MODESTUM
EURASIA Journal of Mathematics, Science and Technology Education, 2023, 19(4), em2247
ISSN:1305-8223 (online)
OPEN ACCESS
Research Paper
https://doi.org/10.29333/ejmste/13066

Research on visualization in probability problem solving

Michael Zorzos [1*], Evgenios Avgerinos [1]

[1] Department of Primary Education, Faculty of Social Sciences, University of the Aegean, Rhodes, GREECE

Received 13 December 2022 ▪ Accepted 03 March 2023

Abstract

The importance of visual representations in education and mathematics is well known. Probabilities are a domain in mathematics that uses many visual representations since their theory consists of a variety of diagrams and graphs. In the past, many studies have shown that the use of various representations in teaching probabilities can greatly improve learning. Of course, the use of a visual representation or a visual tool when teaching or solving an exercise can have a variety of roles. The present work is based on the ancillary and informative role of the image. The following research examines the extent to which students, by solving a probability problem, have the need to use a visual representation or image. Additionally, the differences in student performance are investigated, given the role of the image in the activity. This knowledge can improve the teaching methods of probabilities and, with their appropriate use, school textbooks. The results show that there are more perspectives but also needs for research on the use of visual representations for the teaching of probabilities. The presence of the image works as a motivating factor for children to solve a problem with a lack of probabilistic knowledge, but the role of the image affects their final performance.

Keywords: teaching, probability, visualization, primary school

INTRODUCTION

In various domains of mathematics, charts, graphs, and, in general, images and other visual representations are considered essential for an effective understanding of methods and concepts (Guzman, 2002). Of course, visual representation in mathematics has evolved from the first representation used for didactic purposes (Kadunz & Yerushalmy, 2015). Over the years, technological developments have improved visual media and the details in visual representations (Batanero et al., 2016). All these factors influence the way one makes decisions and deals with problems (de Oliveira, 2021). Consequently, mathematics can highlight the importance and usefulness of optics.

This phenomenon of using images and various visual media in the teaching process has evolved and improved teaching practice for many years. Many teachers and researchers around the world are investigating the phenomenon and have made great discoveries in the advantages of using visualization in teaching. Moreover, as the research has shown, the power of an image cannot be compared to the long description provided by a text. Indeed, images are often and almost automatically created when students or teachers themselves try to understand or explain a situation by creating an imaginary representation (Clements, 2014). In addition, past research proved that visual stimulation aims both at creating mnemonic rules and at easier understanding of the new meaning (Munzner & Maguire, 2015).

Probabilities are slowly being introduced into school mathematics curricula. However, children from a young age, even without basic probability knowledge, can respond to probability activities by creating a host mechanism for understating logically probabilistic definitions (Konold, 2002). For this reason, probabilities can be mentioned as one of mathematics' domains that promotes the use of visual representations. Of course, students' reasoning can take many different forms (Loomes & Pogrebna, 2014), but this does not concern the present investigation. Much research has shown the advantages of representations in teaching practice, yet the field is far from discovering the rank of influence of representations on student learning. In addition, past

© 2023 by the authors; licensee Modestum. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/).
✉ pred18003@aegean.gr **(*Correspondence)** ✉ eavger@aegean.gr

Contribution to the literature

- The present research fills a gap in the literature regarding the influence of the role of a visual representation in a probability problem.
- Research results show that an image can help young students develop probabilistic reasoning to solve a problem.
- The role of the virtual representation affects the students' performance, but it is also a motivation for solving the exercise.

literature lags in two aspects, which this research seeks to cover. More specifically, the influence of the role of the image in solving a probability problem has not been studied. Moreover, there is no research in young students with the advantage of a lack of probabilistic knowledge; this is considered an advantage for the present research because in dealing with the probabilistic problems in the questionnaire, students had to rely on their perceptive ability and the image. Consequently, this research was structured with questions in an attempt to compensate for the lack of international research about visualization and probability problem solving. In the national literature, problems are considered encounters in which the answer and solution pathway are new or unique to the problem solving and hence, not clear at first, invoking a need for sense making (Rahman, 2019). The concept of "problem" in this work is an activity or exercise that deals with probabilistic processes. Of course, the terms "activity" and "exercise" would refer to something known (like a procedure) from previous learning and remain only to be enacted to derive the solution, but children at young ages have no probabilistic knowledge, and for that reason, every probabilistic activity could be described as a problem for them.

The advantages of visualization in probability are well known and are not objects of this research. The present study investigates the extent of advantage or influence that students gain by solving a probability activity in the presence of an image. Therefore, the study was designed based on the following research questions:

1. Do young students have the willingness and ability to use an image to solve a problem that is outside their domain of knowledge?
2. Are pupils' performances differentiated in solving an exercise with data given via a picture or by description in text?
3. Does the role of the image influence the probability problem solving?

Finally, this research will try to demonstrate the extent to which a young child can rely on a visual representation to solve a probabilistic activity.

Representations and Visualization

Representation is traditionally understood as an imaginary or real image that may be influenced by someone's culture or experience and represents a meaning or a situation (Hall, 2013). Often, representations are used by people to store information or understand a concept (Hähkiöniemi, 2006). For that reason, the importance of visual representations in everyday life offers high benefits. A moving or nonmoving image has the power to allow the interpretation of a phenomenon much more simply than a whole text. Visualizations and new technology allow learners to embody experiences and promote a deeper understanding (Schuman et al., 2022). It is a fact, of course, that mathematics, as a demonstrable science, does not require confirmation by a representation. Additionally, the results of Sung's (2017) study, in addition to the direct application of representations in mathematics teaching, demonstrate significant correlations among visualization and problem-solving ability. This reveals the positive effects of visualization on students' problem-solving ability. Consequently, visualization, and therefore representations, help popularize or simplify a complex theory so that it may be understood (Phillips et al., 2010).

Studying the international literature, one can find many classifications of representation (Hähkiöniemi, 2006). For instance, a representation can be internal, external, or blended (Rau, 2020). An internal representation refers to the mental construction of an image; on the other hand, the external representation deals with a physical construction (Rau, 2020). Internal representation is more common and occurs almost automatically in a child's mind when they read or hear information about a new object and create a way of understanding or remembering it (Konold, 2002). External representation is the depiction of the image on paper, a screen, or a table. These kinds of representations can be more easily discussed and shared with all students and for that reason are those that can be used by a teacher. Finally, blended representation is a combination of both of the above. However, each mode of representation exhibits different learning advantages (Rau, 2020). Another classification could be in active images, symbolic images, virtual images, graphics and more, but this is not relevant to the present research. Importantly, a representation is a tool for thinking, understanding, and storing information.

The concept of 'visualization' deals with the formation of a real or imagined picture for the needs of understanding and memorizing new knowledge (Munzner & Maguire, 2015). In other words, it is a process of forming an image and using it to understand and finally to solve a problem. This has the immediate effect not only of visual or spatial perception but also of an intuitive understanding of a problem (Astafieva et al., 2022). This is also confirmed by previous research, and in fact, visualization contributes significantly to enhancing students' creativity (Cioca & Nerișanu, 2020). In addition, in many cases, the use of optical media and new technologies gives new opportunities in teaching and learning.

As various studies have shown, visualization helps readers understand phenomena for which they have no frame of reference (Phillips et al., 2010). A typical example in science is the phenomenon of a microcosm, which one can see only through an image. Visualization also activates neurons that lead to faster and more effective learning. Moreover, it has been demonstrated that 50% of brain neurons are related to vision (McCormick et al., 1987). These findings justify the attitude of the National Science Foundation of America "visualization in scientific computing", which insists on the use of visual representations in teaching. Importantly, the utility of optical thinking in mathematics is increasingly recognized and has aroused interest in further research (Kadunz & Yerushalmy, 2015).

The visualization procedure in mathematics education is evidenced by theories and examples of the past, which characterize learning as a process in which students use their internal mental ability to construct representations that mirror the mathematical structures they are being taught (Minarni et al., 2016). In other words, visualization is a method used to represent mathematical theory (Clements, 2014). For example, a student can think of a shape to remember the Pythagorean theorem. Visualization use appears in many mathematical activities and is directly linked to the history of science (Guzman, 2002). In addition, it has been shown that the use of visualization in mathematics teaching has helped students solve problems (Binder et al., 2020). This method mainly helps students understand and organize the concepts of a problem (Binder et al., 2020). For instance, in geometry, creating a very good and detailed figure greatly contributes to the solution of the problem. Another example of offering visual representations in mathematics could be in algebra, where students in sixth grade can better understand the concept of equation through an example with a scale. For that reason, representations are often constructed and used by the teacher with the aim of promoting understanding (Lowrie et al., 2019). In fact, the students themselves admit the usefulness of visual representations in the teaching process (Lowrie et al., 2019). The thinking process follows the following stages:

(a) initially, the learning object is accepted by the senses,

(b) then the concepts are processed by the brain, and finally,

(c) these concepts are expressed with a representation (Kurniawan et al., 2018).

Elementary students often need visualization to facilitate their understanding (Kurniawan et al., 2018). However, the technique of visualizing a situation is influenced by the social and cultural nature of the activity. In fact, it often leads to recommendations that contradict the goal of learning and understanding (Minarni et al., 2016). Moreover, in many cases, students do not specialize in mathematics, and they cannot visualize the subject, even though this is a necessary step (Shvarts & Chumachenko, 2011). This may happen due to the lack of relevant education (Presmeg, 2006). After all, one's ability to visualize is a result of his or her natural engagement and his or her cognitive load about the subject or his or her perspectives of haptic encoding (Rau, 2020).

Of course, a visual representation may not always be successful. For a visual representation to be good and appropriate, its use must be clear, as well as the knowledge it wants to promote. Furthermore, the success of a visual representation is directly related to the shared experiences of teachers and students (Binder et al., 2020). The teacher of the class knows to choose the appropriate representation that will be understood by most of his or her class in order for learning to be achieved. For example, a teacher who has talked in his class about space and the motions of the planets can use the representation of the motion of the earth around the sun to introduce the ellipse. On the other hand, if a teacher has not talked to his class about this subject, it will be difficult for the students to understand with the same example.

Visual Perception in Teaching Probabilities

The concept of probability seems too abstract for many students. For this reason, mathematical problems based on real ideas and situations are used to teach probability (Lukáč & Gavala, 2019). Examples include gambling, sports competitions, and drug effectiveness testing. After all, probabilities are one of the main domains of mathematics that are directly related to problems in everyday life (MacGillivray, 2018). Their theory has a characteristic impact on various sciences but also on everyday activities, such as decision making (Bonnett & White, 2018). In fact, this theory is characterized by a variety of diagrams, and thus, probabilities are one of the predominant areas of mathematics that can be approached by visual representations. Typical examples are tables and tree diagrams that help students better understand the possible cases and the probability of each case (Binder et al., 2020). In fact, Binder et al. (2020) showed in their research that frequency nets often show better results in probabilistic problems. Another example is the tangram, which can be used to understand the multiplication rule in solving probability problems (Lukáč & Gavala, 2019). To the above examples of visual representations in probabilities, one could also add the unit square with natural frequencies (Böcherer-Linder et al., 2016). This representation was seen in the research of Böcherer-Linder et al. (2016), who found that it is more efficient for calculating conditional probabilities.

Fischbein believed that intuitive models are the starting point for building knowledge of probabilities (Stavropoulou & Gagatsis, 2006). This is a didactic advantage, as visualization allows for a better understanding of the probabilities. An image often leads students to probabilistic reasoning that helps students overcome potential math challenges by connecting math concepts to an intuitive visualization (Binder et al., 2020). Moreover, many past studies have shown that representations in probability activities benefit students of all ages (Sirivianou & Valanidis, 2010). This happens because a visual representation in probabilities usually affects students, similar to experiments in physics classes (Gao et al., 2020).

In 2000, Pratt, based on various research results, stated that the existence of multiple representations in probability teaching is particularly important. These can be either helpful as exercises or for decorative purposes (Stavropoulou & Gagatsis, 2006). Tables, pictures, graphs, diagrams, and other visual tools help students to better understand the experiments and the theory. A direct result of this is the positive impact on problem solving (Stavropoulou & Gagatsis, 2006). For instance, it is easier for a student to obtain information from a pie chart to answer a variety of questions than to read or listen to a long text. The advantages of this method are in the faster understanding of the exercise, its less tedious presentation, and the speed of its solution (Astafieva et al., 2022).

Pictures in Mathematics Textbooks

In mathematics, the term “image” refers to any external representation, such as a shape, table, or a graph, that indirectly or directly helps to solve a problem. Images, depending on the act they perform, influence the understanding of mathematical problems. There are many who argue that the power of an image activates resolution processes and thinking strategies (Phillips et al., 2010).

The presence of images in school math textbooks is common and exhibits a variety of actions. In many cases, the dominance of images in a textbook helps to determine its friendliness to the reader (Kress & van Leeuwen, 2010). This fact justifies the interest in research on the beneficial presence of images in text (Carney & Levin, 2002). Furthermore, an image can help the reader to understand the concept of the text or can add extra information to the subject.

The most important types of images, as proposed by Theodoulou and Gagatsis in 2003, are the following:

(a) decorative images (which offer nothing more than a decorative element),

(b) ancillary images (presenting the problem or part of its data),

(a) ancillary-organizational images (to organize problem data and assist in strategy solving), and

(b) informational images (to provide important information for problem solving).

Consequently, one could divide images that have direct use with mathematical activities into two categories (Phillips et al., 2010). The first category is of the images that appear as complementary to the exercise, and the second category is of images that are an indispensable tool for extracting data in the problem (Van Garderen, 2006). Consequently, in connecting these two categories with the above purposes, one could say that the first two purposes belong to the first category (decorative and ancillary images), while the third and fourth purposes of the images belong to the second category (ancillary-organizational and informational images).

METHOD

The present study was conducted on elementary school students from the 4th through the 6th grade and more specifically at the ages of eight to 12 years. The choice of the specific age range was made due to the lack of basic knowledge of probability. In this way, the role of the image in the solution of a probabilistic activity by the students is strengthened to a certain extent. After all, one of the main purposes of this research is to determine the level and disposition of students’ visualization on probability problems. Concerning the usefulness of the images mentioned in the theoretical framework of the study, the present study deals only with the cases of direct correlation of the images with the activity, and more specifically, with cases of ancillary and informative representations.

The research approach followed by the authors was the quantitative approach since a large-scale survey was required with data collection through a questionnaire. For this reason, the research design was a rigorous sequence of steps. The sampling method used by the researchers was a two-stage sampling from Greece and, more specifically, from Rhodes Island elementary schools. In particular, researchers chose with random sampling the elementary schools, which they had to approach. Then, the researchers, after consultation with the school principals, chose through convenience sampling the classes that would eventually take part in the research. These classes were chosen because of their specific characteristics, such as consistency of students, level of students, and parents' consent for the children's participation in the anonymous survey. Of course, as the size of the sample shows, the expectations of the researchers were surpassed, and the sample reached 346 students. More specifically, there were 126 4th grade, 100 5th grade, and 120 6th grade elementary students. The survey data were collected through a test, and the statistical analysis of the data was performed using the SPSS statistical package. The type of research analysis is descriptive analysis because the aim was to describe, measure and highlight the frequencies and dispersion of the data collected. The research period lasted approximately two months, and the process of selecting the sample and distributing the questionnaires took almost two months.

The tests were constructed by the researchers for the needs of the research, considering the students' level in probabilities. This played an important role, both in the language used in the test and in the selection or presentation of the activities. In fact, the researchers tried not to expand the test to prevent it from becoming long and tedious. This test was shared with the respondents and was accompanied by the personal presence of both the researchers and the teachers of the classes. This helped quite a bit to prevent random choices in the answers. In addition, it is worth noting that all tests that were considered to involve random responses or were completed significantly faster than the average test completion rate were excluded from the total number of 346 participants presented in the current work.

To ensure the validity of their measurement tool, the researchers first assessed the content of the activity questions to be conceptually relevant to the study's objectives. Then, they submitted the measurement tool to an evaluation by a specific group related to the object of the research, and finally, through a pilot application, they compared it with other measurement tools of similar variables. The reliability of the questionnaire was measured by Cronbach's alpha. In this way, the internal consistency and homogeneity of the questionnaire were checked. The value of this coefficient was 0.735, a generally acceptable value.

The questions that made up the test consisted of three probability exercises–problems. As mentioned above, the problems set in this task deal with probabilistic activities that students had not encountered before. The choice of activities was considered particularly difficult in this research. First, the scientific community has shown through research that the process of solving probabilistic problems does not comprise a fixed number of sequential stages (Zahner & Corter, 2010). For this reason, the present study was based on the most common sequence of stages, which is text comprehension, followed by math problem representation and finally strategy formulation (Zahner & Corter, 2010).

According to the above and also the purpose of the research, the three activities that make up the research questionnaire were selected. The selection of the exercises was made with the basic assumption that they were exercises that do not need special probabilistic knowledge to solve them. Specifically, the first two exercises were identical and dealt with equally possible chances, following the classic pattern of examples of various textbooks (Andreadakis et al., 2017). These were almost the same, but one of the two exercises presented the data in text form, and the other consisted of an informative picture. The visual representation in the second activity therefore had an auxiliary role in the exercise and presented the data of the exercise in a visual way. In these two activities, it was expected to observe in which of the two the student had a better performance in terms of his or her correct answers but also which of the two the student engaged in more, a fact that would be observed when the respondents left an answer blank. All activities were chosen to reduce the level of complexity and complex strategies to solve them. Consequently, any interference positive or negative in the exercise should have been a result of the visual representation.

As a result, in activities 1 and 2, which were the same, better performance in the one with the presence of a picture led to a positive consequence of the representation in solving the exercise. Then, an exercise followed with a picture of a wheel of fortune. This activity included an image of a pivotal factor for presenting data to solve the problem. This activity was slightly more difficult than the previous two. The visual representation in this activity had a more important role since it was an informative image that gave the data to the students instead of the text of the exercise. The role of the visual representation in the questionnaire was to highlight the students' ability to extract information from an image and use it to solve the exercise.

Descriptive statistical analysis of the survey data yielded the frequencies and percentages of the results. Additionally, to carry out a comparative analysis between the responses of the respondents of different ages, a normality test of each variable was carried out. The results of the normality test showed that no variable followed a normal distribution, as the significance levels of both the Kolmogorov-Smirnova and Shapiro-Wilk tests were smaller than 0.001. This result led to a nonparametric analysis of variance with the Kruskal–Wallis Test and grouping variable the age of the participants, in order to examine whether there were statistically significant differences in the responses of different ages.

Below are probability activities of the questionnaire.

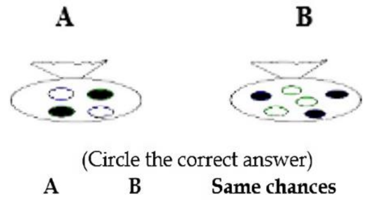

Figure 1. Bag with balls (Source: Authors' own elaboration)

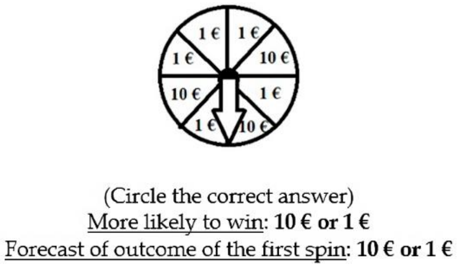

Figure 2. Wheel of fortune (Source: Authors' own elaboration)

Exercise 1

I have two bags with balls. In the first bag (bag A), there are two white balls and two black ones. In the second bag (bag B), there are three white and three black balls. From which bag am I most likely to get randomly a black ball? (Circle the correct answer).

A **B** **Same chances**

Exercise 2

I have two bags with balls. In the first bag (bag A), there are two white balls and two black ones. In the second bag (bag B), there are three white and three black balls. From which bag am I most likely to get randomly a black ball? (**Figure 1**).

Exercise 3

Below is a wheel of fortune. Which of the following amounts is more likely to come up if we spin the following wheel once? What do you think will come up in the first turn? (**Figure 2**).

RESULTS

The analysis of the data collected gave the results presented below. The results are presented in such a way as to answer the research questions raised in the introduction of this work and to favor the presentation of further interesting outcomes that may emerge from the research as secondary findings. Consequently, the researchers found it necessary to present the research results in tables. This can make the reading easier and the comparison more direct.

It is clear from **Table 1** that the correct answers were in small percentages. The important thing in this table, of course, is the difference between the responses to the two exercises. As seen, 41 of the students (11.8%) answered correctly to the first exercise, which had no picture. From the other, 33 of the students (9.5%) answered correctly to the exercise with the image. In contrast, in the first exercise, there were 82 (23.7%) blank answers, and in the second exercise, there were 61 (17.7%) blank answers. Of course, in both exercises, there is a similarity regarding the large percentages of incorrect answers.

Table 1. Answers to the first two exercises

	A	B	Same	Blank
Exercise 1	71 (20.5%)	152 (43.9%)	41 (11.8%)	82 (23.7%)
Exercise 2	96 (27.7%)	156 (45%)	33 (9.6%)	61 (17.7%)

Table 2. Answers to the third exercise

	10 €	1 €	Blank
More likely	77 (22.3%)	146 (42.2%)	123 (35.5%)
First spin	97 (28%)	118 (34.1%)	131 (37.9%)

Table 3. Kruskal-Wallis test with age as grouping variable

	Ex. 1	Ex. 2	Ex. 3a	Ex. 3b
Kruskal-Wallis H	26,227	13,031	29,824	24,182
df	6	6	6	6
Asymptotic Sig.	<0.001	0.043	<0.001	<0.001

Table 2 shows that the picture helped the most students, since most of them correctly answered both questions of the exercise (42.2% and 34.1%, respectively). However, even in this exercise, there were high rates of incorrect or blank answers.

Table 3 shows the results of the univariate analysis of variance given the age of the participants. As seen, there are statistically significant differences at the 0.01 level of significance of the responses of respondents of different ages in exercises 1 and 3. In exercise 2, the differences were significant at the 0.05 level.

DISCUSSION

Summing up all the above, the participants answered a short questionnaire with three probabilistic activities. These activities were mathematical problems for the students, as they had limited knowledge of probability theory, which is a fact that can connect the student's required reasoning for the exercise with the problem-solving process. The results of the research showed that in the first activity that was not accompanied by an image, the students had 82 blank answers (23.7%) and only 41 correct answers (11.8%). In the second activity, it seems that the presence of the image influenced the students' responses. More specifically, the blank answers decreased to 61 (17.7%), while the correct answers also decreased to 33 (9.6%). Then, in the third activity, it seems that the majority of the students answered the question with the informational image correctly, as there were 146 correct answers for the first question (42.2%). Regarding the second question, where there is no specific correct answer, the percentage of students predicting the most likely result as the first result seems to decrease (118 answers, 34.1%), while the blank answers and the estimates that the result will be 10€ slightly increase. Finally, the statistical control of the answers of the students at different ages shows that there are significant differences in the answers of the age groups in all the problems of the questionnaire.

The literature supports the fact that one cannot overlook the power of visualization in interpreting or understanding a phenomenon. For this reason, the utility of visualization in everyday life and especially in mathematics does not need to be verified by the present work. On the other hand, the present work shows that visualization can allow more perspectives and advantages for people with a low level of probabilistic knowledge. This may be because for a child trying to solve a probabilistic problem, the image is the only source of knowledge for creating a "logical" understanding of the subject. Moreover, many earlier studies have demonstrated the benefits of visualization in the probability teaching process (Presmeg, 2006). The results of the present research agree in part with this argument by showing that more children tried to solve the exercises with the images, while there was a marked increase in blank responses for the exercise without the picture. In addition, the results extend the literature data to the knowledge of the rank of this achievement. More specifically, it seems to some extent that the presence of the image confused students, a phenomenon that can be justified because of the lack of knowledge of most children both in visualization and in probabilities. This is also confirmed by the research of Presmeg (2006) and Shvarts and Chumachenko (2011).

Probabilities provide great ground for highlighting and using representations due to the variety of charts and tables they use. Through visual representations, strategic thinking processes are activated, leading the student to solve an exercise (Phillips et al., 2010). In 2002, Konold (2002) showed that children usually look for a visual tool on their own to help themselves understand probability concepts. In this research, visual tools were given to the students in two of the three exercises. However, the question raised was about the use of the given representation. Here, the results show that there was a lack of correct use of the picture information since there were several errors and several blank answers in the activities. Of course, in this is considering the role of the lack of probabilistic knowledge of the sample and the limited experience with stochastic problems but also must consider the research of Presmeg (2006), which states that the learner must be familiar with the use of an optical media in order to decode an image.

The presence of an image in an exercise does not always provide the same facility. As Theodoulou and Gagatsis (2003) showed, the presence of an image in a mathematics exercise has different roles, and for that reason, they could provide different facilities. In the research, it was seen that the role of the image in the exercise influenced both the students' correct answers and the blank answers. The correct answer to each activity is the answer that is scientifically acceptable. The student's correct response to the question and the final selection of a scientifically correct answer is interpreted as success in problem solving, while an incorrect answer is interpreted as a failure. In addition, for the correct answers for the activities, it was assumed that the children were able to use probabilistic reasoning to solve the exercises. The presence of the image was to aid the student's course of reasoning.

The influence mentioned above can be seen by comparing the correct answers and the blank answers for the two activities with pictures from **Table 1** and **Table 2**. The informative image led more children to answer but fewer of them were correct. On the other hand, the ancillary image led fewer children to answer, but more of them were correct. This is explained to some extent by students' lack of experience in dealing with probabilistic problems and situations. After all, the informative image had a greater role in solving the exercise, so its use by the students was inevitable, and consequently, their lack of experience and knowledge was more obvious. On the other hand, one could argue that several children saw the significance of the image in the exercise as decorative rather than helpful. As a result, fewer children engaged in the exercise, but in the end, the presence of the image helped them to answer the problem better.

The fact that the children tried to engage more in the exercise that included the image indicates that the presence of the visual element in the exercise gave the children a greater motivation or courage, contributing to their feeling that they could solve this exercise better. This means that the image helped the children to some extent overcome the lack of knowledge in this field of mathematics and pushed them to try to solve the exercise. Of course, they had neither the experience nor the knowledge to demonstrate great percentages of correct answers. However, this makes us believe that with the appropriate teaching about how to use the images, young children could use probabilities. Moreover, it is worth noting that there were expected high percentages of blank answers, as children at this age had not been taught probabilities. Nevertheless, their intuitive perceptions and visual representations, as demonstrated in the present work, show very interesting results in their performance.

It is worth mentioning here some of the limitations of the study. One of the limitations was the lack of research on students without a cognitive background in probability. This made it difficult for the researchers when designing the questionnaire and choosing the activities, because they had to be very simple so as not to affect the reliability of the measurement. Moreover, there was a lack of research using similar tools to further test the validity of the questionnaire. Finally, it would be interesting for future research to repeat the research with more complex questions and with tree diagrams, which could not be used in the present research due to the young age of the students.

CONCLUSION & RECOMMENDATION

Probabilities are slowly gaining ground in the primary school curricula of many developing countries, and visualization is a way to attract interest and teach probabilities to young children. The present research shows that the presence of a visual representation in the exercise influenced both the correctness of the answer and the percentage of blank answers. Students' lack of knowledge about the theory of probabilities, as well as their young age, creates an unprecedented and unknown situation for students. Consequently, the reversal of this situation in the problems given to them is far from their cognitive fields, and only the presence of the image could be seen as helpful. The image, therefore, seems to have been both a motivating factor for the students to engage more with these exercises and a helpful factor that directed the students' thinking and perception toward a logical path for solving the problem. Another important finding of the research is that visual representation affects students' way of thinking. As mentioned above, the students who took part in the research have no probabilistic knowledge at all. Consequently, their reasoning for solving the exercise was only influenced by the image. The image directs young students to think in a way that is outside of their cognitive domain and prompts them to think probabilistically. All these factors lead to the conclusion that visualization could become a great tool for teaching and learning probabilities.

Visualization has a great impact on probability problem solving, both regarding the role of the image and the rank of its achievement. The results of the present research could contribute to the improvement of teaching in probability but also mathematics in general. More specifically, the findings of the study can be used both when designing lessons in probability and when writing school textbooks. In addition, they can make a significant contribution to the problem-solving process since they reveal that through an image, the reasoning process of the students can be organized to some extent. This, combined with the help of the teacher, can improve the final performance of the students.

Finally, the new knowledge offered by research on the usefulness of the role of image in students with a low cognitive level of probability opens new horizons in education and teaching. This can feed academic knowledge and lead to the development of new teaching methods, which in combination with the visualization capabilities of technology can help to achieve better learning outcomes. As a continuation of the research, it would be interesting to study how an image can change students' reasoning and decision making in a stochastic problem. In other words, the extent to which visual representations influence students' final choice and whether this influences their critical perception could be studied. Furthermore, outside the purposes of the research, a need emerges for further visualization practice in schools because the children showed willingness to use the image but were deficient in the abilities to extract the necessary information from it. This could be an interesting subject for future studies.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: This study could be considered as an extra part of a doctoral thesis. In that case, should be noticed that the implementation of the doctoral thesis was co-financed by Greece and the European Union (European Social Fund-ESF) through the Operational Program "Human Resources Development, Education and Lifelong Learning" in the context of the Act "Enhancing Human Resources Research Potential by undertaking a Doctoral Research" Sub-action 2: IKY Scholarship Program for PhD candidates in Greek universities.

Acknowledgments: The authors would like to thank the teachers of the classes who allowed and helped the authors to conduct the research. The authors would also like to thank IKY Scholarship Program for PhD candidates in Greek universities.

Ethical statement: Authors stated that the study was provided by University of the Aegean. A study-specific approval by the appropriate ethics committee was not necessary, as in Greece it is allowed to carry out studies on teaching by pedagogic departments in schools, with the agreement of the teachers of the classes. Moreover, the research was consistent with national and international standards for conducting research with human subjects.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Andreadakis, S., Katsargiris, B., Papastavridis, S., Polyzos, G., Sverkos, A., Adamopoulos, A., & Damianou, C. (2017). *Algebra and probability elements: High school first class*. ITYE-Diophantus

Astafieva, M., Bodnenko, D., Lytvyn, O., & Proshkin, V. (2022). The use of digital visualization tools to form mathematical competence of students. In *Proceedings of the 16th International Conference on ICT in Education. Research, and Industrial Applications*. Borys Grinchenko Kyiv University.

Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). *Research on teaching and learning probability*. Springer International Publishing.

Binder, K., Krauss, S., & Wiesner, P. (2020). A new visualization for probabilistic situations containing two binary events: The frequency net. *Frontiers in Psychology, 11*. https://doi.org/10.3389/fpsyg.2020.00750

Böcherer-Linder, K., Eichler, A., & Vogel, M. (2016). *Proceedings of* 13^{th} *International Congress on Mathematical Education*. Springer.

Bonnett, L., & White, S. (2018). May the odds be ever in your favor. *Teaching Statistics, 40*(3), 94-97. https://doi.org/10.1111/test.12162

Carney, R. N., & Levin, J. R. (2002). Pictorial illustrations still improve students' learning from text. *Educational Psychology Review, 14*(1), 101-120.

Cioca, L., & Nerişanu, *R.* (2020). Enhancing creativity: Using visual mnemonic devices in the teaching process in order to develop creativity in students. *Sustainability, 12*(5), 1985. https://doi.org/10.3390/su12051985

Clements, M. A. (2014). Fifty years of thinking about visualization and visualizing in mathematics education: A historical overview. In M. Fried, & T. Dreyfus (Eds.), *Advances in mathematics education* (pp. 177-192). Springer. https://doi.org/10.1007/978-94-007-7473-5_11

de Oliveira, A. (2021). When risky decisions generate externalities. *Journal of Risk and Uncertainty, 63*(1), 59-79.https://doi.org/10.1007/s11166-021-09357-6

Gao, Y., Zhai, X., Andersson, B., Zeng, P., & Xin, T. (2020). Developing a learning progression of buoyancy to model conceptual change: A latent class and rule space model analysis. *Research in Science Education, 50*(4), 1369-1388. https://doi.org/10.1007/s11165-018-9736-5

Guzman, M. (2002). The role of visualization in the teaching and learning of mathematical analysis. In *Proceedings of the International Conference on the Teaching of Mathematics*.

Hähkiöniemi, M. (2006). *The role of representations in learning the derivative*. Department of Mathematics and Statistics, University of Jyväskylä.

Hall, S. (2013). *Representation*. SAGE.

Kadunz G., & Yerushalmy M. (2015). Visualization in the teaching and learning of mathematics. In S. Cho (Ed.), *Proceedings of the* 12^{th} *International Congress on Mathematical Education* (pp. 463-468). Springer. https://doi.org/10.1007/978-3-319-12688-3_41

Konold, C. (2002). Understanding students' beliefs about probability. In E. Von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 139-156). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47201-5_7

Kress, G., & van Leeuwen, T. (2010). *Reading images: The grammar of visual design*. Epikentro.

Kurniawan, H., Sutawidjaja, A., As'ari, A., & Muksar, M. (2018). The thinking process of students in representing images to symbols in fractions. *Journal of Physics: Conference Series, 1028*, 012138. https://doi.org/10.1088/1742-6596/1028/1/012138

Loomes, G., & Pogrebna, G. (2014). Testing for independence while allowing for probabilistic choice. *Journal of Risk and Uncertainty, 49*, 189-211. https://doi.org/https://doi.org/10.1007/s11166-014-9205-0

Lowrie, T., Logan, T., & Hegarty, M. (2019). The influence of spatial visualization training on students' spatial reasoning and mathematics performance. *Journal of Cognition and Development, 20*(5), 729-751. https://doi.org/10.1080/15248372.2019.1653298

Lukáč, S., & Gavala, T. (2019). Interactive learning environment supporting visualization in the teaching of probability. *International Journal of Information and Communication Technologies in Education, 8*(1), 48-60. https://doi.org/10.2478/ijicte-2019-0005

MacGillivray, H. (2018). Real probability and probabilistic thinking. *Teaching Statistics, 40*(2), 37-39. https://doi.org/10.1111/test.12159

McCormick, J. B., Webb, P. A., Krebs, J. W., Johnson, K. M., & Smith, E. S. (1987). A prospective study of the epidemiology and Ecology of Lassa Fever. *Journal of Infectious Diseases, 155*(3), 437-444. https://doi.org/10.1093/infdis/155.3.437

Minarni, A., Napitupulu, E., & Husein, R. (2016). Mathematical understanding and representation ability of public junior high school in North Sumatra. *Journal on Mathematics Education, 7*(1), 43-56. https://doi.org/10.22342/jme.7.1.2816.43-56

Munzner, T., & Maguire, E. (2015). *Visualization analysis & design*. CRC Press Taylor & Francis Group.

Phillips, L., Norris, S., & Macnab, J. (2010). *Visualization in mathematics, reading and science education*. Springer. https://doi.org/10.1007/978-90-481-8816-1

Pratt, D. (2000). Making sense of the total of two dice. *Journal for Research in Mathematics Education, 31*(5), 602-625. https://doi.org/10.2307/749889

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutiérrez, & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 205-236). Sense. https://doi.org/10.1163/9789087901127_009

Rahman, M. M. (2019). 21^{st} century skill "problem solving": Defining the concept. *Asian Journal of Interdisciplinary Research, 2*(1), 64-74. https://doi.org/10.34256/ajir1917

Rau, M. (2020). Comparing multiple theories about learning with physical and virtual representations: Conflicting or complementary effects? *Educational*

Psychology Review, 32(2), 297-325. https://doi.org/10.1007/s10648-020-09517-1

Schuman, C., Stofer, K. A., Anthony, L., Neff, H., Chang, P., Soni, N., Darrow, A., Luc, A., Morales, A., Alexandre, J., & Kirkland, B. (2022). Ocean data visualization on a touchable demonstrates group content learning, science practices use, and potential embodied cognition. *Research in Science Education, 52*(2), 445-457. https://doi.org/10.1007/s11165-020-09951-9

Shvarts, A., & Chumachenko, D. (2011). Representations in the development of mathematical concepts. In *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*. PME.

Sirivianou, Y., & Valanides, N. (2010). Science and mathematics education conference. In *Facilitating authentic learning experiences in science and mathematics* (pp. 131-135). CASTeL.

Stavropoulou, S., & Gagatsis, A. (2006). Static and dynamic representations: the case of probabilities. In *Proceedings 9th Conference of the Pedagogical Society of Cyprus*.

Sung, E. (2017). The influence of visualization tendency on problem-solving ability and learning achievement of primary school students in South Korea. *Thinking Skills and Creativity, 26*, 168-175. https://doi.org/10.1016/j.tsc.2017.10.007

Theodoulou, P., & Gagatsis, A. (2003). An image is worth a thousand words ... what kind of image does it help solve a mathematical problem? In *Proceedings of the 2nd Conference on Mathematics in Secondary Education*. Athens EKPA & University of Cyprus.

Van Garderen, D. (2006). Spatial visualization, visual imagery, and mathematical problem solving of students with varying abilities. *Journal of Learning Disabilities, 39*(6), 496-506. https://doi.org/10.1177/00222194060390060201

Zahner, D., & Corter, J. E. (2010). The process of probability problem solving: Use of external visual representations. *Mathematical Thinking and Learning, 12*(2), 177-204. https://doi.org/10.1080/10986061003654240