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Research Framework on Mathematics Teacher Behaviour: Koehler and Grouws' Framework Revisited

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Educational reform centres on changing teacher behaviour, as it is teachers who filter through the curriculum to learners. Educational reform could therefore be viewed as reforming or changing teacher behaviour. This article investigates some of the factors influencing teachers' behaviour namely knowledge, attitude and views and beliefs. The complexity of research on teaching and teacher education is addressed by focusing on the elements of three factors as well as the relationship between these influencing factors. A research framework on teacher behaviour is presented, in an effort to expand the theoretical understanding of the factors influencing teacher behaviour and to guide future teacher education.

Keywords: Teacher Attitude, Teacher Behaviour, Teacher Beliefs, Teacher Knowledge

INTRODUCTION

Many countries are in the midst of educational reform, with the heart of this reform revolving around changes in the curriculum and teacher instructional behaviour (through teacher preparation). Teachers play a central role in bringing about the desired reform as it is the teacher who filters the curriculum through to learners (Jegede, Taplin & Chan, 2000). This implies that educational reform is implemented (in part) by changing teacher instructional behaviour. Adding to this, Jegede et al. (2000) note that for satisfactory and effective public education reform, it is essential that its most valuable human resource (i.e. teachers) must be comprehensively and adequately developed. In order to develop teachers and so change teacher instructional behaviour, it is essential to identify the factors influencing teacher behaviour but also the relationship between them. This article will endeavour to examine the factors influencing instructional behaviour, and in doing so, also suggest a (revised) research framework on teacher behaviour.

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THE COMPLEXITY OF RESEARCH ON TEACHING AND TEACHER BEHAVIOUR

In the 1980s Brophy (1986) noted that despite the remarkable progress made in research, classroom teaching (including research on school mathematics instruction) was in its infancy. Koehler and Grouws (1992) examined research on teaching from the perspective of complexity. Four levels of complexity and presentative models, that reflected the changes and progress made in research on teaching, were presented. The highest level (level 4) reflects current research, where research questions in teaching and learning are being approached from several perspectives, thus having a strong theoretical foundation (Koehler & Grouws, 1992). Koehler and Grouws' proposed model (1992) (see figure 1) postulates that outcomes of learning are based on a learner's own actions or behaviours, which are influenced by a) their beliefs about themselves as learners, b) their beliefs about the discipline of mathematics and c) what the teacher does or says within the classroom. Teachers' behaviour, according to the model, is influenced by the teacher's knowledge (of the content to be taught, how learners learn/understand that specific content and methods to teach that specific content) in addition to teachers' attitudes and beliefs about teaching and mathematics.

Teacher knowledge

Koehler and Grouws (1992) note that teacher behaviour is influenced by the teacher's understanding of the particular content and knowledge of how students might learn (National Research Council (NRC), 2001). This includes knowledge of how students think and learn (National Council of Teachers of Mathematics (NCTM), 2000; Ball, 1993) and, in particular, how this occurs within specific mathematics content (Fennema & Franke, 1992) but also examines sensitivity to the unique ways of learning, thinking about, and doing mathematics that the students have developed (NRC, 2001). Knowledge of how students acquire the knowledge of the mathematics content being addressed, as well as understanding the processes the students will use and the difficulties and successes likely to occur, form part of a teacher's knowledge of student learning (Fennema & Franke, 1992). Ball (1993) rephrases this kind of knowledge as "I must consider the mathematics in relation to the children and the children in relation to the mathematics".

Shulman (1986) presents a framework for discussion of teacher knowledge which postulates that teachers make decisions based on their knowledge. It has been presumed that teachers will develop this knowledge framework as a result of training and experience (Foss & Kleinsasser, 1996). The knowledge mathematics teachers need include knowledge of mathematics itself (subject content knowledge) (Muijs & Reynolds, 2002; Ball & Bass, 2000), and beyond pure subject matter knowledge the teacher needs to know how to teach mathematics (NRC, 2001). This includes knowledge of how to present mathematical topics and ideas (pedagogical content knowledge) and knowledge of mathematics curriculum materials and resources (curricular knowledge) (Shulman, 1986). Ball and Bass (2000) note that understanding and knowing subject matter knowledge is imperative in listening flexibly (hear what they are saying or where they might be heading) but also to be able to create suitable opportunities for learning (Ball, 2000). Ormrod and Cole (1996) report that an increase in knowledge of content could lead to changes in classroom practice that also reflect increased sophistication in pedagogical content knowledge. Knowledge of mathematics (content knowledge) is transformed by means of practical knowledge of mathematics teaching (both pedagogical and curricular) into representations for the classroom use of content knowledge (Ernest, 1989).

Pedagogical content knowledge (also termed pedagogical content knowing by Penso, 2002), can be described as practical knowledge of teaching (knowledge of how to teach that is specific to what is being taught) (Jegede *et al.*, 2000) by blending content and pedagogy (Ball *et al.*, 2001; Shulman, 1987). It includes knowledge of *approaches* to school mathematics topics; teachers' knowledge of teaching *procedures* such as effective strategies for planning, classroom practice, behaviour management techniques, classroom organizational procedures, and motivational techniques; different *ways of presenting* mathematics (Rowan, Correnti & Miller,

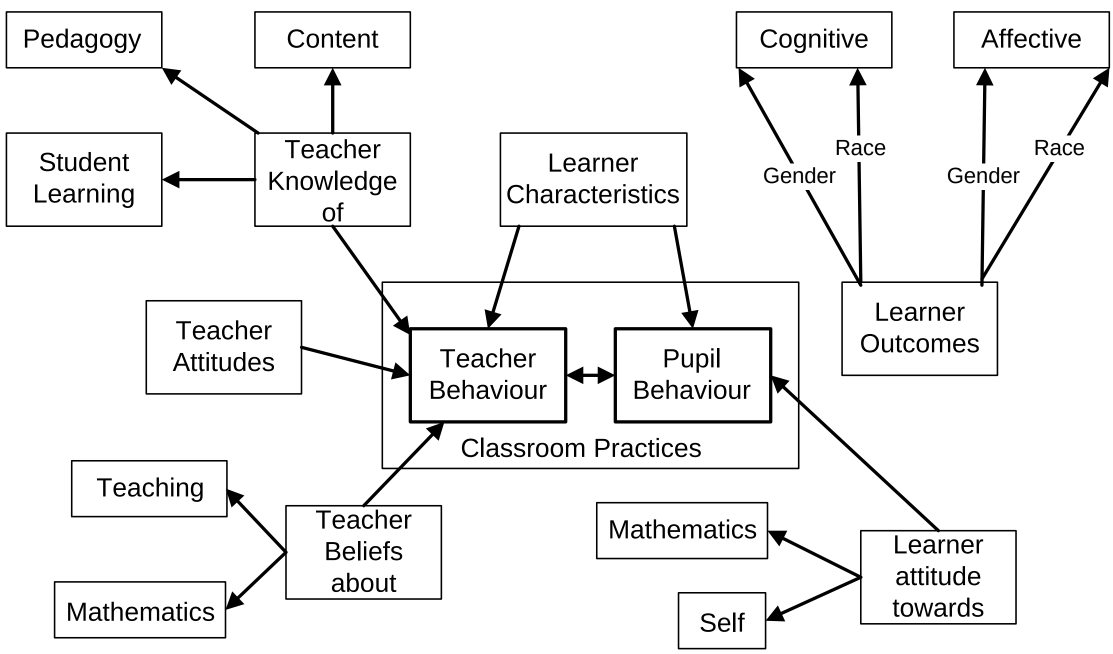

Figure 1. Level 4 research model (Koehler & Grouws, 1992, p.118)

2002; Shulman, 1986) through examples, illustrations, models and simulations (Geddis & Wood, 1997); knowledge of *students* (Penso, 2002) i.e. methods, conceptions, difficulties and common errors (Ball & Bass, 2000); knowledge of mathematical tasks, activities, test items (Fennema & Franke, 1992) and explanations (including alternative instructional methods (Rowan, Correnti & Miller, 2002). It is knowledge that a teacher uses to transform and represent knowledge either directly by the teacher, or by means of instructional media (Ernest, 1989) in order to make the subject matter accessible, comprehensible and compelling to a particular group of learners (Shulman, 1986). Ball and Bass (2000) summarize pedagogical content knowledge as a "unique subject specific body of pedagogical knowledge that highlights the close interweaving of subject matter and pedagogy in teaching". (p. 87)

Curricular knowledge includes knowledge of texts and schemes used to teach mathematics, their contents and ways to use them; school produced curriculum materials; other teaching resources and teaching apparatus; examinations, tests and syllabi (Turner-Bisset, 2001). Shulman (1987) changes the term of curricular knowledge to curriculum knowledge but still defines it as "tools of the trade" (p.8) which could be transcribed as knowledge of the materials and media ("tools") through which mathematics instruction is carried out and assessed (Turner-Bisset, 2001). Harland and Kinder (1997) indicate that this knowledge can have a positive and substantial influence on teachers' classroom practice. Cohen and Ball (2001) note the importance and value of teachers combining their knowledge of content, pedagogy, the selection of suitable curricula (NCTM, 2000) and use of resources.

Fennema and Franke (1992) note that there is a relationship between a teacher's knowledge and beliefs and according to Muijs and Reynolds (2002) these are related to student achievement. Both teacher's knowledge and beliefs have also been viewed as being context specific (Fennema & Franke, 1992). Thompson (1992) theorizes that because teachers treat their beliefs as knowledge, it is difficult to distinguish between knowledge and beliefs, with Manouchehri (1997) noting that teachers translate their knowledge of mathematics and pedagogy into practice through the filter of their beliefs. Turner-Bisset (2001) completes the triadic relationship between teacher knowledge, beliefs and attitude by noting that "one's beliefs about a subject can influence one's attitude towards it". (p.146)

Teacher beliefs

Teacher beliefs is the second factor in Koehler and Grouw's model (1992), as beliefs have a powerful impact on teaching (NRC, 2001) via teacher behaviour (Muijs & Reynolds, 2002; Schoenfeld, 2001) through such processes as the selection of content and emphasis, styles of teaching, and modes of learner learning (Ernest, 1989). Belief systems, according to Muijs and Reynolds (2002), are "dynamic and permeable mental structures, susceptible to change in light of experience" (p.4). A belief consists of the teacher's system of conceptions, values and ideology (Ernest, 1989) and is not consensual and is therefore held in varying degrees of conviction (Thompson, 1992). Studies of teachers' beliefs in mathematics education have investigated teachers' beliefs about the *nature of mathematics* (Ernest, 1989), as well as general conceptions of *mathematics teaching* (Cobb, Wood & Yackel, 1992).

Teachers' beliefs about the *nature of mathematics* are conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the nature of mathematics as a whole (discipline of mathematics) (Ernest, 1989) that appear to affect teacher behaviour (Schoenfeld, 2001). These beliefs or conceptions form the bases of the teachers' own philosophy of mathematics, that teachers may hold consciously or implicitly (Thompson, 1992). Three philosophies/views of mathematics are distinguished due to their observed occurrence in mathematics teaching (Thompson, 1984) but also their prevalence in the academic study of the philosophy of mathematics. Ernest (1989) notes that teachers in practice might combine elements from these views. *Problem solving view*: This view is characterized by a dynamic problem-driven view of mathematics as a continually expanding field of human inquiry. Mathematics is not seen as a finished product, and its results remain open for revision (Ernest, 1989; Thompson, 1984). *Platonistic view*: Mathematics is viewed as a static/ fixed body (NRC, 2001) but a unified body of knowledge and procedures, consisting of interconnecting structures and truths which are to be discovered and not created (Ernest, 1989). *Instrumentalist view*: Mathematics is looked upon as being useful and consisting of an unrelated collection of facts, rules, skills (Ernest, 1989) and processes to be memorized (Leung, 1995).

The second belief system teachers hold is a mental model of *mathematics teaching* that Ernest (1989) views as the key determinant of how mathematics is taught. Kuhs and Ball (1986), as quoted by Thompson (1992, p.136), have identified at least four dominant and distinctive views teachers hold of how mathematics should be taught: *Learner focused*: Mathematics teaching in this view focuses on the learner's personal construction of mathematical knowledge (Manouchehri & Enderson, 2003) – typically underlay by a constructivist view of mathematics learning (Cobb & Bauserfeld, 1995). At the centre of this view is the learners' active involvement in constructing meaning from experiences by doing mathematics (De Jong & Brinkman, 1997) through exploration and formalizing ideas. This view is likely to be advocated by those who have a problem solving view of mathematics, who view mathematics as a dynamic discipline, dealing with self-generated ideas and involving methods of inquiry (Thompson, 1992). *Content focused with an emphasis on conceptual understanding*: Mathematics teaching in this view is driven by the content itself that emphasizes conceptual understanding (Thompson, 1992). This view of teaching would naturally follow the conception of the nature of mathematics that Ernest (1989) labels *Platonist*. In instruction, content is made the focus of classroom activity while emphasising students' understanding of ideas and processes. *Content focused with an emphasis on performance*: Student performance and mastery of mathematical rules and procedures, combined with stress on the use of exact, rigorous mathematical language (Leung, 1995) are emphasized in this view of teaching mathematics. This view of teaching would follow naturally from the instrumentalist view (Ernest, 1989) of the nature of mathematics. This view has the following central premises: a) rules are the basic building blocks of all mathematical knowledge (as mathematics is perceived as a fixed body of knowledge) thus making all mathematical behaviour rule-governed (Leung, 1995); b) knowledge of mathematics is demonstrated by correctly answering and solving problems using the learned rules; c) computational procedures should be "automatized"; d) it is not necessary to understand the source or reason for student errors as further instruction will result in appropriate learning (Kuhs and Ball 1986 as quoted by Thompson, 1992, p.136). *Classroom focused with mathematical teaching based on knowledge about effective classrooms*. Central to this view is the notion that classroom activity must be well structured and efficiently organized according to effective teacher behaviours identified in process-product studies of teaching effectiveness (Thompson, 1992).

Teacher attitudes

Koehler and Grouws (1992) note that teachers' behaviour is not only influenced by their beliefs but also by their attitudes towards mathematics and the teaching of mathematics. Attitudes are defined as internal beliefs that influence personal actions (Schunk, 1996). Gagnè believes (according to Schunk, 1996, p.392) that attitude is learned indirectly through one's experience and exposures.

Teachers' *attitude towards mathematics* itself includes liking (Quinn, 1998), enjoyment and interest in mathematics, teacher's confidence in his or her own mathematical abilities: the teacher's mathematical self-concept, and the teacher's valuing of mathematics (Ernest, 1989). A teacher's self-concept is formed through experiences and interpretations of the environment and depends heavily on reinforcement and evaluations by significant others (Schunk, 1996).

Attitudes to mathematics and its teaching are important contributors to a teacher's make-up and approach, because of the effect they can have on a child's attitude to mathematics and its learning but ultimately on student achievement in mathematics (Ernest, 1989). Teachers' attitude to the *teaching of mathematics* include liking, enjoyment and enthusiasm for the teaching of mathematics, and confidence in the teacher's own mathematics teaching abilities (Ernest, 1989).

Shulman (1987) mentions that teachers should possess knowledge of student characteristics, with Koehler and Grouws (1992) indicating that student characteristics have an influence on the teacher's behaviour, but neither defined which characteristics and how these characteristics influence the teacher's behaviour. The framework put forth for examination and discussion centres on factors influencing teacher behaviour, while attempting to incorporate student characteristics as well as teachers' beliefs on the learning of mathematics. The interactive and dynamic nature of the components as well as how it influences teacher behaviour will be addressed.

A RESEARCH FRAMEWORK ON TEACHER BEHAVIOUR

The proposed model (see figure 2) includes all three factors noted by Koehler and Grouws (1992), namely teacher knowledge, teacher beliefs and teacher attitude. Some additions were made to the model with respect to all three factors. Each addition and/or change will briefly be discussed with some attention given to the interactive nature of the added components.

Teacher knowledge

The author is in agreement with Fennema and Franke (1992) that teacher knowledge is a large, integrated, functioning system and is an important indicator of overall teacher effectiveness (Kanes & Nisbet, 1996). Four components distinguished in teachers' knowledge consist of teachers' knowledge of student learning, subject content knowledge, pedagogical knowledge and *curriculum knowledge*. Curriculum knowledge is explicitly added to the model as knowledge of the subject content (concepts, procedures) and knowledge of different ways of presenting the content (pedagogical knowledge) does not guarantee knowledge of different and effective teaching and assessment resources such as computer software. A teacher with knowledge of various teaching tools may choose to apply a specific tool and combine it with an appropriate teaching style which could lead to learning becoming more effective. For example, a teacher being aware of a specific computer software programme, could design his/her own examples and students could explore their own choices (Kong & Kwok, 1999) to discover theories while constructing their own knowledge, whereas a teacher using only the traditional pen and paper method could be tempted to "transmit" the knowledge to learners. In short, the greater the knowledge of different teaching resources, the more "freedom" a teacher has in the chosen teaching approach. Ernest (1989) notes that this knowledge (curriculum knowledge) is vital to the planning and has a powerful influence on the carrying out of mathematics teaching (Cohen & Ball, 2001) as it is situated in context (Turner-Bisset, 2001). By adding curriculum knowledge, rightful recognition is given to the triadic relationship between subject content knowledge, pedagogical knowledge and curriculum knowledge (as first noted by Shulman, 1986).

Teacher beliefs

Two factors (beliefs about the learning of mathematics and beliefs about students as learners) were added to the original two factors that consisted of teachers' beliefs about mathematics and the teaching of mathematics. Attention will be given only to the two added factors.

Beliefs about the learning of mathematics

Teachers' belief system of the learning of mathematics consists of the teachers' view of the learning process, behaviours and mental activities on the part of the learner, and appropriate and prototypical learning activities, in particular the aims, expectations, conceptions and images of learning activities and the processes of learning mathematics in general (Ernest, 1989). Two key constructs on the learning of mathematics are as follows: viewing learning as the active construction of knowledge as a meaningful connected whole, versus a passive reception of knowledge and the development of autonomy and the learner's own interest in mathematics versus a view of the learner as submissive and compliant. The teacher's model of learning mathematics is a vital factor in the learner's experience of learning mathematics as it influences both the cognitive and affective outcomes of learning experience (Ernest, 1989).

Students as learners

Teachers' beliefs about their students as learners include beliefs about differences in individuals or groups of learners regarding the learners' talent for mathematics and learners' intellectual abilities to successfully learn mathematics. Leung (1995) reports that educational practices of teachers concerned with individual differences amongst students differed significantly from

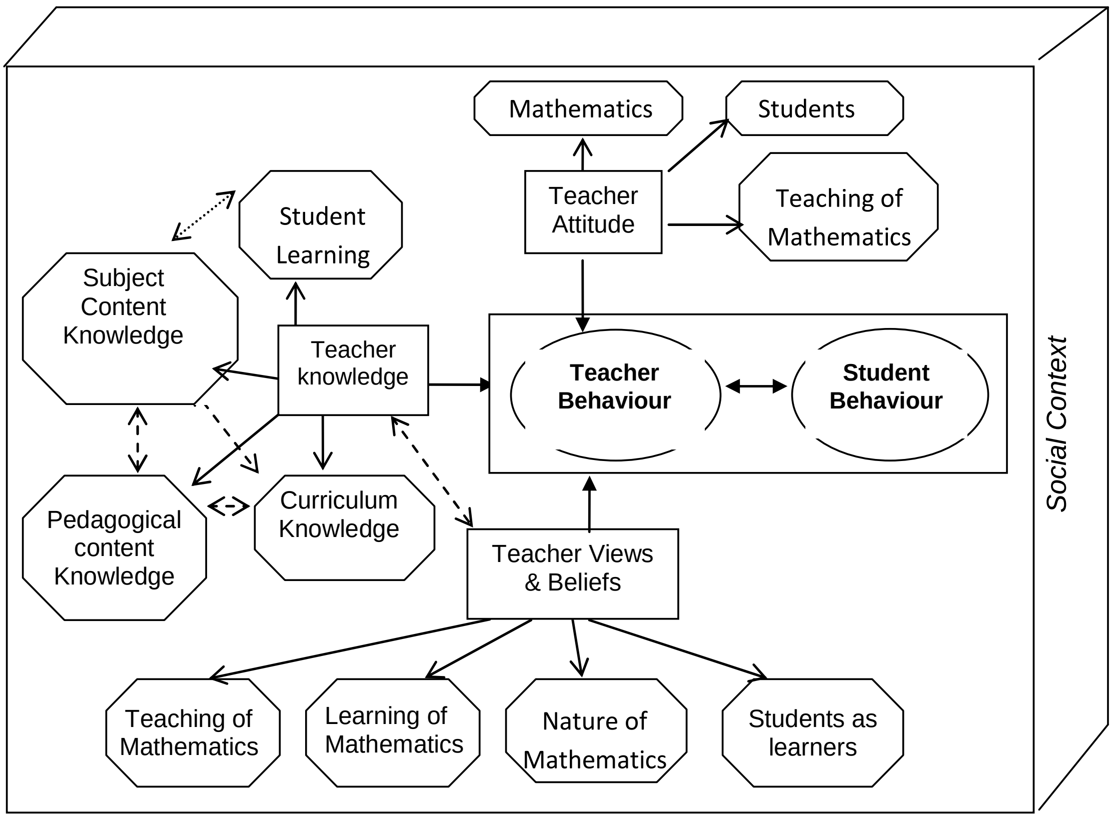

Figure 2. Research framework on teacher behaviour

those of teachers who emphasized conformity. These teachers adopted individual learning programmes in order for students to proceed at different paces. In addition, Penso (2002) notes in her study that prospective teachers believed that most of the learning difficulties were due to the learner's characteristics. Teachers' beliefs about learners could be as varied as believing that some learners are born good learners while others are stuck with a limited ability. Furthermore teachers could believe that some learners have natural talents for mathematics while others do not, so working hard at a problem will only pay off for "smart" learners. If teachers rely on the belief that learners must possess innate knowledge or have a certain type of mind in order to understand mathematics, it could lead to teachers believing that it relinquishes them from their responsibility for applying methods to teach challenging mathematics (Foss & Kleinsasser, 1996).

The relationship between the four factors can be described as follows: The teacher's view of the nature of mathematics provides a basis and is likely to correspond to a teacher's mental models of the teaching and learning of mathematics (Ernest, 1989) which in turn could be influenced by a teacher's beliefs about learners. For example, the problem-solving view of mathematics corresponds to the view that the teacher is a facilitator, the learner is autonomous and learning is the active construction of understanding through problem solving (Ernest, 1989). So too will the view of mathematics as a Platonist unified body of knowledge correspond to a view of the teacher as explainer, and learning as a reception of knowledge, although with an emphasis on the learner constructing a meaningful body of knowledge. The instrumental view of mathematics is likely to be associated with a transmission model of teaching, where the strict following of a text or scheme is advocated (Ernest, 1989). Each of these views (as demonstrated by teacher behaviour) can be influenced by a teacher's belief about the learner's talent, capacity or ability to learn mathematics successfully. If a teacher, for example, believes the learner does not have the (intellectual) ability or talent to learn mathematics in a problem solving manner (by actively constructing own knowledge), the teacher may be tempted to teach mathematics as a set of rules and procedures (instrumentalist view). Learning would then be deemed successful if learners solve problems using the learned rules or procedures (content focused with an emphasis on performance) and present solutions in a fixed format (Leung, 1995).

Teacher attitude

Teachers' attitude is the third factor noted by Koehler and Grouws (1992) and consists of a teacher's attitude towards mathematics and the teaching of mathematics, with the addition of a teacher's attitude towards students. The teacher's attitude to mathematics (for example enthusiasm and confidence) itself may affect the teacher's attitude to the teaching of mathematics, which in turn has a powerful impact on the atmosphere of the mathematics classroom (Ernest, 1989).

Attitude towards students

Attitudes teachers hold regarding students could be attitudes towards individual learners, groups or classes of learners. This could include liking (affection towards learners), enthusiasm to teach these specific learner(s) and familiarity with the culture (e.g. European versus Asian) (Leung, 1995). Teachers are more likely to exhibit more enthusiasm in preparation and presentation of lessons when they are affectionate towards learners than when they are apathetic or indifferent towards these students. This attitude towards student(s) could be formed due to characteristics exhibited by a student or a group of students (such as low socio-economic status, poor discipline, physical appearance or special educational needs e.g. speech difficulties (Dada & Alant, 2002) that teachers personally find "acceptable or unacceptable" or "attractive or repulsive". Penso (2002) reports that prospective teachers explained learning difficulties by focussing on student characteristics, it being convenient "to blame" students, with such remarks as "if only (the students) would listen, they would understand". (p. 34)

Social context

Fennema and Franke (1992) note that teacher's knowledge and beliefs are held within specific contexts as a result of the dynamic interaction of the factors involved in the learning process (Penso, 2002). It could be suggested that due to the relationship between a teacher's knowledge, beliefs and attitude, the teacher's behaviour, as influenced and defined by knowledge, beliefs and attitude, is context specific. Context can vary enormously (Turner-Bisset, 2001) and within a given context, a teacher's knowledge combines with his/her attitude and beliefs about teaching and learning, mathematics and students to create a unique setting that drives classroom behaviour. The teachers' attitude, beliefs and views as well as their knowledge as demonstrated by instructional behaviour are subject to the constraints and contingencies of the social and school context (Fennema & Franke, 1992). Within a given context, teachers' knowledge interacts with their views and beliefs and combines with their attitude to drive classroom behaviour (Fennema & Franke, 1992).

CONCLUSION

Reflecting on the completeness of the presented framework, the author is open to the possibility that other components could be added in future. An aspect that the framework did not include was a possible language differences between the teacher and students (e.g. the teacher teaching or students receiving instruction in their second language). This aspect as well as the applicability of this framework in related subjects (e.g. science) and in other disciplines (e.g. Language instruction) could be areas for future research. This framework could also be applied to the in-service and pre-service training of mathematics teachers to investigate the strength of the interrelatedness of the components.

If educational reform is to be successful, colleges and universities may need to evaluate the suitability of the mathematical topics of courses targeted at mathematics teachers (Kanes & Nisbet, 1996). These courses should ideally encompass elements to improve knowledge (subject content, pedagogy (Quinn, 1998) and curriculum knowledge) but also make teachers and prospective teachers aware of their own beliefs and attitudes as well as the role and impact of their beliefs and attitudes towards mathematics, learner characteristics and the teaching and learning of mathematics within a specific social context. Changes are required in how teachers learn and in the opportunities to learn with course content focussing more on the synergy between knowledge, attitude and beliefs.

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