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Puzzles as a Didactic Tool for Development of Mathematical Abilities of Junior Schoolchildren in Basic and Additional Mathematical Education

Pavel M. Gorev [1*], Nadezhda V. Telegina [2], Lyudmila Zh. Karavanova [3], Stella S. Feshina [4]

[1] Vyatka State University, Kirov, RUSSIA
[2] Kazan (Volga region) Federal University, Kazan, RUSSIA
[3] Peoples' Friendship University of Russia (RUDN University), Moscow, RUSSIA
[4] Financial University under the Government of the Russian Federation, Moscow, RUSSIA

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ABSTRACT

Pedagogical science has always faced the issue of finding effective means for achieving educational results. This problem is especially urgent today, when in the rapidly changing world the tools, which yesterday could be used to support the interest of schoolchildren in study of mathematics and could provide an opportunity for the development of their mathematical abilities, quickly become obsolete. Today it is very important to search for new means that foster the development of students with the help of mathematics and mechanisms for including mathematics in the educational process. Thus, the aim of the article is to analyze puzzles as a didactic tool and study the possibilities of using puzzles in the process of teaching junior schoolchildren mathematics, both in the classroom and extra-curricular activities. The leading method here is the modeling of the methodical training system in general and additional mathematical education of schoolchildren, with the inclusion of a new didactic tool that fosters the students' interest to the subject, develops individual mathematical abilities: logical thinking, abstraction, combining, operating spatial images, critical thinking, mathematical memory, etc. As a result of the research, the authors have determined the place, features and methodological aspects of the inclusion of puzzles in the process of teaching mathematics in general and additional school education. They can be used in the system of classical and creative math lessons and in extra-curricular activities of students: a mathematical club, a system of mathematical competitions, a mathematical camp, etc. The practical use of this model makes it possible to reduce the lack of tools in teaching for the development of students' mathematical abilities, which in its turn, makes it possible to speak of purposefully high results in students' mathematical activities, which is confirmed by the conducted experimental research.

Keywords: teaching mathematics in secondary school, means of teaching mathematics, puzzles, development of mathematical abilities of schoolchildren, increasing schoolchildren's interest to mathematics

INTRODUCTION

Modern pedagogical science considers the humanistic approach to be one of the priority areas for improving the process of education, the main principle of which is the all-round development of the pupil's personality. The wide use of system-activity approach in teaching practice brings to the fore the aspects that determine first of all not a huge amount of knowledge among learners but the ability of pupils to master tools for searching and operating knowledge and the ability to apply these skills in practical activity (Kondakov & Kuznetsov, 2008). The organization

✉ pavel-gorev@mail.ru **(*Correspondence)** ✉ nadya-telegina@yandex.ru ✉ karavanova_l54@mail.ru
✉ Fest1@yandex.ru

Contribution of this paper to the literature

- The authors have analyzed puzzles as a didactic tool in teaching mathematics to junior schoolchildren. They have considered issues that determine the place, possibilities and methodical aspects of the inclusion of puzzles in the process of teaching mathematics in general and additional education of schoolchildren: in the system of lessons, a mathematical club, a system of competitions, a mathematical camp, etc.
- The authors have determined the potential of puzzles in order to increase the pupils' interest to the subject and to develop their mathematical abilities: logical thinking, abstraction, combining, operating with spatial images, critical thinking, mathematical memory.
- A pedagogical experiment based on short-term training courses in the summer mathematical camp and a mathematical club and mathematical competitions during the year, where puzzles were used as one of the basic didactic tools, has proved an increase in the level of pupils' abilities, in particular mathematical ones, measured repeatedly with the help of IQ battery tests.

of such a pedagogical process will be the most effective if we take into account the age and psychological characteristics of pupils (Skilling, Bobis, & Martin, 2016).

Puzzles are one of the didactic tools which allow to provide the above-mentioned positions. The puzzle is a difficult task (in accordance with age and psychological characteristics of pupils), and to solve it, as a rule, pupils need to have intelligence, mastery of certain mathematical abilities, but not some special knowledge. So, for example, the main types of puzzles may be: arithmetic puzzles, magic squares, sudoku, etc.; tasks for rearrangements, for example, tasks with matches or games of 15 and similar; cutting and folding of figures such as tangram, including spatial ones, for example, Soma cubes; puzzles of a topological nature, such as flexagons, tracing figures with one stroke, rope and wire puzzles and others.

Traditionally, puzzles are entertaining mathematics not only because almost any puzzle is based on a certain mathematical concept, but because the solution of a puzzle is close to solving mathematical problems (Dubrovsky & Kalinin, 1990). Thus, puzzles are a powerful tool allowing pupils to develop the culture of mathematical thinking, based on algorithms, logic, abstraction, spatial images.

Puzzles are specially designed to develop pupils' mental abilities, improve and train their thinking, which helps better assimilate and consolidate the acquired knowledge, awaken pupils' interest to mathematics. In general, puzzles should contribute to the development of pupils' memory, attention, creative imagination, capacity for analysis and synthesis, comparison, perception of spatial relationships, development of design skills and creativity, observation, validity of judgments, self-control habit; they teach pupils how to subordinate their actions to the task in view, to bring the work to the end (Minskin, 1982). All these aspects prove the need to include puzzles in educational process as a didactic tool to achieve good educational results, to develop pupils' general and special mathematical abilities.

Thus, the purpose of the study is to determine the place and methodological features of including puzzles in the process of teaching mathematics to junior schoolchildren, and also to model the methodical system of teaching mathematics with the use of new didactic tools in basic and additional mathematical education of schoolchildren on the basis of the identified aspects.

LITERATURE REVIEW

The history of puzzles appearance cannot be separated from the history of mathematics; its roots go deep into antiquity. The first puzzles came to us from the texts of ancient Egyptian papyri, Babylonian clay tablets, manuscripts of Ancient China and Ancient India. So, the "task about cats" from the papyrus of Ahmes has roamed for millennia from one collection of mathematical puzzles to another, and "Tangram" game was popular in China about 4,000 years ago. In Europe, the very first collection of puzzles and logical problems was the book of the second half of the 9th century "Tasks for the Development of a Young Mind" by Alquin, an Irish theologian, scientist and enlightener. Here for the first time a very popular "task about crossing" is met. Medieval mathematicians L. Pisansky (Fibonacci) and N. Tartaglia included puzzles in their scientific research. Pisansky invented the task about rabbits, Tartaglia came up with the task about seventeen horses (Belov, 2002). However, the peak of development and popularization of puzzles was the boundary of the XIX - XX centuries and the first half of the XX century.

It is necessary to note the classical three-volume encyclopedia of entertaining mathematics, which contains almost all known puzzles - "difficult situations", their solutions and mathematical apparatus "In the realm of intelligence" by Ignatyev (1909, 1914, 1915). A great contribution to the popularization of mathematical knowledge, in particular entertaining mathematics and puzzles, was made by our compatriot Perelman (1927, 1940): his books are dedicated to individual puzzles for cutting a square and drawing figures with one stroke (Perelman, 1927, 1940), as well as modern reprints with numerous puzzles (Perelman, 2007). Among more recent authors, we should note

books Kordemsky (2000a, 2000b) and his books "Mathematical Wit" and "Mathematical amusements". They have been republished many times and have a great potential for mathematical education in general, in particular, how to use non-standard tasks in teaching. The well-known books "From Game to Knowledge" by Minskin (1982), "Your Free Time" by Bolkhovitinov, Koltovoy and Lagovsky (1975) and "Puzzles" by Mochalov (1996) are a storehouse of ideas for making and solving puzzles with some methodological explanations. The works of Yaglom (1968), Kordemsky & Rusalev (1994), Ekimova and Kukin (2002) are devoted to separate mathematical ideas related to the problems of cutting. The book "*Numerical puzzles and their solutions*" by Likhtarnikov (1996) is devoted to numerical puzzles with the methods of their solving.

Translations of foreign authors are also of great interest. Undoubtedly, books "Mathematical miracles and mysteries", "Mathematical puzzles and entertainment", "Mathematical leisure hours" and others by genius popularizer of science Gardner (1978, 1999, 2000) are worth mentioning. They contain dozens of puzzles, their description and original solutions. Classics of the genre are books "Mathematical mosaic" by Loyd, creator of puzzles (1995) and "520 puzzles" by Dyudeni (2000), as well as popular books by Barr (1987), Carter and Russell (2007), Hardy (1998), which contain original solutions and mathematical analysis of many puzzles.

The above-mentioned books are not the whole list of literature on the subject of puzzles and their solution. Here are the most well-known works of outstanding scientists. In general, there are more than 200 books on this subject by Russian and foreign authors. Puzzles invariably inspire interest in modern periodical literature. Puzzles have been studied mainly in three directions.

The first direction is a direct study of the mathematical side of solving puzzles. For example, Haraguchi (2016) examines the mathematical computer model of the three-dimensional puzzle «Eight Blocks to Madness». Lin et al. (2014) consider optimal models for solving a one-dimensional puzzle with coins. Ortiz - García et al. (2007) offer models of solving logical puzzles with images. Majia, Janab and Palc (2013) give optimal algorithms for solving Sudoku puzzle. It should be noted that the solution and mathematical justification of puzzles repeatedly became the basis of a new mathematical theory. The most striking examples of this are the emergence of graph theory, derived from the problem of the Konigsberg bridges, proposed by the eminent mathematician Euler (1741), or the development of probability theory based on the solution of gaming problems that came from practice.

The second direction of research is the use of puzzles to develop individual mathematical abilities of pupils. For example, Lin and Chen (2016) studied the influence of computer puzzle games on the development of spatial imagination and the operation with spatial images of primary school pupils. The study of Milková (2014), devoted to the use of puzzles for the development of logical thinking and abstracting, offers some methodological approaches and recommendations for teachers.

In the third direction, researchers suggest to use puzzles as a tool for assessing the abilities of different categories of pupils. In particular, Aral, Gursoy and Yasar (2012) investigated the influence of puzzles on cognitive, linguistic, motor, social and emotional development of preschoolers. Vakil and Heled (2016) studied the cognitive abilities of adolescents when solving tasks using Hanoi Towers puzzle, taking into account the accuracy of the task solution, the speed of coming to the right solution and planning the time before starting the task. Sengul and Argat (2015) attempted to use various teaching aids, in particular puzzles, to examine students with low academic achievements, and accordingly, developed appropriate recommendations. Sargin et al. (2015) considered the method of "puzzle" to increase the interest of students in vocational schools, their motivation, attendance of lectures and better understanding of the theoretical parts of the courses. They came to the conclusion that thanks to this method, there was an increase in the connection with the level of socialization student-student and student-teacher, and the reduction of their worries about future job search.

We should say a few words about the methodological aspect of using puzzles in teaching mathematics to children and adolescents. Thus, in the course "Visual Geometry" for pupils of the 5-6th grades of the general school, Sharygin and Erganzhieva (2001) build most of the lessons on particular entertaining tasks and puzzles, but this course, despite of its strong sides, has not been included in general school course practice and remained only a good support for additional mathematical education of pupils. Separate questions related to the development of combinatorial abilities at the lessons of mathematics were studied by the famous Russian methodologist Zaikin (1996). In his manual "Mathematical Training", he reveals the gradual development of these abilities through systems of specially selected puzzle exercises. However these studies are of a highly specialized nature and have not been included into practice of teaching math at school. The most significant contribution concerning the use of puzzles in educational process was made by Zinovkina, a teacher who offered to use a system of continuous development of a creative person, where at each stage of the system implementation puzzles were used consistently as one of the steps of a lesson or tutorial (Utomov, Zinovkina and Gorev (2013)). Thus, one can note the interest in puzzles as a special phenomenon in mathematical education. However, well-known sources describe either generalized theories, that include the use of puzzles in teaching mathematics as an element, or extremely narrow specialized issues related to puzzles. But modern school needs the development of applied aspects of didactic tools

related to puzzles, and its methodological support in basic and additional mathematical education of children and adolescents.

MATERIALS AND METHODS

The methodology of our study is the subject of puzzles as a didactic tool used in teaching pupils in general and additional mathematics education, characteristic of the purposeful development of pupils' interest in mathematics and the development of mathematical skills such as ability to think logically, ability to powerful abstract thinking, combinatorial abilities, abilities to spatial imagination and manipulation with spatial images, ability to critical thinking, mathematical memory as a specific ability of mathematical activity.

We carried out an analysis of puzzles that allowed to construct a model of methodological system for interdependent and complementary teaching of junior schoolchildren in the framework of general and additional mathematical education, where the puzzles are included in the educational process as a didactic tool at all stages of its implementation. The developed model of methodological system is intended to include puzzles in teaching mathematics to pupils: 1) at the stages of classical lesson of mathematics; 2) at the stages of creative lesson in CFCT-TIPS system, which are included in each studied topic of the course; 3) in extra-curricular activities of pupils (additional mathematical education).

Puzzles as a Teaching Tool in a Classical Lesson of Mathematics

Classical methods of teaching mathematics have a clear structure of the lesson steps. The introduction of new elements changes the type of lesson and the lesson becomes innovative. However, in the structure of classical lesson of mathematics you can find a place for puzzles as a tool for development mathematical abilities. This, in our opinion, may occur at one of three stages of the lesson.

At the *actualization of knowledge stage,* one can organize the work with pupils aimed to prepare them to the study of new material. Thus, for example, in studying the characteristics of divisibility in the 6-th grade, we can use arithmetic puzzles. They will not only help to prepare pupils for the necessity to learn divisibility criteria but will also contribute greatly to the development of logic, combinatorial abilities, critical thinking, help to develop mathematical memory. In addition, the puzzles allow to train the application of tables of addition and multiplication.

Another example of using puzzles at the stage of actualization can be any puzzle on compiling the figures from the parts ("Tangram'', "Columbus' egg", "Stomachion", "Pythagoras", "Mongolian game", "Pentamino" et al.) as a prerequisite for the study of plane figures areas and their properties in the 5-th grade. The use of this kind of puzzles determines the development of both combinatorial abilities, logical thinking and also allows to develop the capacity for operating with spatial images.

Modern requirements to the lesson define the need for special breaks for the psychological, emotional and physical unloading of pupils, especially for younger ones. At stage of *the pause – warm-up*, children can be offered simple puzzles to relieve psychological inertia (Gorev & Utomov, 2014, 2015, 2016).

Puzzles are a very time-consuming tool to be fully used at any particular stage of the lesson of mathematics. However, there is potential for solving puzzles at home with a further presentation of the results at the *stage of checking homework*. Thus, for example, in studying the concept of an algorithm in the 6-th grade, when introducing algorithms of search for the greatest common divisor or the least common multiple of two or more integers, and algorithm for finding prime numbers ("Eratosthenes' Sieve "), you can offer the already mentioned old puzzle "Towers of Hanoi" for moving disks. Discussion of this kind of puzzles at the stage of checking homework allows to show pupils the beauty of mathematics, to increase their interest to the subject and to develop their logical thinking, combinatorial abilities, critical thinking.

Of course, an experienced teacher will be able to find a place for puzzles at other stages of a lesson of mathematics. We have described only the most effective of the possible applications of puzzles at the classic lesson. But we should not forget that a puzzle, as any other tools of teaching, should not become an end in itself in the teaching of mathematics. It should be a tool that gives impetus to the development of pupils, in particular their mathematical abilities. In addition, the selection of puzzles to a particular topic of mathematics lesson is quite a time-consuming process, so it is not possible and not necessary to do this for each lesson.

Using Puzzles at a Special Stage of the Creative Lesson in Cfct-Tips System

In conducting non-traditional math lessons, we focus on the implementation of CFCT-TIPS system (continuous formation of creative thinking based on the theory of inventive problem solving), developed by Professor Zinovkina (2008). Here the author describes the subsystems of creative pre-school, school, secondary vocational,

higher and postgraduate education. The structure of these subsystems includes the stage of puzzle solving as one of the main ones. There is a so-called creative math lesson in *school education*. We have recently reported about the structure, content and possibilities of the pupils' development at such lessons, which take place in the study of each school mathematics topic (Gorev & Kalimullin, 2017). We should note that the selection of puzzles to the appropriate stage should be done very carefully: they must not fall out of the general context of lesson and must have the educational effect. To do this, the teacher conducting the creative math lessons occasionally, must possess a huge stock of different puzzles with their methodological description. Today we are doing a lot of work in this direction in the school museum of entertaining science, which was created a few years ago on the basis of Kirov city lyceum № 21, where puzzles have found their special decent place.

It should also be noted that CFCT-TIPS system is intended to include puzzles as a separate stage of lesson in the educational process even in preschool education. Our manual "Exciting game with Sovionok" for pre-school education (Zinovkina, Gorev, & Utomov, 2015) presents a system of such lessons. Puzzles are used in each lesson, composing a system of increasingly complex tasks embodied in real objects. We should also note that puzzles at the creative lesson may be used at the stage of motivation ("Meeting with a miracle"), since many of the object puzzles are unique ideas based on physical or mathematical principles that have always struck the imagination of the people, and not only at pre-school age. For example, at the motivation stage, children can be demonstrated wire or jigsaw puzzles made of wood or plastic, which have the idea of ingenious assembly or hinge puzzle like Rubik's Cube. All this creates a strong interest to the study of mathematics with the effect of a miracle at the same time, which later can lead pupils to the serious study of puzzles as mathematical problems.

Puzzles as a Didactic Tool in Additional Mathematical Education of Pupils

Additional mathematical education of children and adolescents, perhaps, is exactly the environment in which one can use didactic tools associated with puzzles most efficiently for the comprehensive development of the student's personality, in particular for the development of mathematical abilities.

Basic structural unit of additional mathematical education for pupils is a *mathematical club* (elective, studio). During club activities we may consider puzzles as separate math problems and a series of puzzles. For example, the lesson can be devoted to the study of Mobius band properties or flexagon, topological puzzles with rope and buttons, or be constructed on effects of impossible figures and optical illusions. For a younger age, it is advisable to devote a series of lessons to some concrete puzzle: "Tangram", pentamino, "Towers of Hanoi", tracing figures with one stroke, etc. It allows pupils to focus on the problem of the puzzle, to understand its structure. And these aspects allow to give a boost not only to understanding the mathematical essence of the puzzle, but also to develop logical thinking, combinatorial abilities, strengthen the capacity of abstract thinking and operating with spatial images, instill critical thinking and develop mathematical memory. It should be noted that our mathematical club program "Lessons of developmental mathematics", worked out and tested for pupils of 3-6-th grades (Gorev et al., 2017), presents a technological model of club lessons by modules. Each module comprises units, based on solving puzzles. The mechanism of including the developed technological model in additional mathematical education of 5-6-th grades pupils is described in the article (Gorev & Novoselova, 2017), and the didactic support of technological model is presented in books "Lessons of developmental mathematics. 5-6-th grades: mathematical club tasks" (Gorev & Utemov, 2014) and "Twenty intricate riddles by Sovionok "(Gorev & Utomov, 2015).

Another important component of additional mathematical education are *mathematical competitions*. Puzzles can become their integral part as specific tasks. So, we have successfully used puzzle challenges in mathematical battles, team competitions, such as "Mathematical Carousel", "Mathematical skirmish", "Mathematical auction" and others. However, separate competitions based on puzzles may also be organized.

Another area of additional mathematical education for schoolchildren is the *work of the summer mathematical camp*. Kirov city lyceum № 21 has organized the camp "Mathematics. Creation. Intellect" for pupils of 5-10-th grades since 2001. It is aimed first of all to develop mathematical abilities of pupils by various means: from unusual tasks to project activities. In the 5-6-th grades, the work with puzzles is performed in two main directions.

Firstly, as a rule, the program includes a special course of 5-6 sessions focused specifically on puzzles, their construction and solution. The course sessions presented in the experimental part of this study are: 1) puzzles with matches; 2) tasks for cutting and putting figures together; 3) disappearance of figures; 4) optical illusions; 5) Mobius band; 6) Soma cubes.

Secondly, schoolchildren of 8th grade conduct a series of project clubs and workshops in the camp program for pupils of 5-6-th grades. Each of them is aimed to obtain a specific result on one of the entertaining mathematics topics connected with puzzles solving. As a rule, there are 80-100 younger pupils in the camp and 6-10 clubs are organized for them. The purpose of project clubs and workshops is to increase interest in the subject through joint creative work with senior school pupils, to develop mathematical abilities of younger schoolchildren. Examples of such clubs are presented in **Table 1**. Every day junior pupils work in the chosen workshop and at the end of the

shift they present results of their work. This form of work activates the interest of younger schoolchildren to mathematics, helps to acquire new mathematical knowledge, giving opportunities for creative expression of skills.

Table 1. Project clubs and workshops for students of 5-6 grades

Name of the club	***Project objectives***	***Final product***
Do not believe your eyes	To explore and present geometric illusions, non-existent in the real-world figures that can only be pictured	Album with illusions
Hot cockles	Explore the history and features of the game "hot cockles" and create your own games like this one	Set of games
Magic Tetris	Consider different types of polyominoes (domino trimino, pentamino with square, triangular, hexagonal elements) and tasks associated with them	Sets of polyominoes, wall newspaper
Supercubes	Explore the history of the emergence and creation of supercube, different versions of the game with it and putting figures together	"City" of the supercube's parts
Finding Minotaur	Explore the history of legends and myths associated with labyrinths, their types and algorithms of escape from labytinths	Labyrinths, wall newspaper
3D tasks	To study and learn how to assemble a variety of three-dimensional object puzzles	Set of puzzles and wall newspaper about them
Flexagons	To study different types of flexagons, the history of their origin and creation	Sets of flexagons and wall newspaper about them

In general, it should be noted that within the framework of additional mathematical education one can find endless potential for the development of mathematical abilities of pupils by various means, in particular using didactic tools, based on puzzles.

MATERIALS AND METHODS

The most significant in the study of the theoretical basis were the works of Russian psychologists, devoted to the study of general and special mathematical abilities. In Russian and foreign science, allocation of abilities by type of activity is the most common. Thus, abilities are the stable properties of people that determine success they have achieved in various activities. So, we can talk about special mathematical abilities that manifest themselves in a specific type of human activity - mathematical, in particular, the educational mathematics of children and adolescents. A significant contribution to the study of mathematical abilities was made by V.A. Krutetsky. He considered abilities to study mathematics to be individual psychological features (especially the features of mental activity) that meet the requirements of academic mathematical activity and conditions, and provide the success of creative mastery of mathematics as a subject of study, in particular the relatively rapid, easy and profound mastery of knowledge and skills in the field of mathematics (Krutetsky VA, 1968). In his book "The Psychology of the Mathematical Abilities of Schoolchildren," he has structured and given mathematical abilities the following classification: 1) clear logical thinking, correct use of logical methods; 2) power of abstraction; 3) combinatorial ability; 4) ability to spatial representation and operation with spatial images; 5) criticality of thinking, ability to abandon the erroneous course of thought; 6) mathematical memory (generalized memory for mathematical relations, typical characteristics, schemes of reasoning and proofs, methods of solving problems and principles of their approach). It should be noted that the study of mathematical abilities and mathematical thinking is still of great interest. Thus, a recent study of D. Jeannotte and C. Kieran has provided an opportunity to construct a conceptual model for the development of mathematical thinking, consisting of two main aspects: structural and procedural, which should facilitate the restructuring of curricula taking into account their conclusions (Jeannotte D. & Kieran C., 2017).

Approbation, generalization and introduction of the results of the conducted research are carried out on the basis of the regional innovative educational platform "Interconnection of content, forms and methods of basic and additional mathematical education of schoolchildren" (2012-2017):

- by conducting an experienced teaching in the main course of mathematics in 5-6grades and a mathematical club in 3-6 grades by mathematics teachers in the basic educational organization of the Institute for the Development of Education of the Kirov region - lyceum No. 21 in the city of Kirov (more than 120 pupils annually since 2012);
- by implementing individual educational courses in the summer camp "Mathematics. Creation. Intellect" since 2009 (80-100 pupils of 5-6th grades annually);
- by conducting a pedagogical experiment based on tests to assess abstract thinking, mathematical, perceptual, verbal and spatial-visual abilities of students of 5-6 grades (Eysenk H. & Evans D., 1998), who

had experimental training during the school year and in the school mathematical camp (2 groups of 20 students each);

– by discussing methodological aspects at seminars and round tables with teachers of mathematics of the Kirov region within the framework of the advanced training courses at the Institute for the Development of Education of the Kirov region (120-150 participants annually since 2015); in the form of reports and speeches at scientific conferences and seminars of various levels, including international ones, publications in collections of scientific articles and scientific and methodical periodicals.

The research had three stages. At the first stage, the state of the problem was revealed in the theory and practice of teaching students in basic and additional mathematical education, implemented in primary and basic education. For this purpose, the study and analysis of psychological, pedagogical and methodological literature on the research problem, observation and analysis of the experience of teachers of mathematics and primary school were carried out in order to explore possible ways of including puzzles in the process of teaching mathematics and the likely organizational forms of organizing such activities with children and adolescents for effective continuous formation of personality, who possess basic types of mathematical abilities: logical thinking, abstraction, combination, operation of spatial images, criticality of thinking, mathematical memory. At the second stage, there were developed methodical approaches to the implementation of the didactic tools based on puzzles and the mechanisms for its implementation within the framework of the summer mathematical camp, as well as during the academic year in the classroom and extra-curricular activities. Discussion of their implementation was carried out and continues to be carried out during the work of seminars and round tables with teachers of mathematics in the region, reports at conferences and seminars of various levels, which leads to a consistent improvement of the methods of working with puzzles in the practice of teaching mathematics to children and adolescents. In parallel with the second, the third stage is realized, during which the experienced teaching is conducted on the proposed methodological aspects on the basis of lyceum No. 21 of the city of Kirov.

RESULTS

40 pupils of Kirov city lyceum № 21 aged 11 - 12 years (5-6-th grades) took part in the experiment. They were divided into two groups of 20 pupils who participated in all stages of the experiment. Pupils of 5-a class were Group A in the 2015 / 2016. They studied in the lyceum from the first grade and during the education process in 4-th grade they attended a mathematical club where the method "Lessons of developmental mathematics" worked out by the authors (Gorev et al., 2017) was used and puzzles were one of the main didactic tools. Pupils of 5-g class were Group B. They didn't attend this mathemical club.

Diagnostics at all stages of the experiment was carried out using a battery of five tests in oreder to determine the level of cognitive development (intelligence quotient IQ): tests for abstract thinking (AT); tests to determine the mathematical abilities (MT); perceptual tests (PT); verbal tests (VT); tests to assess the ability to spatial-visual thinking (SV). Brief designations of tests are shown in tables and diagrams. Input diagnostics on IQ tests was conducted at the beginning of the 5th year of study in 2015 / 2016. **Table 2** and **Figure 1** show its results in a proportion of the number of pupils who have reached a certain level of certain types of abilities formation, and the average score (the conversion tables are presented in the book (Eysenk & Evans, 1998)) for each test category. Scoring was carried out on a scale for 11 year-old children; the average score is rounded up to the nearest whole number by the accepted rules of rounding.

Table 2. Results at the beginning of the experiment 2015 / 2016, 5-th grade (14/09/2015)

	Group A (20)					Group B (20)				
	AT	**MT**	**PT**	**VT**	**SV**	**AT**	**MT**	**PT**	**VT**	**SV**
Perfect	0	0	0	0	0	0	0	0	0	0
Excellent	1	3	2	1	0	1	2	1	0	0
Very good	3	4	2	3	4	3	4	2	4	3
Good	6	7	4	10	8	4	6	6	8	6
Average	5	3	8	3	5	7	4	7	6	7
Satisfactory	5	3	4	3	3	5	4	4	2	4
The average score of the group	**60**	**43**	**58**	**40**	**50**	**54**	**40**	**57**	**40**	**46**

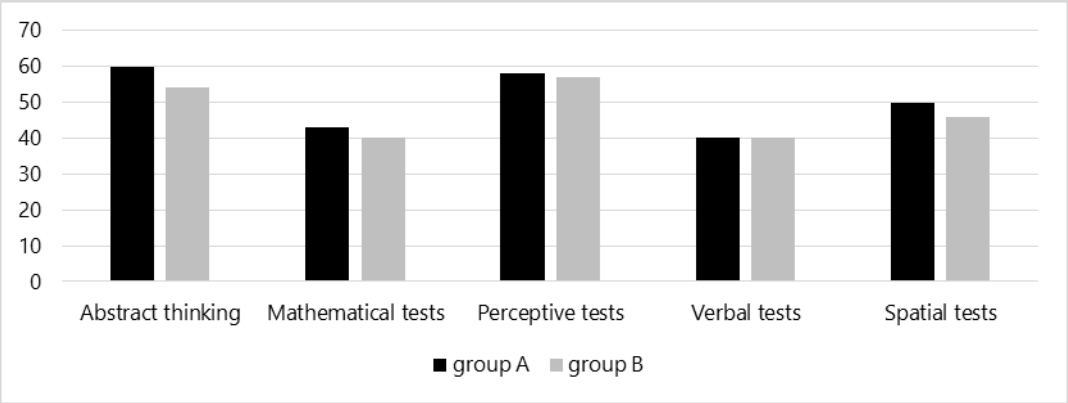

Figure 1. Comparison of average scores in the groups at the beginning of the experiment

As we can see from the chart, there is little difference in the results of perceptual and verbal tests. This is likely due to the fact that the pupils of the 5-th grade after 4 years of primary school have little distinctions in perception and verbal presentation of information. The difference in solving mathematical tests, abstract tests and tests on spatial-visual thinking we refer to the results of the work with group A in the previous period with the use of didactic tools, based on puzzles.

During the next academic year both groups had had the same teacher and the same textbooks. Pupils of 5-a class attended a mathematical club based on method "Lessons of developmental mathematics" (1 hour a week), where puzzles were used as a didactic tool. All pupils of both classes were encouraged to attend a mathematical camp "Mathematics. Creation. Intelligence" during the first two weeks of June. **Table 3** shows IQ results at the beginning of the shift (i.e. at the end of the analyzed academic year). Scoring was carried out on a scale for 12 year-old pupils.

Table 3. Results at the beginning of summer shift in 2015/2016, 5-th grade (01/06/2016)

	Group A (20)					Group B (20)				
	AT	**MT**	**PT**	**VT**	**SV**	**AT**	**MT**	**PT**	**VT**	**SV**
Perfect	0	0	0	0	0	0	0	0	0	0
Excellent	1	3	2	4	1	1	2	1	1	1
Very good	5	5	3	4	6	4	5	2	5	5
Good	9	7	6	6	7	7	7	7	8	6
Average	3	3	6	4	4	7	4	7	6	7
Satisfactory	2	2	3	2	2	5	4	4	2	4
The average score of the group	**66**	**50**	**63**	**47**	**56**	**54**	**41**	**55**	**42**	**46**

Let us compare the results of each group. They are presented in **Table 4** and in **Figures 2** and **3**.

Table 4. Comparison of the results of three group works

	Group A (20)					Group B (20)				
	AT	**MT**	**PT**	**VT**	**SV**	**AT**	**MT**	**PT**	**VT**	**SV**
Average score at the beginning of the school year	60	43	58	40	50	54	40	57	40	46
Average score at the beginning of the shift	66	50	63	47	56	54	41	55	42	46

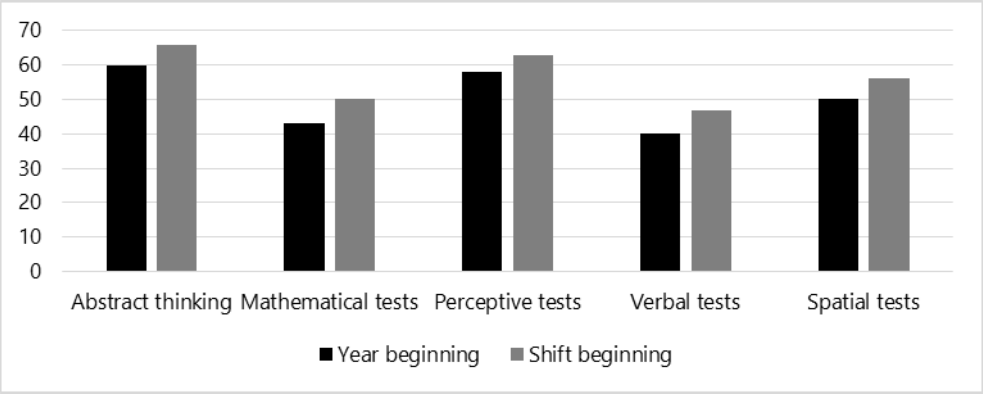

Figure 2. Comparison of average scores in group A

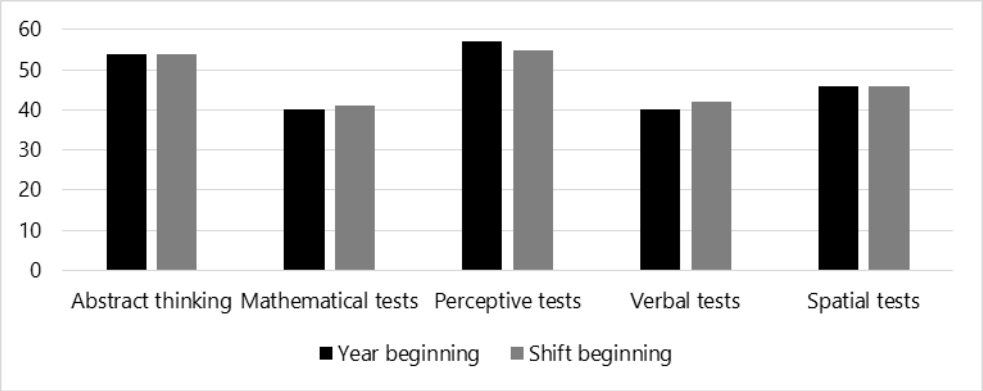

Figure 3. Comparison of average scores in group B

Let us note that group A had a significant increase in all indicators. We explain it by the fact that pupils of group A were taught by the program "Lessons of developmental mathematics", and pupils of group B didn't have this opportunity.Potentially both groups initially showed good capacity for intellectual activity, which was confirmed by the results of the check. Let us pay attention to the fact that the same type of materials (different options) were offered for tests. However, the children did not purposefully solve the test tasks during the learning process. Instead of them different types of puzzles were used, the solution of which was very similar to certain tasks solution in IQ tests.

Thus, we came to the conclusion that the proposed methodology "Lessons of developmental mathematics", which uses didactic tools based on puzzles, improves mathematical abilities of pupils: ability to think logically, ability to abstract thinking, combinatorial abilities, abilities at spatial representation and manipulation with spatial images, critical thinking abilities. Moreover, the influence of other teaching tools undoubtedly has an influence on the results. However, we have not seen such growth in similar classes earlier, when our worked out methodological model of using puzzles in the educational process has not been used.

DISCUSSION

Today there is a significant number of works devoted to puzzles and solving them in Russian and foreign literature. Many authors relate puzzle-solving with direct mathematical activity that explains their place in mathematical education of children and adolescents (Milková, 2014).

Any methodological system of teaching mathematics is inconceivable without a specific set of tasks. Tasks are used as a very effective means of mastering concepts and methods, as the most effective means of developing pupils' thinking culture, as an irreplaceable means of inculcating skills in practical applications of mathematics (Zaikin, 1996). Puzzles, being a kind of mathematical tasks of higher complexity (in accordance with the age and psychological characteristics of pupils), determine not only the subject skills of pupils, but also serve as a powerful tool for the development of mathematical skills: ability to think logically, ability to abstract thinking, combinatorial abilities, aptitude for spatial representation and manipulation with spatial images, critical thinking abilities, mathematical memory as a specific ability of mathematical activity (Gorev & Utomov, 2016).

Analysis of psychological, educational and methodical literature, experience of teachers of mathematics and primary school teachers, teachers of preschool education show that the formation of mathematical abilities of children and teenagers, is of great importance today. In addition, it is necessary to create the condition for pupils that contribute to the emergence of cognitive needs in acquiring knowledge, mastering skills of using it, and influencing the formation of independent thinking activity skills, which would enable them to succeed in life using internal potential, both intellectual and creative.

Thus, the proposed methodological approaches to the use of didactic tools based on puzzles give the possibility to include them in both additional and basic mathematical education of children and adolescents, which is a new trend in the methodological work of teachers of mathematics allowing to create conditions for the achievement of high educational results.

CONCLUSION

In the process of analyzing different points of view on the development of mathematical abilities of children and adolescents, there have been developed and implemented in educational practice methodological approaches, based on the teaching possibilities of a new teaching tool - mathematical puzzles. Testing of various puzzles as a didactic tool in general and additional education of schoolchildren and preschool children for the development of their mathematical abilities allowed to build methodological approaches to their inclusion in the process of teaching mathematics. Methodological approaches have been established and key ideas on the use of puzzles in the educational process have been identified as a result of studies and experienced teaching, conducted during the last three years by the creative group of mathematics and primary school teachers. Practical use of puzzles as a didactic tool allows to observe pupils' progress in learning the subject.

Using puzzles as a didactic tool improves pupils' results in mastering program material. Monitoring of pupils' learning results shows positive dynamics. When solving learning and extracurricular tasks students demonstrate creative thinking, initiative, resourcefulness, activity, ability to emotional perception of math problems and arguments, take responsibility for the choice of solving method and answer. So, puzzles may be the means to achieve not only learning, but also personal results.

It should be also noted that during last two years a school museum of entertaining sciences has been actively filled up with exhibits created by pupils, among them is a huge number of puzzles. Thus, the direct experience of puzzles use in additional education lets us come to the conclusion that their harmonious combination with the lessons activity provides realization of basic requirements for the results of mastering the basic educational program of basic general education, formulated in national educational standard. The indicators of this are participation and victories of our pupils in contests and Olympiads of various levels, and also the fact that students themselves are initiators of new mathematical activity based on solving and creating puzzles.

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