German-speaking countries. The connection between the frequency of technology use in mathematics lessons and procedural knowledge indicates to what extent the digitization of our society probably will lead to a change in the types of knowledge acquired in school. The current situation in Austria (so far unrestricted use of technology in the school leaving exam, but a technology-free part from 2025/26 on) offers unique framework conditions for research into the change in procedural knowledge and its dependence on the frequency of technology use.

## LITERATURE REVIEW

### Procedural Knowledge in Large-Scale Studies

In order to survey students' procedural knowledge, appropriate tasks are needed. Of course, there are many studies that test students' mathematical knowledge in general, e.g., PISA and TIMSS. A closer look at the items of PISA (OECD, 2018) or TIMSS shows that students should apply their mathematical knowledge in an embedded context, and, hence, it is a mixed query of conceptual and procedural knowledge. Besides the fact that 15-year-old students are tested and due to the mixture described, the items cannot be used to investigate procedural knowledge. However, the upper secondary level study TIMSS advanced provides some suggestions for item creation. Its framework distinguishes between content domains and cognitive domains. In the cognitive domain "knowing", the category "compute" is listed. The description of the tasks of this category is very close to the understanding of procedural knowledge presented here.

> "Carry out algorithmic procedures (e.g., determining derivatives of polynomial functions, and solving a simple equation)" (Grønmo et al., 2014, p. 14).

Solely nine countries took part in the last survey (TIMSS advanced 2015), and there were no German-speaking countries among them. In conclusion, the Russian federation (intensive courses) achieved the highest and Sweden the lowest average score among the nine participating countries (Provasnik et al., 2016). A detailed analysis of the results in the category "compute" was not published, an exception to this is Norway. Students from there achieved poorly on items with high demands on symbolic manipulations. In contrast, however, they achieved very good results in PISA tasks, which are set in an application-oriented context (Pederson, 2015). As a consequence, Pederson (2015) calls for more time to be devoted to algebraic activities in mathematics classes. Generally, a decreasing tendency of numerical and arithmetic skills can be observed in the Scandinavian countries. Some blame application orientation, others the use of technology in mathematics teaching (Österman & Bråting, 2019). In summary, it can be said that there are hardly any studies that survey purely procedural knowledge (Rolfes et al., 2021).

### Measuring Procedural Knowledge of University Entrants

Close to the target group of the present study, students in their last year of school, are university entrants. Such students often face difficulties in mathematics in the transition from secondary school to higher education (Di Martino & Gregorio, 2019). For this reason, many universities test the mathematical knowledge of the students in the introductory phase (e.g., Germany: Greefrath et al., 2015; Poland: Kopńaska-Bródka et al., 2015; UK: Cambridge Assessment Admissions Testing, 2019). According to Hoever and Greefrath (2018) and Knospe (2008), entrants' mathematical knowledge has decreased over the years. A closer look at such tests at German universities shows that they mostly ask for procedural skills, in concrete terms, this means the fast and correct execution of known procedures and a confident handling of standard representations (Heinze et al., 2019). Considering the test of Hoever (2018) at the FH Aachen University of Applied Science, it is a paper-pencil test with no aids (neither technology nor formula booklet) allowed. The items can be described as follows: the text length of the items is kept short, they are of procedural nature, students are asked for mathematical contents known from school and the correctness of the students elaborations determines the total score (Greefrath et al., 2015).

### Procedural Knowledge and Use of Technology

The use of digital tools (such as CASs, dynamic geometry systems, etc.) in mathematics teaching led and leads to changes in teaching in many ways. The associated effects on students' mathematical abilities and skills have been researched extensively over the years. Essentially, a positive but rather small effect ($d \approx 0.2$) in mathematics performance could be shown (Cheung & Salvin, 2013; Drijvers et al., 2016 based on Li & Ma, 2010; Rakes et al., 2010). Cheung and Slavin (2013) summarized these results with "it's a help but not a breakthrough" (p. 102). Otherwise, the PISA study from 2012 revealed a negative correlation between mathematics performance and computer use ("index of computer use in mathematics lessons"), which led to the following statement

> "[…] there is little solid evidence that greater computer use among students leads to better scores in mathematics […]" (OECD, 2015, p. 145)

Ronau et al. (2014) nevertheless summed up positively: "[...] digital technologies such as calculators and computer software improve student understanding and do no harm to student computational skills" (p. 974). This is confirmed by an Austrian study, according to which the regular use of more advanced technology is beneficial for the general mathematical learning outcome (Liebscher et al., 2013). These (meta) studies refer to the mathematics performance of students, which include conceptual and procedural knowledge, though, the present study is interested in the use of digital tools in connection with procedural knowledge exclusively. Long time ago, Wynands (1984) already examined the effects of an ordinary (scientific) calculator on the arithmetic skills of students at the end of secondary school without using a theoretical model. In this study, it was shown that students who reported using the calculator did not perform any worse in a non-calculator test as students who did not use the calculator in class. According to Wynands (1984), the students' success rate was on a rather low level but comparable to results from the time when there were no pocket calculators. The currently frequently expressed skepticism (see introduction) regarding nowadays used technology or digital tools (CAS, DGS, etc.) in mathematics classes is reminiscent of that towards the ordinary (scientific) calculator at that time. A few studies indicate that paper-and-pencil skills or procedural skills, respectively, can also be learned using technology (e.g., CAS) in the classroom (Ingelmann, 2009; Kieran & Drijvers, 2006; Kieran & Yerushalmy, 2004). According to Kieran and Yerushalmy (2004), "[…] the presence of technology does not eliminate symbolic manipulation from algebra, but it does change it" (p. 142). Regarding students' paper-and-pencil-techniques, it seems that what matters is not whether technology is hardly used in mathematics education, but how it is taught. The studies do not answer whether there is a connection between paper-and-pencil skills or procedural knowledge and the frequency of use of technology, and in general there are no large-scale studies that examine this.

## THEORETICAL FRAMEWORK

In order to create a well-founded theoretical reference point and to be able to operationalize procedural knowledge by creating corresponding tasks, the most important terms are to be discussed and defined here.

## Procedures

Hiebert and Lefevre (1986) did some fundamental work on the distinction between procedural and conceptual knowledge. They stated that the central property of procedures is that they "are executed in a pre-determined linear sequence" (Hiebert & Lefevre, 1986, p. 6). One can deduct from this, when creating suitable test items for procedural knowledge, an attempt must be made to limit the requirements as far as possible to carrying out sequences of processing steps known from the mathematics lessons in the usual way. Knowledge about the selection of a specific solution method, strategic planning knowledge for task solving as well as conceptual knowledge (which ultimately provides reasons why procedures work) should explicitly not play a role. Even if there is a controversial discussion in the literature to what extent procedural (knowing how) and conceptual (knowing why) knowledge for solving tasks are mutually dependent (Kieran, 2013; Star, 2005), first indications point out that these types of knowledge can be separated empirically (Lenz et al., 2019, for the topic of fractions).

We work with the following definition: "A procedure is a step-by-step instruction that prescribes how a task is to be solved." (Hiebert & Lefevre, 1986) An example is solving a quadratic equation without technology. Recognizing the type of equation, grasping the structure of the equation, and selecting an appropriate procedure require conceptual knowledge and are therefore not part of the procedure. Conversely, if the task specifies that it is a quadratic equation that is to be solved using one of the solution formulas, the individual processing steps are predetermined. The procedure then consists of writing down the formula, reading the coefficients from the quadratic equation, inserting them into the formula, carrying out the necessary arithmetic calculations, applying fractional calculation rules if necessary and writing down the solution set.

## Procedural Knowledge

The execution of procedures requires a specific type of knowledge, which will be examined in more detail in this section and finally laid down in a definition. Of the different definitions of procedural knowledge in the literature, we focus in particular on the formulations of Star et al. (2015, p. 45): "Procedural knowledge refers to having knowledge of action sequences for solving a problem" and Rittle-Johnson and Schneider (2014, p. 5): "[…] procedural knowledge is the ability to execute action sequences (i.e., procedures) to solve problems." While the first description gives attention to the knowledge of the procedure (or the algorithm), the second formulation focuses on the concrete execution of the procedure (i.e., the aspect of skill). Altieri (2016) takes these two aspects into account and integrates them into a common and therefore more differentiated definition. In this respect, procedural knowledge is the combination of "knowledge of the procedure" and "procedural skills":

- **Knowledge of the procedure:** Knowledge of symbols and the formal language of mathematics as well as knowledge of rules and procedures for solving mathematical problems.
- **Procedural skills:** Skills required to apply the knowledge of the procedure in a case-specific and targeted manner in a way that leads to a correct result in a reasonable time, especially in the case of procedures (Altieri, 2016, p. 25, translated).

In the case of a quadratic equation, the reproduction of the solution formula is part of the knowledge of the procedure, all further processing steps are part of the procedural skills. In general, knowledge of the procedure usually refers to specific procedures, while procedural skills usually consist of (elementary) mathematical skills that can be used in a wide variety of mathematical topics (Altieri, 2016).

## Task Characteristics

In the following, the dimensions "number of procedural steps of the task", "curricular grade level" and "content area" are described, according to which tasks on procedural knowledge can be created and systematically classified. The corresponding model is taken up again when the creation of the tasks is reported.

The number of procedural steps of a task can be determined based on the number of different decisions that are necessary to solve the task according to the intended solution method. Decisions are to be understood as small solution steps, in this case the steps in the procedure under consideration. Typically, the length of a task is described by categories such as low, medium, and high (Jordan et al., 2006, p. 61). In our case, it makes sense not to count several analog solution steps that occur more than once. We will report on this in the methods section in more detail.

The curricula of Austria's high schools (Curriculum, 2021) and the concept of the standardized written school leaving exam (Concept SRP, 2021) serve as the basis for the dimensions "content area" and "curricular grade level". Since visual representations of mathematical objects and connections to the non-mathematical world play a central role in the content areas of functional dependencies and probability and statistics per se, the procedural part of the knowledge required for solving corresponding tasks is difficult to isolate. In the content areas of algebra and analysis, on the other hand, there are a lot of tasks that can be carried out in a formalistic and non-visual way (e.g., calculating a scalar product, deriving a function using the product rule). We therefore limit our test items to these two areas.

**Table 1.** Tasks of the study & associated model parameters (Class: Grade level, content (sub)area [AG: Algebra, AN: Analysis], number of different procedural steps, & the rating of importance on a scale from 0 to 3)

| No. | Description | Gr. | Content subarea (content area) | Steps | Rating |
|---|---|---|---|---|---|
| PA01 | Transform a formula | 8 | Terms & formulas (AG) | 3 | 2.79 |
| PA02 | Use a binomial formula[T] | 7 | Terms & formulas (AG) | 5 | 2.86 |
| PA03 | Solve a linear equation[T] | 6 | Linear equations & systems of equations (AG) | 4 | 2.93 |
| PA04 | Conduct a polynomial division | 11 | Terms & formulas (AG) | 6 | 1.00 |
| PA05 | Use certain exponentiation rules[T] | 10 | Exponentiations, roots, & logarithm (AG) | 3 | 2.86 |
| PA06 | Solve a fractional equation | 8 | Linear equations & systems of equations (AG) | 9 | 1.86 |
| PA07 | Solve a 2x2 system of equations (elimination method)[T] | 8 | Linear equations & systems of equations (AG) | 12 | 2.93 |
| PA08 | Solve a quadratic equation | 9 | Nonlinear equations (AG) | 11 | 2.21 |
| PA09 | Calculate a cross product[T] | 10 | Vectors (AG) | 4 | 1.71 |
| PA10 | Calculate a different quotient | 10 | Differential calculus (AN) | 7 | 2.64 |
| PA11 | Differentiate a polynomial function[T] | 11 | Differential calculus (AN) | 5 | 2.93 |
| PA12 | Conduct an integration by parts[T] | 12 | Integral calculus (AN) | 6 | 1.00 |
| PB01 | Add two fractions[T] | 6 | Arithmetic (AG) | 5 | 2.71 |
| PB02 | Divide two decimal numbers[T] | 5 | Arithmetic (AG) | 6 | 2.00 |
| PB03 | Conduct a partial root extraction[T] | 10 | Exponentiations, roots, & logarithm (AG) | 3 | 2.07 |
| PB04 | Transform in floating-point notion | 9 | Exponentiations, roots, & logarithm (AG) | 3 | 2.29 |
| PB05 | Solve 2x2 system of equations (substitution method) | 8 | Linear equations & systems of equations (AG) | 12 | 2.86 |
| PB06 | Solve an equation with square roots | 10 | Nonlinear equations (AG) | 8 | 2.21 |
| PB07 | Solve a biquadratic equation[T] | 11 | Nonlinear equations (AG) | 13 | 1.50 |
| PB08 | Calculate a dot product | 9 | Vectors (AG) | 3 | 2.93 |
| PB09 | Calculate a linear combination of vectors | 9 | Vectors (AG) | 7 | 2.64 |
| PB10 | Differentiate using the product rule[T] | 11 | Differential calculus (AN) | 4 | 2.36 |
| PB11 | Differentiate using the chain rule[T] | 11 | Differential calculus (AN) | 3 | 2.43 |
| PB12 | Calculate a definite integral[T] | 12 | Integral calculus (AN) | 9 | 2.93 |

## RESEARCH QUESTIONS

As reaction to the problem areas outlined above and the identified research gaps, the present study (which was carried out as part of the OFF project "Operational skills and abilities without the use of technology") is intended to collect corresponding data and thus answer the following research questions:

1. What is the relationship between success in completing curriculum-related procedural tasks (without technology and formula booklets) and the frequency of more advanced technology use in mathematics education?
2. What is the relation between success in completing curriculum-related procedural tasks (without technology and formula booklets) and the intention to use technology for these tasks?
3. What is the relation between success in completing curriculum-related procedural tasks (without technology and formula booklets) and the self-assessed technology knowledge for solving these tasks?

A formula booklet was not permitted because we also wanted to survey the students' "knowledge of the procedures" (as a part of procedural knowledge). We do not want to be misunderstood with these research questions by giving procedural knowledge in mathematics education a higher priority than before. On the contrary, we do not want to make any statements about the importance of procedural knowledge but create a database that is as descriptive as possible. Although we collected our data in Austria, the results of our study are also interesting for other regions of the world due to our study design presented in next section.

## METHODS

### Item Development and Validation

Despite thorough literature research, the authors are not aware of any suitable, validated item packages for testing the procedural knowledge of high school students in their final year. In part, Hoever's (2018) tasks of a test in the introductory phase at the University of applied sciences Aachen (Germany) seem suitable for the study at hand. According to Heinze et al. (2019), these tasks mainly cover procedural knowledge, but do not comprehensively cover AHS curricula. For these reasons, a separate collection of items was designed for the present study. Originally, this included 30 items and due to validation processes the number was reduced to 24. A total of 15 people (five mathematicians, five mathematics education researchers, five teachers) rated the procedural knowledge evoked by the tasks as part of an expert validation (procedural score on a scale from 0 "no procedural knowledge" to 3 "exclusively procedural knowledge"), gave additional feedback on the task formulations and rated the importance of tasks. The latter means, that the experts assessed, whether a student should be able to solve the task without using technology and formula booklets (3 "yes", 2 "rather yes", 1 "rather no", 0 "no") (**Table 1**).

Find the solution set of the system of equations

$$\begin{aligned} 2x + 3y &= 1 \\ -4x + 5y &= -13 \end{aligned}$$

in $\mathbb{R}^2$ by addition (elimination) method.

**Figure 1.** Prototypical task of the present study, task PA07 of test booklet A, system of linear equations (translated) (Source: Authors' own elaboration)

This question was asked in a general educational sense, more concretely, regardless of which educational or professional path the students will later choose.

The final 24 items have an average procedural score of $\bar{x}$=2.6 (s=0.2). In the context of a pilot study carried out in January 2021, a correlation (r≈0.66 and p<0.01) to selected procedural tasks by Hoever (2018) could be demonstrated (constructive validation). In order to identify conceptual processing steps when solving the items, four students were asked to solve the tasks while thinking aloud (type-1 verbalizations according to Ericsson & Simon, 1980, 1993). Finally, we can assure that (almost) no conceptual knowledge is being tested. A prototype task from our testing can be seen in **Figure 1**. In the case of task PA07, the solution procedure was deliberately specified because, according to the theory, the decision for an appropriate solution method must be assigned to conceptual knowledge.

When processing the task, the reproduction of the requested procedure (knowledge of the procedure) and its concrete implementation (procedural skills) remains.

In the following, we want to illustrate the counting of the "number of different procedural steps" for solving PA07 (**Figure 1**), which is one of the task characteristics. Before we do that it must be noted, we only consider the shortest possible solution. Of course, some students (see percentage at the end of the paragraph) deviate from the presented path below, which can lead to an increase in number of the different procedural steps.

When using the prescribed counting method, we multiply the first equation by two (step 1). After applying the distributive law on the left-hand side (step 2), the term $2x$ and the term $3y$ must be multiplied by two. In this counting system, they only count as one step (step 3), because they are analog calculations. This system is based on the consideration of preventing a task with many analogous steps, but overall, only few different processing steps from being coded as a long task (e. g. differentiation of a polynomial function, see item PA 11 in **Table 1**). In the next step, the elimination method is applied (step 4). Then, one must add two terms (step 5) and two integers (step 6). Performing the equivalence transformation (division) leads to the next step (step 7), as well the division of two integers (step 8). Then we insert the result -1 of the equation into one of the given equations (step 9). The solving process of this new established equation contains a multiplication of integers (step 10), which is followed by an addition as equivalence transformation (step 11). According to the presented rules, the division does not contribute to increasing the number of different procedural steps. Finally, for a complete processing of the task, the solution set must be written down (step 12). According to this counting method, item PA07 has 12 different procedural steps, which is considered as long task. The above-mentioned validation measures confirm the assumption of the straightforward solvability of the procedural tasks. If there is a deviation from the specified solution procedure, the student's processing was not considered as correct. This does not include deviations from the prototypical solution, provided that the procedure specified in the task is carried out. Considering the main survey, this was the case in 42 of a total of 5,460 student processing (less than 0.8%).

**Table 1** provides an overview of the tasks used in the survey. In order to shorten students' processing time during the survey, the 24 tasks were divided into two test booklets (see designation PA and PB in **Table 1**), a balanced distribution of the model parameters (number of procedural steps of the task, curricular grade level and content area) was considered when splitting.

## Use of Technology

To answer the research questions, which combine procedural knowledge with the use of technology, two additional questions were asked at the end of 14 selected procedural tasks, see marking "T" in **Table 1**. These surveyed on the one hand the self-assessed knowledge of technology and on the other hand the intention to use technology, both of which relate to the (ideally previously processed) procedural task. The first question is formulated, as follows: "Would you know exactly what you would have to enter or do in order to get the result of the task using technology?" with the possible answers "yes", "no, but I would know how to try it", "no, I would try it somehow" and "no, I would have no idea at all". The second reads, as follows: "If you had the opportunity – would you use technology to solve it? (multiple answers possible)" with the possible answers "no", "yes, ordinary scientific pocket calculator (e.g., TI 30, pocket calculator on the mobile phone)", "yes, graphics calculator with CAS (e.g., Casio ClassPad, TI-Nspire)" and "yes, GeoGebra (or a comparable software)". As part of the first evaluation, we used only a distinction between "no" and "yes" for the second question, i.e., the three last-mentioned categories were combined. At the end of the booklets, as part of a questionnaire section, there are three questions on technology usage frequency:

> **Tea:** How often did your high school math teacher typically use more advanced technology (e.g.,

GeoGebra, Casio ClassPad, and TI-Nspire) in class during upper secondary level? (Ordinary pocket calculators are not meant here).

**Stu:** How often did you typically use more advanced technology (e.g., GeoGebra, Casio ClassPad, and TI-Nspire) in the school exercises during upper secondary level? (Ordinary pocket calculators are not meant here).

**Hw:** How often did you typically use more advanced technology (e.g., GeoGebra, Casio ClassPad, and TI-Nspire) for homework in upper secondary level? (Ordinary pocket calculators are not meant here).

The answer options are: "(almost) every math lesson" (for questions "Tea" and "Stu") or "(almost) every homework" (for question "Hw"), "approx. once per week", "approx. once or twice per month", "less than once per month" and "never". Such a gradation in the answer options can be found in other surveys of this type as well (e.g., Yao & Zhao, 2022). The first option listed received the value 0 during data entry, the second the value 1, and so on. The teachers of the students participating in the survey answered the first two questions in almost identical form in order to check the reliability of the students' answers.

### Sample, Data Collection, and Type of Data Evaluation

The tasks above were tested on a representative sample, which was drawn according to the recommendations for carrying out the field trial for the tasks of the Austrian centralized school leaving exam (Bartok & Steinfeld, 2015). All students in final classes at AHS represent the entire target population according to the research questions. For organizational reasons, it was not possible to draw the sample at student level, which is why schools were drawn at random. For the sampling frame, the list of school codes kindly received from the Austrian Federal Ministry of Education, Science and Research was supplemented by the stratification variables federal state, school type and degree of urbanization. Schools were drawn from each stratum using the probability-proportional-to-size method (including substitute schools whose turn it was when the schools originally drawn were canceled).

After the consent of the schools and the approval of the education departments of all nine federal states, the approval of the legal guardians was obtained. 538 students declared their willingness to take part in the survey in April 2021. In the end, we received 455 (test booklet A: 230) completed test booklets and 25 teacher responses to questions concerning the frequency of technology use. The teachers participating with their class were free to choose a 50-minute survey time (duration of one lesson) in April. In the pilot study, it was checked by recording the processing time that the estimated 50 minutes are sufficient for processing one test booklets and in order to guarantee a uniform procedure in the different classes, each teacher received an accompanying letter with precise instructions on the sequences. The students were not allowed to use a formula booklet or any technology when working on the test booklets.

Due to the here used definition of procedural knowledge according to Altieri (2016), procedural knowledge is only available if both knowledge of the procedure and procedural skills are available and hence the procedural task is solved correctly. The authors coded the students' processing of each task of the completed test booklets by awarding one point for successful completion and zero points otherwise. To counter the objection that longer tasks count just as much as shorter tasks, the success rate does not depend on the number of procedural steps (linear model: $\beta$=-0.022, p=0.234, other models were also calculated, which did not provide a better fit of the given data). The results below are based on descriptive data analysis and, in order to take class structure into account, linear mixed effects models. In doing so (model 0 in **Table 2**), the procedural performance $p_{ij}$ of student $i$ in class $j$ can be written as the sum of the arithmetic mean $\beta_{0j}$ of the procedural performances of students in class and an error term $\varepsilon_{ij}$, so it applies: $p_{ij}= \beta_{0j}+ \varepsilon_{ij}$ and the above-mentioned mean value can be again represented as sum of the overall mean value $\mu_{00}$ of student performances and an error term $a_{0j}$, then we have: $\beta_{0j}= \mu_{00}+ a_{0j}$. Based on these considerations, the intraclass correlation coefficient (ICC) can be calculated. Due to the use of two test booklets, we checked their interaction, in other words, whether the effects were the same in both test booklets. This was achieved by calculating a separate linear model with the feature "test booklet" and the corresponding two-way interaction (one of the variables "Stu", "Tea", or "Hw" and the categorical variable "test booklet"). For the calculations of the models, reference category "A" was chosen for the categorical variable "test booklet".

## RESULTS

**Figure 2** shows the students' success rate of each item. On average, the arithmetic mean of these rates is 36%, with some almost never being solved (PA12: 0.04%) and some by almost everyone (PA03: 94%). The median of the above-mentioned success rates amounts to 30% and the corresponding quartiles are 17% and 52%. Overall, the two exercise books hardly differ, the difference in the average students' success rate is 3.7 percentage points (exercise booklet A: 34.3%, exercise booklet B: 38%). According to the Austrian curriculum, the items PA01, PA02, PA03, PA06, PA07, PB01, PB02, and PB05 can be assigned to lower secondary level. On average, these items have higher students' success rates

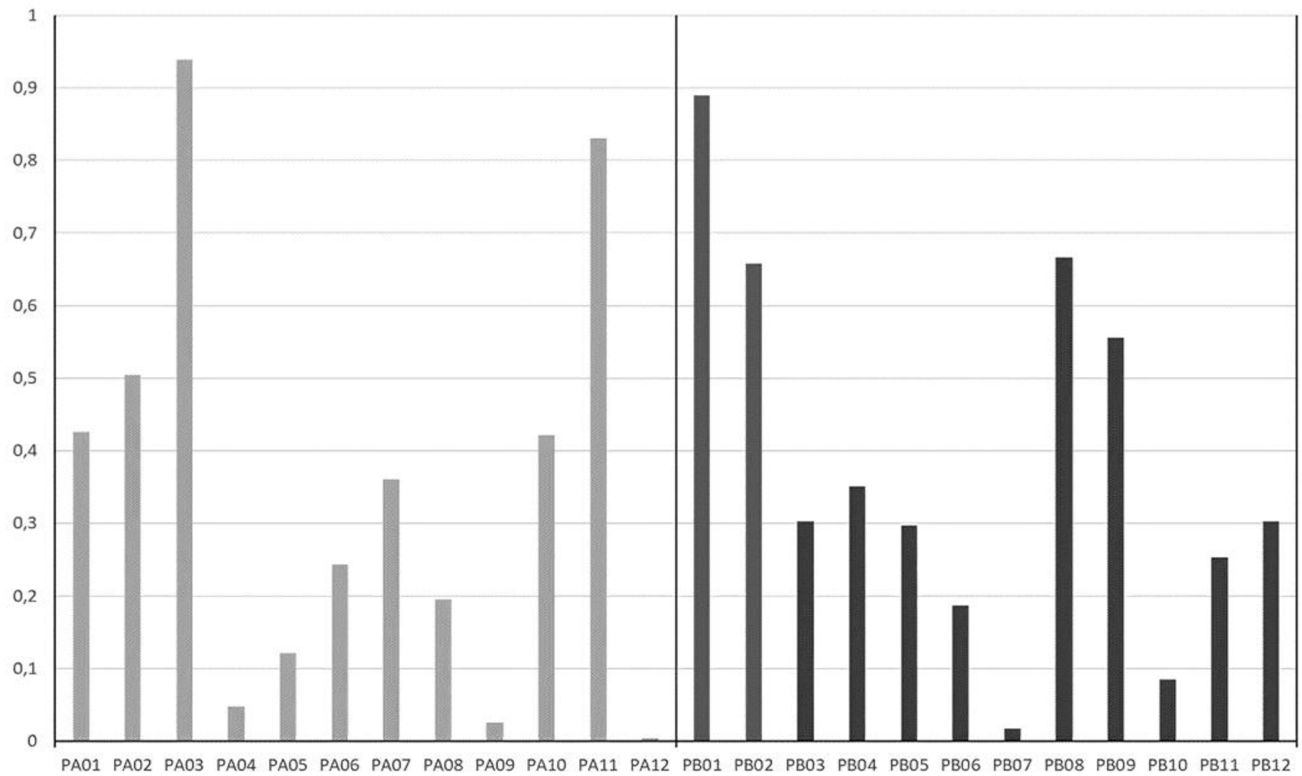


**Figure 2.** The relative students' success rates for each item in the test booklets (Source: Authors' own elaboration)

**Table 2.** Linear mixed effects models to predict students' procedural knowledge through technology usage frequency ("Tea", "Stu", & "Hw").

| | Model | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| | Intercept | **0.362** | **0.345** | 0.333 | 0.332 |
| **Main effects** | Tea | | | -0.004 | 0.011 |
| | Stu | | | 0.011 | -0.003 |
| | Hw | | | **0.021** | 0.004 |
| | Test booklet (A) | | 0.035 | | 0.006 |
| **Interactions** | Tea | | | | -0.033 |
| | Stu | | | | 0.022 |
| | Hw | | | | **0.040** |
| | ICC | 28.17% | 27.58% | 24.50% | 24.01% |
| | logLik | 135.95 | 136.51 | 147.84 | 154.06 |

Note. All parameters are not standardized; for the variable "test booklet" the reference category is given in brackets (A); & parameters in bold are significant ($p<0.05$).

than those at upper secondary level (sec I: $\bar{x}$=0.54 and s=0.25, sec II: $\bar{x}$=0.27 and s=0.24).

The following can be reported for the questions on the frequency of technology use: The mean value of the coded answers to the question estimating how often the mathematics teacher used more advanced technology ("Tea") is $\bar{x}$=1.00 (s=1.03). This corresponds to the answer option "approx. once a week". Considering the second question, how often the students themselves typically used more advanced technology during high school ("Stu"), here the average value amounts to $\bar{x}$=1.21(s=1.21). The third question focuses on the use of technology in the homework ("Hw"), here the value is $\bar{x}$=0.92 (s=1.2). The mean values of the teachers' answers to the first two questions ("Tea" and "Stu") are almost identical to the values of the students ($\bar{x}$=1.08, s=0.76 and $\bar{x}$=1.33, s=0.85).

In the following paragraph, we consider the dependency of procedural knowledge on technology usage frequency (**Table 2**).

For procedural knowledge, the hierarchical structure of the data is considered and, hence, linear mixed effects models must be used because the class affiliation for itself could already explain 28.17% (ICC) of the variance of procedural knowledge (model 0). This variance explanation is only slightly reduced by the addition of the explanatory variables "Tea", "Stu" and "Hw". However, the test booklet has no effect on the procedural knowledge (model 1). When looking at the aforementioned variables, only one effect was shown for the use of technology in homework (model 2). The value of 0.021 means that for every one unit increase on the technology use scale ("Hw", e.g., from "(almost) every homework" to "approx. once per week"), the procedural score increases by 2.1 percentage points. Though, this effect varied with the test booklets. However, the absolute values of the numbers in **Table 2** are very small, even for the significant variable "Hw", for this reason the frequency of technology use has only little influence.

**Figure 3** visualizes the intention to use technology for the respective task. A distinction was made between successful processing, marked with 0, and unsuccessful processing, marked with 1. The bottom bar shows the relative frequency of students who would use technology, measured by the number of students who successfully completed task PA02, here 55%, and the

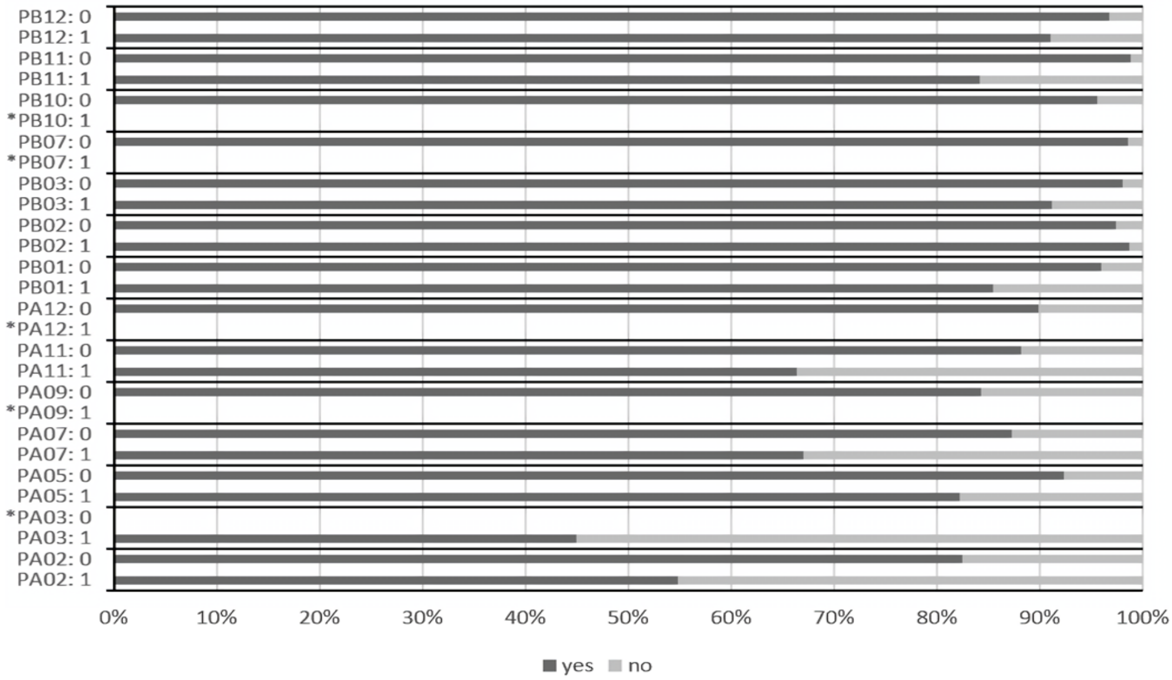


**Figure 3.** The intention to use technology in the respective task, separately visualized in successful (1) & unsuccessful (0) students' processing (Source: Authors' own elaboration)

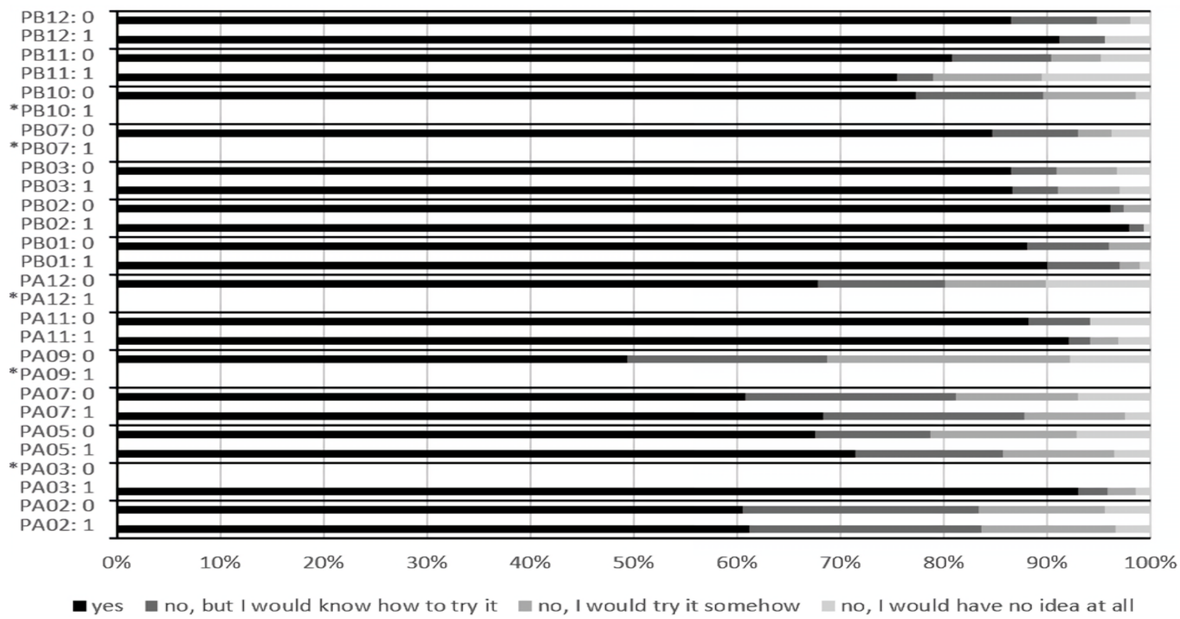


**Figure 4.** The self-assessed knowledge of technology in the respective task, separately visualized in successful (1) & unsuccessful (0) students' processing (Source: Authors' own elaboration)

relative frequency of students who would not use technology, measured by the number of students who successfully completed task PA02, amounts to 45%. The second bar from the bottom refers analogously to the unsuccessful students in this task, the numbers are 82% and 18%. This means that 82% of the students who did not complete the task successfully would use technology. For tasks whose solution rate is not between 10% and 90% (marked with an asterisk in the legend), the corresponding bar was omitted due to little information value (e.g., PA03: 0).

Remarkably, the intention to use technology in the respective procedural task is lower in almost all tasks for successful processing than for unsuccessful ones (single exception PB02). However, this difference is only significant for the tasks PA02, PA07, PA11, PB03, and PB10. Apart from this, with only one exception (PA03), for each of these items, more than 50% of those students who solved the task without any aids would also use technology to solve the task if given the opportunity.

The results of the self-assessed technology knowledge question are shown in **Figure 4**. The bottom bar shows the relative frequencies of students who successfully completed task PA02 and "yes", "no, but I would know how to try it", "no, I would try it somehow" and "no, I would have no idea at all". Here, 61% of the students stated that they knew what they had to do to get the result of the task. The values of the other categories are 22%, 13%, and 3%. The second bar is to be interpreted analogously, just considering the unsuccessful students' processing. Again, for tasks whose students' success rate is not between 10% and 90% (marked with an asterisk in the legend), the corresponding bar was omitted due to little information value (e.g., PA03: 0). With the exception of task PA09, the relative proportions of the category "yes" for the items show values of over 60%, regardless of the students' success. It is also noticeable that the bars for successful processing are similar to the bars for unsuccessful processing. There are no significant differences in the distribution of answers for any of the tasks (Chi-squared test for homogeneity, $p>0.05$).

## DISCUSSION, CONCLUSION, AND LIMITATIONS

The present study sees itself as an inventory and thus as a possible data basis for current educational policy decisions in Austria and as a basis for comparing the procedural knowledge of high school students in upcoming graduation classes. Apart from the concrete political situation in Austria, our results are also interesting for the international scientific community. While it is generally assumed that more frequent use of technology leads to less procedural knowledge, the present study shows that there is no substantial statistical correlation. Hence, there does not seem to be a contradiction between using more advanced technology in the classroom and still acquiring mathematical procedural knowledge. The class structure of the data better explains students' performance in procedural knowledge than the frequency of technology use. From this, one can cautiously conclude that the acquisition of procedural knowledge depends on the way teachers design their lessons. In the literature, there are already indications in this context that the presence of technology in the classroom does not impede the acquisition of procedural skills (Ingelmann, 2009; Kieran & Drijvers, 2006; Kieran & Yerushalmy, 2004).

For the time of the survey in 2021, it can generally be stated that the students' success rates when processing procedural tasks can be classified as rather low. This confirms the observations reported in the literature (Di Martino & Gregorio, 2019; Matyas & Drmota, 2018; Open Letter, 2017; Pederson, 2015) and is reminiscent of the results of Wynands (1984), who, however, examined the computational skills of students at the end of lower secondary school. In this regard, it should be mentioned that in the present study, procedural tasks from lower secondary level have, on average, higher success rates than those from upper secondary level. However, it cannot be deduced from our data to what extent this is related to the frequency of technology use in lower secondary school, to the intensity with which certain types of tasks are trained or to the level of complexity of certain tasks.

The myth of the omnipresent use of technology in Austrian mathematics education mentioned in the introduction (ÖMG, 2019) cannot be confirmed from the representative data collected. The results of the survey show that more advanced digital tools (CAS, graphic calculators) are used on average only once a week by mathematics teachers or their students in class and once a week by students for homework. Conversely, this means that it is not used in the remaining two to three weekly lessons or for the remaining homework. The relatively rare use of technology in mathematics lessons may have different reasons, such as a lack of infrastructure in schools or appropriate teacher training. However, this requires further research. In general, analyzes and investigations are needed to find the reasons for the relatively low frequency of solving some of the tasks.

Unsurprisingly, we observed a somewhat greater intention to use technology for certain tasks among students who were unsuccessful in the technology-free processing of these tasks. In general, however, it can be said that a large number of successful students also appreciate the opportunity to have technology available, probably for control purposes or to save computing time and effort. Restricting technology in the classroom would deprive them of that opportunity.

The data on technological knowledge shows that, to a large extent, the students feel capable of solving the procedural tasks with the help of technology. More precisely, this depends on the specific task you look at (e.g., for the cross product (vector product, PA09) only 50% of the students were sure what they had to enter, while for the division task (PB02) almost all were sure). For each of the 14 tasks for which the technology questions were asked, the percentage of students who had no idea how to use technology was less than 10%. Overall, it can be said that the students rate their own technological skills as quite high. This applies equally to students with high and low success rates in technology-free work. If one assumes that these self-assessments correspond approximately to their actual knowledge of technology, one can speak of a contribution of mathematics lessons to digital education. Nevertheless, at this point we cannot really be sure whether the students have a good self-assessment concerning this question.

We surveyed the frequency of technology use. Of course, classroom observations could provide more objective data. In addition, statements about the type and quality of technology use could also be made in this way. Unfortunately, this possibility was not available in our study. However, through the additional questioning of the teachers, we were at least able to confirm the statements of the students.

All the procedures in our test are curriculum-related (Curriculum, 2021). However, we do not know whether all procedures were actually taught in class. In any case, there is hardly any data that indicates that someone solved certain tasks with conceptual knowledge (e.g., by developing a suitable solution method her- or himself during the test). In short, if someone was able to solve a task, then it happened with procedural knowledge in almost all cases. A deviation from the intended step-by-step sequences could only be recognized in approximately 0.8% of all student processing.

It should be emphasized that our data represent a snapshot from which no conclusions can be drawn about the years and decades before. Low success rates for individual tasks in no way mean that corresponding success rates have been higher in the past. Nor can we make any statements about in what way the procedural knowledge of the students will change by the time they start studying mathematics or other STEM courses at a technical college or university.

In the light of our results, it would of course be interesting to see what success rates students would have when using technology for processing the tasks. A corresponding survey is already being planned.

As part of the OFF project, it is planned to continue collecting data on the procedural knowledge of high school students in final classes in the coming years. This enables statements to be made about the development of procedural knowledge of students at Austrian high schools, especially in view of the forthcoming technology-free part of the school leaving exam.

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**Ethical statement:** This study design fully complies with the ethical standards of the involved institutions and the Austrian Federal Departments of Education. According to the rules in Austrian universities and research foundations, formal ethical approval is not necessary for the current type of studies. Written informed consent was obtained from all students and their legal guardians before the study, as well as from the teacher and the headmaster of the school.

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**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

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