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# Prospective Primary School Teachers' Proficiencies in Solving Real-World Problems: Approaches, Strategies and Models

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This study investigates approaches, strategies and models used by prospective primary school teachers in responding to real-world problems. The research was carried out with 82 participants. Data were collected through written-exam and semi-structured interviews; and they were analysed using content and discourse analysis methods. Most of the prospective teachers did not pay attention to the realities of the contexts in which the problems are situated. Most of them displayed non-realistic approaches excluding their real world knowledge and experiences from their solutions. Many participants tended to use rules and procedures in a straightforward way leading to failures at interpreting salient aspects of the problem situations. Majority of the participants lacked the ability to use appropriate strategies that could scaffold their realistic considerations. Some constructed models of the situations; nevertheless, many of them lacked the ability to use these instruments as a conceptual tool to identify key aspects of the problem contexts.

*Keywords*: Models, primary education, problem solving strategies, prospective teachers, real-world problems, realistic and non-realistic approaches.

## INTRODUCTION

Problem solving is considered the most significant cognitive activity in professional and daily life (Jonassen 2000). Due to its unifying role in mathematics curricula educators suggest that problem solving should be used as a general teaching and learning approaches (Cai, 2003; Cockroft, 1982; National Council of Teachers of Mathematics [NCTM), 1989). It is believed that such approaches would assist students develop much deeper and better understanding of mathematics. Problem could be defined as an unfamiliar situation for which an individual does not know how to carry out its solution (Schoenfeld, 1992). It refers to a situation in which desired goal has to be attained, but the direct path towards the goal is blocked (Krulik & Rudnick, 1985). A problem cannot be resolved using routine and familiar procedures (Carlson & Bloom, 2005); thus, a situation is regarded as a problem if it causes cognitive conflicts in the mind of individuals. Problem solving, on the other hand, is a dynamic process through which individuals try to understand a situation, make a plan, develop or select methods, apply all these heuristics to get the solutions, and finally check out the answers that they obtain (Barnet, Sowder & Vos, 1980; Polya, 1973; Suydam, 1980). In this process one may use various strategies (making a list, working backwards, etc.) and could establish models of the problem situations (Polya, 1973; Posamentier & Krulik, 1998).

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Traditionally mathematical problems are classified under two major categories: routine and non-routine problems. Routine problems could be resolved by the application of rules and procedures that the problem solvers already know (Arslan & Altun, 2007; Mahlios, 1988). However, non-routine problems do not have a straightforward solution; they request creative and


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*State of the literature*

- Solution of non-routine problems request utilising metacognitive strategies that include self-regulatory actions, such as decomposing the problem, monitoring and regulating the solution process, and evaluating and justifying the results.
- In their interpretation of real-world problems students seem to follow rules and procedures without reflecting upon what these routines imply in the specific context in which they are used.
- When faced with the real-world problems not only school children but also prospective teachers tend to act without much apparent concern what would be realistically meaningful outside of the classroom.

*Contribution of this paper to the literature*

- This study attempts to scrutinise their capability at using realistic considerations, problem solving strategies and mathematical models.
- Research findings would provide timely feedback to curriculum developers and teacher training institutions concerning the extent to which prospective teachers are ready to implement the new curriculum.
- The findings inform international community about the capability of prospective Turkish teachers in using realistic consideration, models and strategies in responding to real-world problems.

critical thinking, alternative approaches, different methods and strategies, and using appropriate mathematical models (Inoue, 2005). Non-routine problems also requires utilising metacognitive strategies that include self-regulatory actions such as decomposing the problem, monitoring and regulating the solution process, and evaluating and justifying the results (Hartman, 1998; Nancarrow, 2004; Schoenfeld 1992; Verschaffel, De Corte & Vierstraete, 1999). Previous studies indicated that flexibility and adaptivity are two essential cognitive skills that are positively related to students' performances in solving non-routine problems (Elia, Heuvel-Panhuizen & Kolovou, 2009). Flexibility refers to quantity of variations displayed by an individual while carrying out mental operations in responding to a problem (Demetriou, 2004). It entails an ability of employing multiple strategies and shifting between them. Adaptivity refers to capability of using problem solving approaches consisted with the socio-cultural context in which the problem is situated (Verschaffel, Luwel, Torbeyns & Van Dooren, 2009). It enables students to adjust approaches and strategies in ways that meet the conditions posed by the problem contexts.

Mathematics provide tools for describing, analysing and predicting behaviour of the systems in the real world. One major goal of mathematics education is to equip students with the necessary knowledge and skills to cope with the problems that they encounter in such situations. Word problems are intended to develop students' skills in knowing when and how to apply their knowledge of mathematics to various kinds of problems in everyday settings (De Corte, 2000). They are useful instructional tool to provide students with the opportunity to apply their knowledge of mathematics to real world situations. Nevertheless, word problems applied in mathematics education research differ in their particular focus and cognitive demands. They could be classified as simple arithmetic word problems and realistic word problems. Those in the former group can be executed by the application of basic operations. Their textual representations contain no information or objects from everyday life; thus, students do not need to build imageries of such problems. They could manipulate them by searching for numbers in the texts and carrying out the operations invoked by the problem statements (Mayer & Hegarty, 1996; Verschaffel & De Corte, 1997).

Realistic word problems are connected to real life. Their textual representations contain information and non-mathematical objects from everyday settings. It is for this reason solution of realistic word problems request building mental imageries of the objects given in the problem story. Due to their relations with the real world situations realistic word problems are also called real-world problems (from now we shall use the term real-world problem) and considered in the category of non-routine problems (Verschaffel et al., 1999). Real-world problems often contains irrelevant data or lack of information; thus, they might have no solution or more than one solution. Thus, students need to regulate their knowledge of mathematic in ways that satisfy requirements of the problems contexts. Real-world problems are genuine tasks in that they provide opportunities for the transfer of knowledge from mathematics to real life or vice versa. They provide activity based teaching-learning environments in which students could develop meaningful learning by acting upon mathematical notions. It is suggested, thus, that real-world problems should be incorporated into mathematics curricula and students should be engaged in such tasks in and out of school (Carpenter, Lindquist, Mathews & Silver, 1983; Verschaffel, De Corte & Lasure, 1994; Yıldırım, & Ersozlu, 2013).

Several researches have been conducted to investigate students' performances in solving real-world problems (Chacko, 2004; Greer, 1993; Reusser & Stebler, 1997; Verschaffel et al., 1994; Yoshida, Verschaffel & De Corte, 1997). The general outcome is that students mostly exclude their real-world knowledge and experiences from their solutions (Chacko, 2004; Greer, 1993; Verschaffel et al., 1994). They do not pay attention to the relationships between real-world situations evoked by the problem statements and the mathematical operations they carry out. Students tend to follow rules and procedures without reflecting upon what these routines imply in a specific context in which they are used. As a matter of clarity we present three problems of this kind (Verschaffel et al., 1994):

1. Bus problem: 450 solders must be bussed to their training site. Each army bus can hold 36 solders. How many buses are needed?
2. Planks problem: Steve has bought 4 planks each 2.5 meters long. How many planks 1 meter long can he saw from these planks?
3. Runner Problem: John's best time to run 100 meters is 17 second. How long will it take him to run 1 kilometer?

These problems have in common the potential for the students' responses to include some realistic considerations. A realistic answer is taken to mean one which pays some attention to just those sorts of realistic considerations that might characterize problem solving outside of the classroom. A realistic answer for the first problem is 13. A realistic answer to the second one is 8; because in reality one can saw only 2 planks of 1 meter from a plank 2.5 meters long. The third problem does not have a single correct answer; a wide range of answers considerably larger than 170 would be a realistic response. Verschaffel et al. (1994) reported, however, that 49% of the students (the research sample included 75 students at the ages of 10-11 years) displayed realistic considerations when responding to the first problem. This figure declined to 13% for the second problem and further to 3% for the third one.

These findings were replicated by many researchers who reported that not only elementary school students (Chacko, 2004; Greer, 1993; Yoshida et al., 1997) but also prospective teachers (Verschaffel, De Corte, & Borghart, 1997) attempt to solve real-world problems without apparent concern for what would be realistically meaningful outside of the classroom. Verschaffel et al. (1997) reported that only 48% of 332 prospective elementary school teachers displayed realistic considerations in their responses to seven real-world problems that included the tree items above. Almost all of them activated their real-world knowledge and experiences when dealing with the interpretation of the outcome of a division with a remainder as it is the case in the bus problem. The percentages of realistic answers declined when they confronted with the situation involving linearity illusion, and 31% of the prospective teachers claimed that John needs more than 170 seconds to run 1 kilometer distance. The participants struggled more when the problem context included realistic modeling difficulties and an interpretation of additive situations involving sets with disjoint elements. For instance, only 29% of the prospective teachers gave realistic answers to: "Carl has 5 friends and Georges has 6 friends. Carl and Georges decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party?" (Verschaffel et al., 1997, p. 341). Educators argue that sstudents' lack of competence in solving real-world problems is a consequence of traditional way of teaching and learning mathematics in schools (Greer, 1993; Verschaffel et al., 1997).

### Research Objectives

In Turkey, a new primary school mathematics curriculum (grades 1 to 4; ages: 6-10) has been introduced to be used in schools beginning from 2005-2006 academic years (Talim ve Terbiye Kurulu Başkanlığı [TTKB], 2009). In this document, problem solving is conceived as a means of interpreting real life situations. A major goal of mathematics education has been stated as to raise students who are able to use different methods and strategies, and establish mathematical models to solve real-world problems. Teachers are encouraged to create learning environments in which students could develop realistic considerations (TTKB, 2009). To achieve such a vital task teachers need a strong subject-matter understanding and an expertise in the field. Thus, this study aims to investigate prospective teachers' proficiencies in solving real-world problems. It seeks answers to the following questions:

1. How capable prospective primary school teachers are at solving real-world problems?
2. Do they use realistic or non-realistic approaches when dealing with such problems?
3. Do they use problem solving strategies and mathematical models? If they do so, how capable they are at using these instruments?

## RESEARCH METHOD

### Research Design and Data Sources

The research employed a qualitative case study (Yin, 2003), and it was carried out with 82 prospective teachers in an Education Faculty. The participants were at the third and fourth year of their undergraduate studies (see Table 1). Data were collected through the end of schooling year by which the participants had taken all the courses about mathematics and mathematics education. In Turkey, in their first year

**Table 1.** Number of Prospective Teachers at Each Grade Level

| Grade level | 3rd year | 4th year | Total (n) |
|---|---|---|---|
| Number of participants | 31 | 51 | 82 |

**Table 2.** Eight Real-World Problems Used in the Research

| Name | Problems |
|---|---|
| Picnic | A bus can carry 36 students and 450 students are to be transported for a picnic. How many busses are needed? (Carpenter et al. [as cited in Verschaffel et al., 1997]). |
| Runner | Kemal's best time to run 100 meters is 17 seconds. How long will it take to travel 1 kilometer? (Greer [as cited in Verschaffel et al., 1997]). |
| Planks | Onur has bought 4 planks of 2.5 meter each. How many planks of 1 meter can he get out of these planks? (Kaelen [as cited in Verschaffel et al., 1997]). |
| Jar | A conic-shaped jar is being filled from a tap at a constant rate. If the depth of the water is 3.5 cms after 10 seconds, how deep will it be after 30 seconds? (Greer [as cited in Verschaffel et al., 1997]). |
| Course | Mustafa goes 3 days to art and 2 days to guitar courses in week. How many days he does not have a course (Verschaffel et al., 1997). |
| Party | Gökçe has 5 friends and Ayça has 6 friends. Gökçe and Ayça decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party? (Nelissen [as cited in Verschaffel et al., 1997]). |
| Queue | Nihat and Aykut stand in a queue. Nihat is the 8th person from the beginning and Aykut is the 12th person from the end. There are also three people between them. How many people are there in the queue? (Authors, 2013). |
| School | Gökhan and Orhan go to the same school. Gökhan lives at a distance of I7 kilometers from the school and Orhan at 8 kilometers. How far do Gökhan and Orhan live from each other? (Treffers & De Moor [as cited in Verschaffel et al., 1997]). |

prospective primary school teachers take a subject-matter course (General Mathematics) which has two hours credits in a week and encompasses the first and the second term. In this course, they learn basic concepts including, fractions and decimals, data handling, statistics and functions. During the third year they get another course (Teaching Mathematics) with three hours credits and encompassing again both terms. This module is concerned with the pedagogical aspects of teaching and learning mathematics. It aims to help prospective teachers gain necessary knowledge and skills that they will need to be effective in teaching mathematics.

Data were obtained from written exam and semi-structured interviews. First, the participants were given a questionnaire that included eight real-world problems (see Table 2). These problems were taken or adapted from the literature to ensure validity and reliability issues. In addition, they were checked and revised through a pilot study. The written exam was completed in 45 to 60 minutes during which the participants were encouraged to provide reasons for their answers. After the exam in-depth clarification interviews were carried out with four participants and these were selected considering the sort of approaches, strategies and models that they used in the exam. The interviewees were invited to solve the problems one by one; then, the line of inquiry developed in accord with their responses. The aspects of clinical interview (Gingsburg, 1981) were considered and the interviewees were prompted through 'why' and 'how' questions and requested to explain underlying reasons of their solutions. All that they wrote during the interview was collected and properly identified. The interviews were audio-taped and annotated field notes were taken for later consideration.

### Data Analysis

Literature about real-world problems (Chacko, 2004; Lesh & Harel, 2003; Reusser & Stebler, 1997; Verschaffel et al., 1994; Verschaffel et al., 1999; Yoshida et al., 1997) provided a theoretical basis for the data analysis. Content and discourse analysis methods (Miles & Huberman, 1994; Philips & Hardy, 2002) were used to discern meaning embedded in the written and verbal expressions. Data analysis started with an examination of exam papers and writing up a summary of participants' responses to each question. Then, in-depth examination continued and codes were established to identify approaches, strategies and models that they used. In the last phase of analysis a pattern coding was applied and the previous codes were collected under two major categories: Realistic Answer (R-A) and Non-Realistic Answer (N-R-A). Table 3 gives a summary of R-A and N-R-A for each problem used in this research.

Realistic answers included rational contemplation of the real-world situations that the problems are related to. Non-realistic answers included no understanding of the real-world situations elicited by the problem statements whatsoever. Such responses resulted from using rules, procedures and arithmetical operations in an uncritically way. For instance, the *course problem* has three solutions. If a participant obtained only one of these as invoked by the problem statement (such that, 3+2=5, so Mustafa does go to course 7-5=2 days in a week) his/her answer was classified as N-R-A. If he/she illustrated one of the remaining alternative solutions his/her answer was collected under the category of R-A. In addition, a third category was added, namely *No Answer* (N-A). Categories as to the use of strategies, models and other kind of manipulative instruments (e.g., arithmetical operation, cross-product algorithms) were also established. These were combined with the general problem solving approaches and presented as the sub-categories of R-A, N-R-A.

The interview data were also subjected to qualitative analysis. Interviews were fully transcribed and considered line by line. Then, a summary of participants' responses to each problem was written up. These documents were read thoroughly and codes were established to distinguish sort of approaches, strategies and models used. Repeated on different copies of the text this eventually led to the creation of two major categories: *Realistic Answer* (R-A) and *Non-Realistic Answer* (N-R-A). The results are presented in the coming section.

## RESULTS

The research findings indicated that most of the prospective teachers revealed non-realistic approaches. They mostly employed rules, procedures and factual knowledge without adjusting them to accommodate realities of the problem context. Many lacked the ability to use appropriate strategies and mathematical models. It is inferred from their written and verbal responses that the prospective teachers' familiarity with the real life context in which a problem is situated, the number of operational steps and the mathematical notions requested for the solution of the problems were crucial factors influencing their realistic considerations. The overwhelming majority (93%) activated their knowledge of real world in responding to picnic problem that requested dealing with the interpretation of an outcome of a division with a remainder. Yet, their performances declined when responding to the problems that included linearity illusions and challenges associated with the use of proportional reasoning. The runner problem yielded 22% realistic answer. The remaining 78% applied the idea of direct proportion in a straightforward way and these all used cross-multiplication algorithms. They were unable to identify that a runner slows down after a

**Table 3.** Realistic Answers (R-A) and Non-Realistic Answers (N-R-A) for Each Problem

| Name | R-A/N-R-A Responses for each problem |
|---|---|
| Picnic | R-A: 450 divided by 36 is 12.5. So 13 buses are needed.<br>N-R-A: 450 divided by 36 is 12.5. So 12.5 buses are needed. |
| Runner | R-A: It is impossible to answer precisely what Kemal's best time on 1 kilometer will be.<br>N-R-A: 17:10=170. Kemal's best time to run 1 kilometer is 170 seconds. |
| Planks | R-A: Onur can saw 2 planks of 1 meter from 1 plank of 2.5 meters. 2x4=8. So, he can saw 8 planks.<br>N-R-A: 4x2.5 =10 meters. 10:1=10; Onur can saw 10 planks of 1 meter. |
| Jar | R-A: It is impossible to give a precise answer.<br>N-R-A: 3x3.5=10.5. After 30 seconds, the level of the water will be 10.5 cm |
| Course | R-A1: Mustafa can go 1 day only to art course, two days to both courses; in this case he gets 7-3=4 days off.<br>R-A2: He can go 2 days only to art course, one day to both courses, and one day to only guitar course. In this case he gets 7-4=3 days off in a week.<br>N-R-A: 3+2=5 and 7-5=2; so Mustafa has 2 days off in a week. |
| Party | R-A: You cannot know how many friends there will be at the party.<br>N-R-A: 6+5= 11. There will be 11 friends at the party. |
| Queue | R-A: Nihat is $8^{th}$ person from the beginning; Aykut is $12^{th}$ person from the end and $4^{th}$ person from the beginning; so, there are 15 people in the queue.<br>N-R-A: 8+3+11=23 |
| School | R-A: You cannot know how far Gökhan and Orhan live from each other.<br>N-R-A1: 17-8=9. Gökhan and Orhan live at 9 kilometers from each other.<br>N-R-A2: 17+8=25. Gökhan and Orhan live at 25 kilometers from each other. |

**Table 4.** Teachers' Responses to Planks Problem

| Approaches | Model/strategies/manipulative tools | Frequency | Percent |
|---|---|---|---|
| R-A | Model | 9 | 11,0 |
| | Arithmetical operations | 5 | 6,1 |
| N-R-A | Model | 30 | 36,6 |
| | Arithmetical operations | 32 | 39,0 |
| N-A | | 6 | 7,3 |
| Total | | 82 | 100,0 |

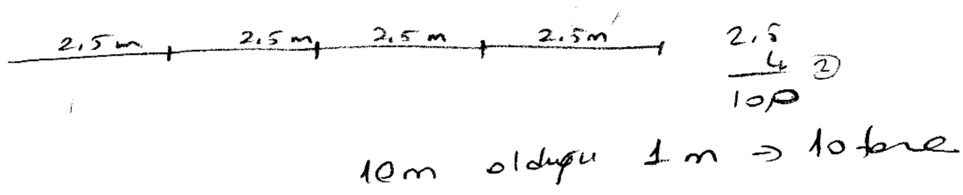


**Figure 1.** Non-realistic model constructed for the solution of planks problem [T78]

**Table 5.** Students' Responses to Jar Problem

| Approaches | Model/strategies/manipulative tools | Frequency | Percent |
|---|---|---|---|
| R-A | Model | 8 | 9,8 |
| | Verbal explanation | 7 | 8,5 |
| N-R-A | Model + Cross-multiplication algorithm | 9 | 11,0 |
| | Cross-multiplication algorithm | 41 | 50,0 |
| N-A | | 17 | 20,7 |
| Total | | 82 | 100,0 |

while; thus he/she needs more than 170 seconds to finish 1 km distance. As to the planks problem only 17% of the participants displayed realistic considerations while the remaining 76% acted non-realistically following a sequence of operations elicited by the problem statement (see Table 4). It is noticeable that although 47,6% of the participants incorporated modelling activities into their solution more than half of them (36,6%) constructed non-realistic models (see Figure 1).

Only 18% of the participants gave realistic answers to the Jar problem (see Table 5), and these all provided comments such that since a conic-shaped jar gets narrowed the depth of water will be more than 10,5 cm in 30 seconds. Using cross-multiplication algorithms 50% claimed that the depth of water reaches to 10,5 cm in 30 seconds. An interesting result is again that although 21% of the whole participants constructed realistic models by sketching a conic-shaped flask (see Figure 2) half of them failed to use these instruments as a conceptual tool to understand the salient aspects of the problem context. Instead, they manipulated these models as part of the routine that included application of the idea of direct proportion in a straightforward way.

The remaining four items also required realistic interpretations. Various approaches, several strategies and different kind of mathematical models could be used; and each of them has more than one solution. The course and party problems were additive situations in that they required considering alternative combinations of the variables given in the problem statements. 29% of the participants gave N-R-A in responding to course problem (see Table 6). Just like school children they followed strictly a sequence of operations elicited by the problem statement and argued that 'since Mustafa goes 3 days to art and 2 days to guitar courses he takes two days off in a week with the accompanying operations such that 3+2=5, 7-5=2'. Only those who utilised *making a list strategy* illustrated precisely three of the alternative solutions. The remaining 59% also recognised that the problem has more than one solution but was able to illustrate one of the two alternatives apart from the one invoked by the problem statement (3+2=5, one gets 7-5=2 days off in a week).

As to the party problem 30,5% of the participants acted realistically. Having illustrated the situation through a couple of cases – such that if Gökçe and Ayça have a common friend there are 10 guests in the party – they concluded the idea that 'it is impossible to give a precise answer because we do not know how many common friends Gökçe and Ayça have'. Those who displayed non-realistic interpretation made arithmetical calculations as invoked by the problem statement and argued that there are 11 guests in the party (see Table 7).

Construction of mathematical models could

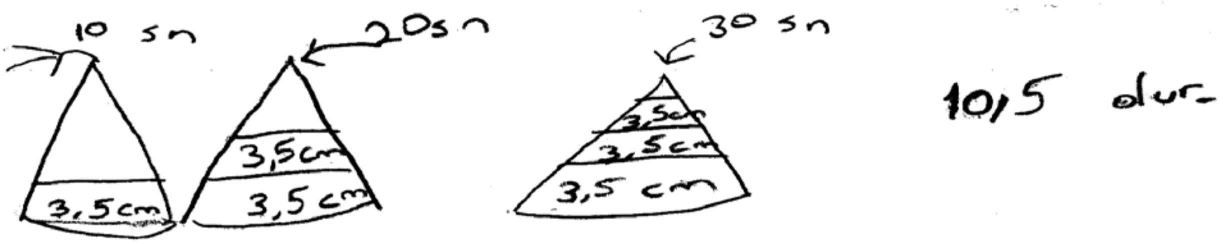


**Figure 2.** A realistic model misused in the solution of jar problem [T7]

**Table 6.** Students' Responses to Course Problem

| Approaches | Model/strategies/manipulative tools | Frequency | Percent |
|---|---|---|---|
| R-A | Making a list | 9 | 11,0 |
| | Arithmetical operation + Verbal explanation | 48 | 58,5 |
| N-R-A | Arithmetical operation + Verbal explanation | 24 | 29,3 |
| N-A | | 1 | 1,2 |
| Total | | 82 | 100,0 |

**Table 7.** Students' Responses to Party Problem

| Approaches | Model/strategies/manipulative tools | Frequency | Percent |
|---|---|---|---|
| R-A | Verbal explanation | 25 | 30,5 |
| N-R-A | Arithmetical operation + Verbal explanation | 57 | 69,5 |
| Total | | 82 | 100,0 |

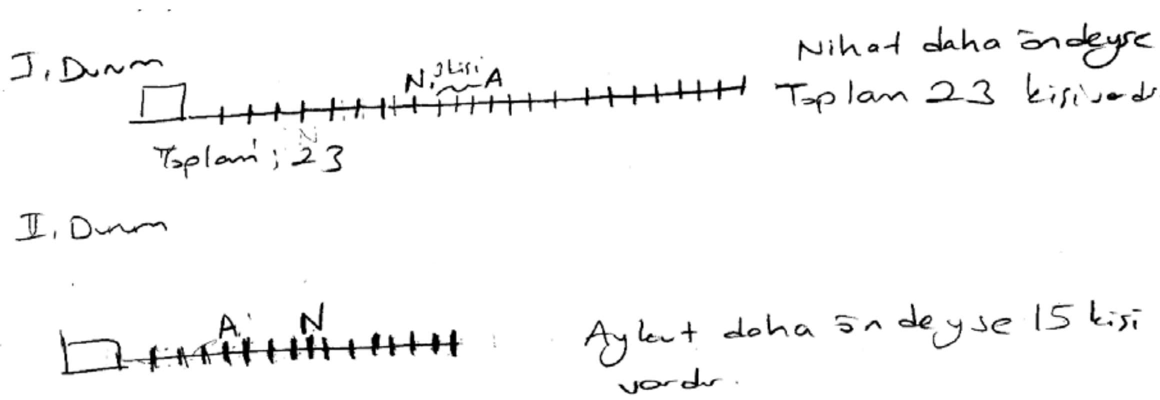


**Figure 3.** Realistic model constructed for the solution of queue problem [T56]

**Table 8.** Students' Responses to Queue Problem

| Approaches | Model/strategies/manipulative tools | Frequency | Percent |
|---|---|---|---|
| R-A | Model | 47 | 57,3 |
| | Arithmetical operation + Verbal explanation | 4 | 4,9 |
| N-R-A | Model + arithmetical operations | 25 | 30,5 |
| | Arithmetical operation + Verbal explanation | 3 | 3,7 |
| N-A | | 3 | 3,7 |
| Total | | 82 | 100,0 |

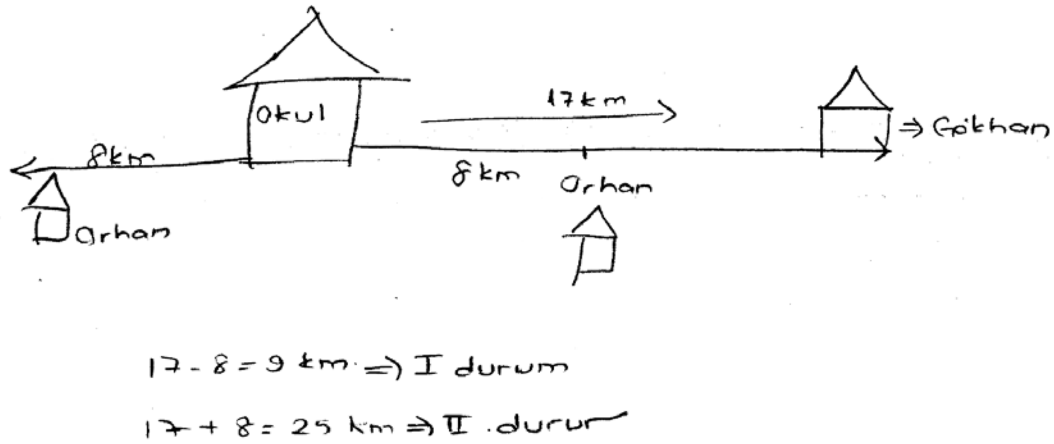


**Figure 4.** Non-realistic model constructed for the solution of school problem [T57]

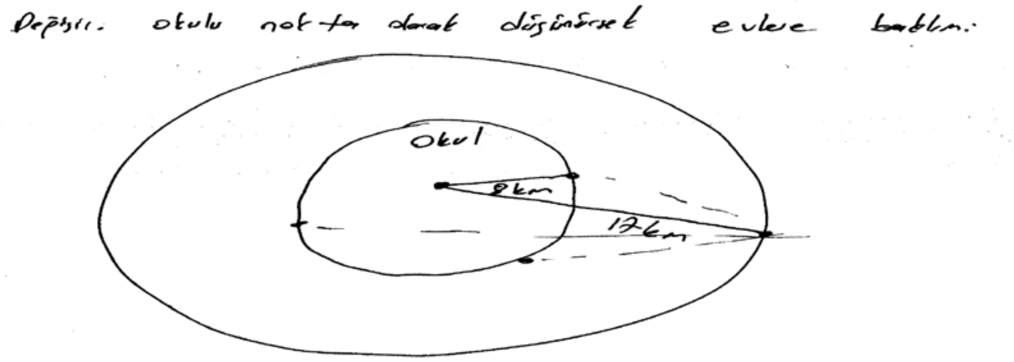


**Figure 5.** A realistic model for the solution of school problem [t38]

enhance realistic considerations to overcome challenges caused by the queue and school problems. This is verified, to some extent, by the research findings. In responding to queue problem a great majority of the participants constructed models that included a series of drawings on a straight line each of which representing a person in the queue. Most of them utilised these instruments as a conceptual tool and illustrated two of the alternative solutions as it is seen in Figure 3. Yet still a considerable number of them (30,5%) manipulated the models as part of the routine – arithmetical operations such that 8+3+12=23 – suggested by the problem story.

The number of participants who incorporated modelling activities into their solution of school problem was also considerably high (see Table 9). However, more than half of them produced inappropriate models that included linearity constraint in that the problem solvers placed Orhan's and Gökhan's houses on a straight line either on the same side or on the opposite side of the school (see, for instance, Figure 4). These teachers produced two solutions (17+8=25 or 17-8=9) that were also obtained by 20,7% of the participants through arithmetical calculations. Less than one third of them produced realistic models (see an example of realistic model in Figure 5) and claimed that that nobody knows how far Gökhan and Orhan live from each other, because their

**Table 9.** Students' Responses to School Problem

| Approaches | Model/strategies/manipulative tools | Frequency | Percent |
|---|---|---|---|
| R-A | Model + Verbal explanation | 22 | 26,8 |
| | Verbal explanation | 9 | 11,0 |
| N-R-A | Model + Verbal explanation | 34 | 41,5 |
| | Arithmetical operation + Verbal explanation | 17 | 20,7 |
| N-A | | 0 | 0,0 |
| Total | | 82 | 100,0 |

**Table 10.** A Summary of the Interview Results

| Name | Çağrı | Emre | Cengiz | Büşra |
|---|---|---|---|---|
| Picnic | R-A | R-A | R-A | R-A |
| Runner | R-A | R-A | R-A | N-R-A |
| Planks | N-R-A | R-A | R-A | N-R-A |
| Jar | R-A | R-A | R-A | N-R-A |
| Course | R-A | R-A | R-A | R-A |
| Party | R-A | R-A | R-A | N-R-A |
| Queue | N-RA | R-A | R-A | R-A |
| School | N-RA | R-A | R-A | N-R-A |

houses could be located in many different ways around the school.

The follow-up interviews drew better results (a summary of interview results is presented in Table 10). Two interviewees, Emre and Cengiz, revealed realistic considerations in responding to all the problems. The remaining interviewees displayed realistic considerations when responding to situations that requested an understanding of the division with a remainder (as in the picnic problem) and a combination of disjoint elements as in the *course problem*. Yet, they disregarded realities of the problem contexts when the problems included illusion of linear proportionality (as in the runner and planks items) and required an understanding of additive situations involving many disjoint elements as in the *party problem*.

The interview results suggest that prospective teachers displayed non-realistic considerations not because of their cognitive deficiencies but due to pedagogical and cultural norms that they possessed – beliefs about problem solving and perceptions about the relations between mathematics and the real world. In the beginning some revealed non-realistic considerations; yet, on our probing they revised their thinking in ways that met the requirements of the problem contexts (*approaches presented in the above table were revealed until our probing*). For instance, Büşra's first reaction to the runner problem included: "…here 1000 meters is ten times more than 100 meters; we know that if the distance increases ten times the running time should increases ten times… I mean, the increase in both things [variables] should be proportional". Then the following exchange occurred between Büşra and the researcher:

Int: Put yourself in Kemal's place; and tell me can you run 1000 meters in 170 seconds?

Büşra: I cannot run; I am not good at running; …it is too long for me.

Int: Do you mean that you cannot, but Kemal could finish it in 170 seconds?

Büşra: ...[Silence]...yes, he can, he might complete it in 170 seconds. …

Int: Do you think Kemal could maintain his speed, his performances does not decline?

Büşra: …[Slience]… He cannot maintain his performances… I think he gets tired and slow down… Thus, it would take more time… I did not think of this point… I am sorry…

Int: What would you say now? How long does it take for Kemal to finish this running?

Büşra: … I am saying it takes more time; I mean more than 170 seconds. If you are asking a certain time I could not give it… It still depends on his effort. …. Actually, we do not give attention to the relations between real life and the problems…we are not used to think this way… I ignored that Kemal is a human being; I treated him as if he was a car…

The interviewees also incorporated modelling activities into their solution. Many of the models that they constructed at the beginning lacked the content validity. However, on our probing the interviewees revised their initial models or produced a new one that potentially delineated key aspects of the problem contexts. Büşra was one of these students. In responding to school problem she sketched a linear model (on the left side, Figure 6) and claimed that Gökhan and Orhan live 25 km or 9 km far from each other.

Afterwards the following exchange occurred between Büşra and the researcher:

Int: Is this the only solution? Are there any other alternatives?

Büşra: ... Let me think... Yes, there might be; let me put Gökhan's house here and Orhan's house here [sketching a new model in circular shape]... [Silence]...

Int: What are you thinking? Could you tell me your opinion?

Büşra: I could locate them [the houses] like this; but, in this case I cannot calculate the distance between the two. This is not a proper triangle; I mean I know just two sides, nothing else; if it had been a right-angled triangle [sketching a right angled triangle] I would calculate how far Gökhan and Orhan live from each other...

Int: Do you have to find…a numerical solution?

Büşra: Do not we have to? … If we do not, I could locate them in many different ways by changing their positions on these circles... If we think this way I could say that there is a number of solution but we cannot calculate the distance...

It is seen in this exchange that Büşra insists to make calculation using the data given in the problem and her speech suggests that this is driven by her belief system that every problem has a numerical solution. It emerged during the interviews that pedagogical and cultural norms that the prospective teachers had developed during their former training were affecting their problem solving approaches. The interviewees put forward several comments suggesting that their non-realistic approaches were grounded in their beliefs about problem solving. We provide a couple of them to allow the discussion:

Episode 1: (Çağrı; Planks problem): I did not think of it… Each piece of plank is 2,5 meters; as we put them side by side it makes 10 meters; so we could sow 10 planks of 1 meter each… This was my thinking; yet on your probing I got the point; it does not work out in the real-world… I think I am not the only one...[in the questionnaire] many of my friends would have acted like me.

Episode 2: (Büşra; Jar problem): I simply employed the idea of direct proportion… I did not even think of that the jar is in conic shape; of course the water rises more quickly as it gets narrowed… This is our usual reaction to this kind of problems… We always used direct proportion or inverse proportion while solving pool problems [in the classes]… This is the similar one…I thought that the idea of direct proportion works out in this task.

Episode 3: (Çağrı, Queue problem): … I thought that in the queue Nihat should always come before Aykut. I did not even think of that the problem might have another solution. … This is a very tricky task; as you give it to other students I guess all of them solve it like I did… Teachers do not challenge their students through such tricky problems; I know it from school experiences… Unfortunately we are coming from the same background; we got the same education in our schools [in the past]. ... We were...always engaged with the problems whose solution was

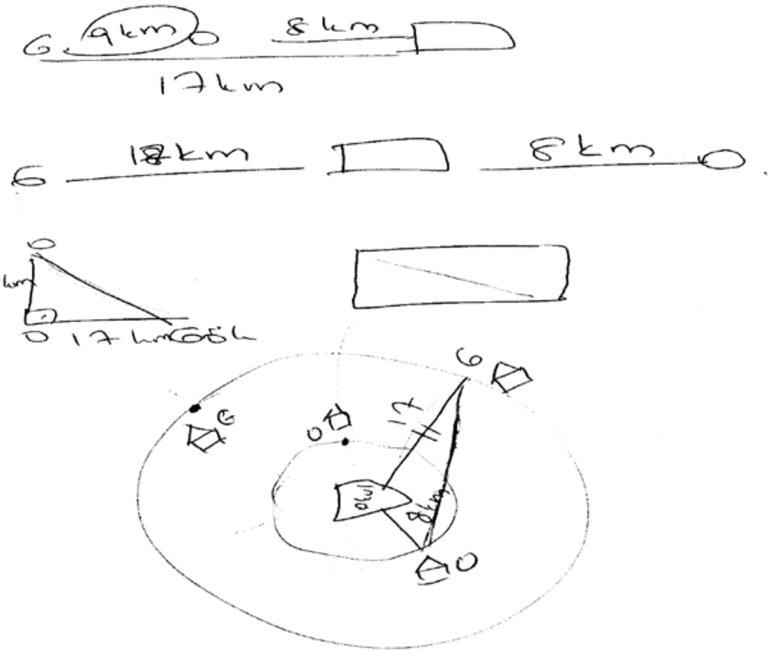


**Figure 6**. A model constructed by büşra for the solution of school problem

straightforward. I do not remember any of our teachers gave us this kind of problem...

These quotations suggest that that the participants possess a result-oriented, not a process oriented, problem solving behaviours. They reveal, implicitly though, a belief that every problem has only one solution and this is obtained by calculating the data in the problem statements. They do not check out whether the results are plausible in the real world situations. Also the participants explicitly point out their background trainings to excuse why they ignore realities of the real world situations that the problems are related to.

## DISCUSSION AND CONCLUSION

The purpose of this study was to examine sort of approaches, strategies and models that the prospective primary school teachers used when solving real-world problems. The research findings complement the results of previous studies (Chacko, 2004; Reusser & Stebler, 1997; Verschaffel et al., 1994; Verschaffel & De Corte, 1997; Yoshida et al., 1997). Overall, the results indicated that the prospective teachers do not consider realities of the daily life that the problems are connected to. They exclude their real world knowledge and experiences from their solutions. They do not pay attention to the factual relationships between the defining aspects of the problem context and the operations they carry out.

Our results indicate that teachers' acquaintance with the contexts in which the problems are situated, the degree of complexity of the tasks, the number of operational steps and the mathematical notions requested for the solution of the problems are the crucial factors that potentially influence their realistic considerations. It can be concluded from our results, and was also reported by previous researchers (Verschaffel et al., 1997), that it is relatively easier for students to activate their real world knowledge and experience in dealing with a real-world problem that include division with a remainder. Nevertheless, they mostly disregard the realities of the problem contexts when the problems include linearity illusions, modelling difficulties, and the complications associated with combination of disjoint elements. When confronted with such situations they tend to use rules, procedures, and factual knowledge without paying attention to their underlying meanings. In such situations for most of the participants the focus reflection was a sequence of operations invoked by the problem statement not the factual relationships between these operations and the defining aspects of the problem contexts. They showed no attempts to adjust their knowledge of mathematics to meet the requirements posed by the real-world situations.

The findings provide us with some insight into models and modelling activities. Models provide an effective tool for mathematisation that includes translating a reality into mathematical terms (Freudenthal, 1991; Polya, 1962). This can be carried out in many different ways including, for instance, writing up an algebraic equation or drawing some pictures that potentially embed key aspects of the real-world that the problems are connected to. Teachers need to be capable of using most appropriate models to bridge the gaps between situated knowledge (knowledge about real world) and formal mathematics (Gravemeijer, 1994). They need this to support the development of situational reasoning and realistic consideration in their students (Blum, 1993; Blum & Ferri, 2009; Gravemeijer, 1994; Zbiek, 1998). In our stud y a number of prospective teachers incorporated modelling activities into their solutions. However, more than half of the models were non-realistic in that they lacked the content validity to represent the situations in which the problems were situated. For instance, two third of the models produced for the *planks problem* included linearity constraint in that the participants obtained a plank of 10 meters by putting four pieces of 2,5 meters each side by side (see Table 4 & Figure 1). Three quarter of the participants produced models for the *school problem*; yet again most of them included linearity constraint and two houses were located on a straight line either on the same side or on the opposite side of the school (see Table 9, Figure 4 & 6). These models were devised and used with the intention of carrying out operations invoked by the problem statement. They were not utilised as a conceptual tool to understand fully the problem situations.

The qualitative evidences indicate that prospective teachers' conception of the problems influence quality of the models that they produce. In responding to school problem Büşra started with an initial model that contained linearity constraint (see Figure 6). On our probing she revised her conception and produced an appropriate model representing realities of the problem situation. On the other hand, some participants established appropriate models, yet they did not reflect upon these instruments to enhance their realistic considerations. Models were manipulated as if they were a sort of arithmetic or algebraic rule, and this can be seen clearly in Figure 2. It can be concluded from these evidences that a realistic consideration is required to construct appropriate models. Also, construction of appropriate models would be essential but not sufficient to create realistic answers to the real-world problems. One needs to be capable of reflecting upon these instruments in cooperation with the information in the problem statements. One needs mental flexibility to shift back and forth between his/her *cognitive models* (an understandings of the problem situation) and the *conceptual ones* (Greca & Moreira, 2002) that he/she sketches.

Flexibility and adaptivity at using problem solving strategies is considered to be crucial skills (Elia et al., 2009; Verschaffel et al., 2009). Strategies could enable individuals to display more systematic and analytic approaches towards the solution of real-world problems. They could facilitate one's reflection on his/her solution process and, thus, enable him/her to activate metacognitive skills including self-monitoring and self-regulation. Nonetheless, the majority of the participants did not use problem solving strategies at all. The strategy of '*making a list or a table*' could promote realistic consideration for the solution of course and party problems. If the participants had used these strategies they could establish various combinations of the data in each of the tasks and recognise that each problem has more than one solution. However, in responding to the party problem none of the participants used the strategy of making a list or a table. As to the course problem nearly two third of the participants revealed realistic consideration (see Table 6); yet only one sixth of them, who utilised making a list strategy, illustrated three of the alternative solutions. The rest of them obtained only one of the alternative solutions apart from the one that could be obtained following a sequence of arithmetical operations elicited by the problem statement. Based on these evidences we argue that one needs to be capable of using most appropriate strategies to understand the salient aspects of the problem situation so that he/she could obtain more plausible and all the alternative solutions.

In conclusion, most of our participants displayed result-oriented, not process-oriented, problem solving approaches. Those who displayed non-realistic approaches did not check out plausibility of the answers that they obtained; that is, they skipped one crucial step of Polya's (1973) problem solving stages –*looking back*. Their written responses included almost no indication that they made attempts to incorporate critical and creative thinking into their solutions. It was not the focus of this research, but emerged from interview data that the participants excluded their real-world knowledge and experiences from their solutions mainly because of pedagogical and cultural norms that they possessed. The interviewees point out the ways mathematics is taught and learned in Turkey as the basic factors hindering the development of critical thinking and realistic considerations (see Episodes 1, 2, & 3). For instance, Büşra speculated: "I did not even consider that the jar is in conic shape... ...this is our usual reaction to this kind of problem… We always used either direct proportion or inverse proportion while solving pool problems…". Çağrı was more explicit in his accusation of the traditional way of teaching and learning mathematics in Turkey: "Teachers do not challenge their students through such tricky problems; I know it from school experiences... We are coming from the same background; we got the same education in our schools [in the past]. ... I do not remember any of our teachers gave us this kind of problem…". It is inferred that as a result of being immersed in a traditional teaching-learning environments the participants of this study appears to have developed cultural and pedagogical norms (appreciating rules and procedures and believing in that such routines could lead to the correct answers etc.) and these were influencing their problem solving behaviours. The idea of didactical contract (Brousseau, 1984) should be invoked here, meaning the rules and expectations that reciprocally evolve between teachers and their students. Assessment system contributes significantly to the development of didactical contract because it signals what is valued and expected (Caldwell [as cited in Greer, 1993]). In Turkey, students pass through several exams during their primary and high school education; and they have to pass a university entrance exam to get a position in teacher training departments in Education Faculties. These are all centralised exams in which students are assessed by means of multiple-choice tests. Students' fluency in using rules, procedures and factual knowledge is highly appreciated and considered to be essential skills to succeed in these exams. So, the impacts of didactical contract on the prospective teachers' problem solving behaviours might be greater than that noted in this study and could have many other sources including the centralised exam system and the ways mathematics is taught and learned in *teacher training programs* in Turkey. Thus, a follow-up study is suggested to explore all the possible factors contributing the development of didactical contract and the impacts of these on the prospective teacher' problem solving approaches.

## REFERENCES

Arslan, Ç., & Altun, M. (2007). Learning to solve non-routine problems. *Elementary Education Online*, *6*(1), 50-61.

Barnet, J. C., Sowder, L., & Vos, K. E. (1980). Textbook problems: Supplementing and understanding them. In S. Krulik, & R. E. Reys (Eds.), *Problem solving in school mathematics: 1980 yearbook* (pp. 334-371). Reston, VA: National Council of Teachers of Mathematics.

Blum, W. (1993). Mathematical modelling in mathematics education and instruction. In T. Breiteig, I. Huntley, & G. Kaiser-Messmer (Eds.), *Teaching and learning mathematics in context* (pp. 3-14). Chichester, UK: Ellis Horwood.

Blum, W., & Ferri, B. (2009). Mathematical modelling: Can it be taught and learned? *Journal of Mathematical Modelling and Application*, *1*(1), 45-58.

Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In H. G. Steiner (Ed.), *Theory of mathematics education* (pp. 110-119). Germany, Bielefeld: IDM.

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: An exploratory study. *International Journal of Mathematical Education in Science and Technology*, *34*(5), 719-737.

Carlson, M., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, *58*, 45-75.

Carpenter, T. P., Lindquist, M. M., Mathews, W., & Silver, E. A. (1983). Results of the third NAEP mathematics assessment: Secondary school. *Mathematics Teacher*, *76*, 652-659.

Chacko, I. (2004). Solution of real-world and standard problems by primary and secondary school students: A Zimbabwean example. *African Journal of Research in SMT Education*, *8*(2), 91-103

Cockroft, W. H. (1982). *Mathematics count*. London, UK: Her Majesty's Stationary Office.

De Corte, E. (2000). New perspectives for research and practice in mathematics education. *Pythagoras*, *52*, 33-46.

Demetriou, A. (2004). Mind intelligence and development: A cognitive, differential, and developmental theory of intelligence. In A. Demetriou, & A. Raftopoulos (Eds.), *Developmental change: Theories, models and measurement* (pp. 21-73). Cambridge, UK: Cambridge University Press.

Elia, I., Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM Mathematics Education*, *41*, 605-618

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Gingsburg, H. (1981). The clinical interview in psychological research on mathematical thinking: Aims, rationales, techniques. *For the Learning of Mathematics*, *1*(3), 57-64.

Gravemeijer, K. P. E. (1994). *Developing realistic mathematics education*. Utrecht, The Netherlands: CD-B Press/ Freudenthal Institute.

Greca, I. M., & Moreira, M. A. (2002). Mental, physical, and mathematical models in the teaching and learning of physics. *Science Education*, *1*, 106-121.

Greer, B. (1993). The mathematical modelling perspective on world problems. *Journal of Mathematical Behaviour*, *12*, 239-250.

Hartman, H. J. (1998). Metacognition in teaching and learning: An introduction. *Instructional Science*, *26*, 1-3.

Inoue, N. (2005). The realistic reasons behind unrealistic solutions: The role of interpretive activity in word problem solving. *Learning and Instruction*, *15*, 69-83.

Jonassen, J. (2000). Toward a design theory of problem solving. *Educational Technology Research and Development*, *48*(4), 63–85.

Krulick, S., & Rudnick, J. A. (1985). Developing problem solving skills. *Mathematics Teacher*, *79*(9), 685-692.

Lesh, R., & Harel, G. (2003). Problem solving, modelling and conceptual development. *Mathematical Thinking and Learning*, *5*(2), 157-189.

Mahlios, J. (1988). Word problems: Do I add or subtract. *Arithmetic Teacher*, *36*(3), 48-52.

Mayer, R. E., & Hegarty, M. (1996). The process of understanding mathematical problems. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 29-53). Mahwah, NJ: Lawrence Erlbaum.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. London, UK: Sage Publications.

*Nancarrow, M. (2004). Exploration of metacognition and non-routine problem based mathematics instruction on undergraduate student problem solving success (*Unpublished Doctoral Thesis*).* The Florida State University, Florida.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Phillips, N., & Hardy, C. (2002). *Discourse analysis: Investigating processes of social construction*. London, UK: Sage Publications.

Polya, G. (1962). *Mathematical discovery*. New York, NY: Wiley.

Polya, G. (1973). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton University Press.

Posamentier, A. S., & Krulik, S. (1998). *Problem solving strategies for efficient and elegant solutions – A resource for the mathematics teacher* (2nd ed.). Thousand Oaks, California: Corwin Press.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution: The social rationality of mathematical modelling in schools. *Learning and Instruction*, *7*(4), 309-327.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grouvs (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334-370). New York, NY: MacMillan.

Suydam, M. N. (1980). Untangling clues from research on problem solving. In S. Krulik, & R. E. Reys (Eds.), *Problem solving in school mathematics: 1980 yearbook* (pp. 34-50). Reston, VA: National Council of Teachers of Mathematics.

Talim ve Terbiye Kurulu Başkanlığı (2009). *İlköğretim matematik dersi 1–5. sınıflar öğretim programı ve kılavuzu [Curriculum guidebook for elementary school mathematics course: Grades 1 to 5]*. Ankara: Ministry of Education.

Verchaffel, L., De Corte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the real-world knowledge in mathematical modelling of school word problems. *Learning and Instruction*, *7*, 339-359.

Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modelling in the elementary school: A teaching experiment with fifth graders. *Journal for Research in Mathematics Education*, *28*, 577-601.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modelling of school arithmetic word problems. *Learning and Instruction*, *4*, 273-294.

Verschaffel, L., De Corte, E., & Vierstraete, H. (1999). Upper elementary school pupils' difficulties in modelling and solving nonstandard additive word problems involving ordinal numbers. *Journal for Research in Mathematics Education*, *30*(3), 265-285.

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education*, *3*, 335-359.

Yıldırım, S., & Ersozlu Z. N. (2013). The relationship between students' metacognitive awareness and their solutions to similar types of mathematical problems. *Eurasia Journal of Mathematics, Science & Technology Education*,*9*(4), 411-415.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). London, UK: Sage Publications.

Yoshida, H., Verschaffel, L., & De Corte, E. (1997). Realistic considerations in solving problematic word problems: Do Japanese and Belgian children have the same difficulties? *Learning and Instruction*, *7*, 329-338.

Zbiek, R. M. (1998). Prospective teaches' use of computing tools to develop and validate functions as mathematical models. *Journal for Research in Mathematics Education*, *29*(2), 184-201.

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