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Rise or Resist: Exploring Senior Secondary Students' Reactions to Challenging Mathematics Tasks Incorporating Multiple Strategies

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Learning to solve more challenging mathematics problems using multiple strategies has been promoted by research and is a feature of numerous curricula. Yet teachers have been found to be reluctant to incorporate these tasks into their lessons predicting resistance from students. This study examined 87 senior secondary students' reactions to a challenging quadratics task as part of their teachers' participation in a design-based research project. It sought insights into students' own perspectives on challenge and multiple strategies in mathematics teaching and learning, with the intent of understanding more about the cognitive, affective, and motivational aspects of this teaching approach. In responding to open-ended prompts, the students reported diverse but generally positive reactions related to their interest in the task and its relevance for their learning. Two thirds reported that they liked learning strategies from peers, and two thirds liked learning multiple strategies from their teacher. Implications for considering secondary students' preferences for learning, engagement, and motivation in mathematics lessons are discussed.

Keywords: challenging mathematics tasks, multiple solutions, achievement goals, engagement; motivation, quadratic functions

INTRODUCTION

Being able to reason and think critically by evaluating different ways to solve a problem is an important skill in today's world across many domains. In mathematics, learning multiple strategies for solving problems has been promoted by research and is incorporated in some high-performing international education

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systems. It is an important feature of challenging tasks, defined in this study as those tasks that "require students to process multiple pieces of information simultaneously and make connections between them, and for which there is more than one possible solution or solution method" (Sullivan et al., 2014). Many curriculum and education policy documents advocate the use of multiple-strategy tasks (e.g., Australian Curriculum Assessment and Reporting Authority, 2009; National Council of Teachers of Mathematics, 2000; Woodward et al., 2012). A number of theoretical models for mathematical problem solving and thinking incorporate them (e.g., Fraivillig, Murphy, & Fuson, 1999; Hufferd-Ackles, Fuson, & Sherin, 2004; Stein, Engle, Smith, & Hughes, 2008; Zazkis & Leikin, 2009). Silver and colleagues described their value for connective learning:

> *It is nearly axiomatic… that students should have experiences in which they solve problems in more than one way. Doing so has the potential advantage of providing students with access to a range of representations and solution strategies in a particular instance that can be useful in future problem-solving encounters. Moreover, different solutions can facilitate connection of the problem at hand to different elements of knowledge. (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005, p. 288)*

Although studies on younger students learning multiple strategies are quite prevalent in the literature there is an increasing interest in how they are or might be used at secondary levels of schooling. The pervasiveness of teaching with multiple strategies in different education systems is not well understood (Lynch & Star, 2014). It is suspected that to utilise multiple strategies, suitable teaching approaches at secondary levels will differ from those at primary levels. There is not only the need to consider effective approaches for learning at secondary levels (Sullivan & Murnane, 2014) but also how to support their sustainable use by secondary teachers. Research has highlighted the constraints teachers can experience when trying to increase the task demand on their students by providing opportunities to solve challenging problems and explore different solution methods (Swan, 2007; Tzur, 2008).

This article discusses the findings of a sub-project within a larger project investigating the potential of challenging tasks for effective learning by examining teachers' experiences of posing such tasks, strategies for helping students persist, the effect on students' learning, and students' responses to them[1]. The sub-project

State of the literature

- Challenging mathematics tasks, which encourage students' development of multiple strategy use, have been demonstrated in studies as effective for conceptual learning.
- Studies have found that secondary teachers are reluctant to use challenging tasks because they experience resistance from students or pressure to simplify the task; they are also reluctant to teach multiple strategies because high-achieving students are perceived as being bored by them and low-achieving students as being confused by them.
- Limited research exists on students' perceptions of challenging tasks in relation to their motivation and engagement, since studies have focussed more on teachers' views.

Contribution of this paper to the literature

- Novel and challenging tasks that secondary students viewed as relevant to their learning goals seemed to promote situational interest, engagement, and a positive affective response.
- High-achieving students who have experienced cognitive improvement on challenging tasks in an environment emphasising progress rather than performance in mathematics seemed spontaneously to evidence mastery goals rather than performance goals.
- Some high-achieving students expressed the desire to understand multiple strategies for solving one problem simultaneously, rather than learning about multiple strategies to choose one to adopt from among them. This finding is at odds with studies of teachers' reasons for teaching multiple strategies.

[1] The *Encouraging Persistence Maintaining Challenge* project, funded through an Australian Research Council Discovery Project (DP110101027) and involving collaboration between Monash University and

focused on senior secondary students' reactions to a challenging task after having experienced a number of them throughout the year, and examined their views on challenge in general and on learning multiple strategies. The central research questions for this study were:

1. What do senior secondary students focus on in describing their reactions to challenging tasks and their views on learning multiple strategies?
2. How do students' perspectives relate to affective, cognitive, and motivational aspects of their mathematics learning?

The following section provides details on the context for the research and discusses theoretical perspectives from the literature, which provided a framework for the study.

RELATED RESEARCH AND CONTEXT

In the past, learning mathematics was viewed as an essentially cognitive process, but more recently the affective dimensions – emotions, attitudes, beliefs, and values – are being considered more seriously as important influences on learning (DeBellis & Goldin, 2006). Researchers examining intellectual functioning from different paradigms nonetheless advocate the integrating the *cognitive* domain with the *affective* and *motivational* domains (Dai & Sternberg, 2004). This study examined the students' expressions of their views about challenging tasks and multiple strategies to see how they might relate to cognitive, affective, and motivational aspects of their learning. The following sections provide details on each of these three aspects.

Cognitive aspects of learning with challenging tasks incorporating multiple strategies

Involving students in wrestling with important mathematical ideas is considered an effective way of facilitating their conceptual understanding (Hiebert & Grouws, 2007). Therefore tasks that make higher cognitive demands on students elicit more opportunities for restructuring and reconfiguring their connections between facts, ideas, and strategies (Stein & Lane, 1996). The types of challenging tasks explored in this project involve finding more than one solution or strategy, which encourages students to grapple with key ideas simultaneously and make connections between them. The literature highlights three approaches to multiple strategies, which are presented in Table 1. For ease of reading, each approach has been categorised (Types 1 – 3).

Table 1. Three approaches to teaching and learning with multiple strategies

Type 1: Generated by learners
Individuals or groups are provided with the task and asked to solve the problem in more than one way. This supports students in inventing their own strategies based on their current conceptual and intuitive understanding of the mathematics involved. "The lesson then concludes with the whole class discussion and summary of various student-generated approaches… for the whole class to view and discuss" (Stein et al., 2008, p. 316).
Type 2: Demonstrated by teachers
Teachers introduce and explain different methods for solving the problem. They might use this approach for helping a diverse range of students find one way to learn the particular concept successfully (Lynch & Star, 2014). A step beyond this is helping students develop logical thinking through comparing and evaluating the efficiency of different strategies to make connections between concepts (Rittle-Johnson & Star, 2007).
Type 3: Provided as worked examples
Learners are provided with written examples of a type of problem, the solution steps for multiple strategies, and the final solutions. Teachers might aim to relieve learners of the struggle of having to find a solution on their own, to concentrate effectively on comparing the different strategies (Große & Renkel, 2006).

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There is much in the research literature about the Type 1 approach at primary levels of schooling and is a feature of reform mathematics teaching in the United States. Mathematics research on Japanese secondary mathematics classrooms has also highlighted that a key aspect of their problem-solving-oriented lessons is having students find and then present their solutions so that the class can discuss them (Fujii, 2014; Ohtani, 2014).

Some research describes combinations of different approaches, for example eliciting students' solution attempts before providing worked examples (e.g., Fraivillig, Murphy, & Fuson, 1999) or elaborating on students' initial solutions and generalising them further with the class (Ohtani, 2014). In a review of studies on the use of multiple strategies for problem-solving, Woodward et al. (2012) concluded that all three types of approaches were recommended in the research literature for helping students learn.

One study considered two different approaches and their effect on students' frequency and type of use of multiple strategies. Star and Rittle-Johnson (2008) evaluated 132 Year 6 students' responses to prompts to discover strategies for themselves (Type 1 approach) and to direct teaching on multiple strategies (Type 2 approach) when solving linear equations. They found that prompts to solve the equation in two different ways led to greater *use* of multiple strategies whereas direct teaching led to the use of more *efficient* strategies. They asserted that direct instruction on multiple strategies after students had first experienced exploring different strategies for themselves provided "reliable gains in problem-solving flexibility" (p. 575).

Another study investigated the approach of providing multiple worked examples of solutions to task (Type 3 approach) to consider the effect on students' procedural knowledge and conceptual knowledge. Rittle-Johnson and Star (2007) studied the use of multiple-solution examples with Year 7 students to solve linear equations in algebra. They found that comparing different solutions method side-by-side rather than simply learning each method sequentially produced greater gains in the students' procedural knowledge and flexibility. They speculated that identifying similarities and differences was important, as well as the evaluation of each method's efficiency. They cautioned that students would need sufficient prior knowledge to be able to do this effectively, which relates to concerns raised in another study about the cognitive load on students. Rittle-Johnson, Star, and Durkin (2009) found negative responses to and effects of the Type 3 approach for students with low or no prior knowledge of algebraic methods in problem solving. They speculated that the benefits of comparing alternative solution methods depend on sufficient prior knowledge, otherwise novices in a particular domain of mathematics could experience working memory overload and find the task demand too high. This could result in negative affect, and lowered engagement and motivation.

In another study, a schema-based instruction approach (provided by the teachers) aimed to help students interpret the problems, and compare and contrast multiple strategies (Type 3), with explicit attention paid to reducing the cognitive overload for low-achieving students. Jitendra et al. (2009) studied the effects of an intervention with 148 Year 7 students learning to solve ratio and proportion word problems (intervention and control groups randomly assigned). The intervention also included metacognitive strategies to help students with self-monitoring. The assessments – post-test and delayed post-test – demonstrated higher problem-solving performance for students who experienced the intervention. Yet both groups performed comparably on state standardised achievement testing. They speculated that a 10-day intervention was not long enough to encourage students to transfer their skills to other types of problems. They cautioned that it was difficult to establish which particular aspects of the intervention were responsible for the success or whether it was a combination of approaches.

Research with secondary teachers highlighted substantial reluctance to experiment with multiple-strategy tasks, even after professional learning (Leikin et al., 2006). From a group of 30 participants, all of whom indicated no prior experience with these types of tasks, only seven agreed to trial the tasks with their students. Of these teachers, only two agreed to implement the tasks and to also ask their students to complete post-task questionnaires about their perceptions. Discussions with the teachers revealed concerns about the inappropriateness of such tasks for low-performing students and about group-based learning involving students sharing solutions with each other. Leikin and Levav-Waynberg (2007) studied 12 secondary mathematics teachers and found that their lack of knowledge about and use of multiple-strategy tasks were related to their school curriculum, which for the most part did not include such tasks. Some teachers described the difficulty of having to respond to students' alternative solutions (different to the one set out in the textbook). Leikin and Levav-Waynberg asserted that there is a gap between teachers' theoretical valuing of multiple-strategy tasks for learning and their actual teaching practice – that they find it difficult to teach with multiple strategies and therefore do so rarely.

Another study examined middle and high school mathematics teachers' perceptions of the advantages and disadvantages of using multiple strategies for teaching algebra (Lynch & Star, 2014). All of the teachers had reported using multiple strategies previously in their teaching; 90 out of 92 teachers indicated that they found the approach useful. A majority of the teachers described multiple strategies as helping a diverse range of students find *at least one way* to solve a problem that might eventually "stick" and therefore access the mathematics somehow. Lynch and Star compared this response to other studies on elementary teachers and noted that it was different in that elementary teachers were more likely to describe the developmental benefit of students learning to progress from intuitive ('invented') strategies to more efficient formal strategies. They speculated that either secondary teachers might have a "limited or incomplete view of the practice of teaching mathematics with multiple strategies" (p. 101) or that perhaps some approaches to multiple strategies in elementary grades such as strategy invention may not be appropriate at secondary levels. They suggested that the use of multiple strategies in elementary grades through inventing them might aim to build students' intuitive knowledge, whereas their use at secondary levels might aim to help students "develop a more connected understanding of mathematics and to develop fluency" through knowing, comparing, and evaluating different strategies (p. 102).

Students' affective responses to challenging tasks and multiple strategies

The interdependent nature of interactions between teacher and learner means that students make judgements about how they are taught and take an active role in determining the effects of teaching on their own learning. Student "interest and achievement mutually influence one another" (Shiefele & Csikszentmihalyi, 1995, p. 177) and teachers adjust their teaching to match their perceptions of students' cognitive and affective reactions to their teaching and choice of tasks.

There is substantially more in the research literature on the perceptions of teachers about their students' reactions than on students' own expressions of their reactions. A few studies in the research literature focussed specifically on teachers' perceptions of younger students' reactions to learning with multiple-strategy tasks. Fennema, Carpenter, Levi, Jacobs, and Empson (1996) described elementary teachers' perceptions that their students became eager to share their thinking and were more enthusiastic about their learning when encouraged to solve problems using their own strategies to share them with the class (Type 1).

Research has highlighted that teachers believe students with higher ability resist opportunities to learn with challenging tasks because their "desire for immediate comprehension is very strong and may ultimately be debilitating" at more advanced levels of mathematics study (Davis, Hersh, & Marchisotto, 1995, p. 315). Studies on senior secondary mathematics students found that in response to being given challenging tasks involving "high-level cognitive processing or when the answers were not readily available", "there was a tendency for students to resist task engagement or negotiate the task demands downwards" (Anthony, 1996, p. 42). Recent research with middle-school students also found that students exerted pressure on their teachers to over-explain the task or to provide simpler ones (Sullivan et al., 2009). In response to students' resistance with challenging tasks, teachers are more likely to adapt their teaching by reducing the learning demands on them (Anthony, 1996; Swan, 2007; Tzur, 2008). This would then lead to more passive learning behaviours and teachers' increased reluctance to incorporate cognitive demand in their lessons by using multiple strategies for learning.

Sullivan and Mornane (2014) sought junior secondary students' own perceptions of challenging tasks and found mixed reactions, but all of the students reported preferring such tasks to textbook exercises or teachers writing questions on the board. Some students connected feeling less stressed about the task with knowing that there was more than one way to tackle it. Leikin, Levav-Waynberg, Gurevich, and Mednikov (2006) studied both Year 10 mathematics teachers' perceptions and their students' reactions to challenging multiple-solution tasks. The students were from only lower-level and intermediate-level classes. Initially the teachers (none of whom had used such tasks before) predicted that their students would dislike the activities because they would be confused by the different strategies and would not be able to explain their solutions to others. Yet the students subsequently reported positive reactions, related to appreciating being able to look for easier solutions, to think and understand the tasks in their own way, and expressing situational interest in the variety of the tasks. They found that some students who were in lower-level mathematics classes did express the view that the use of the tasks was confusing and unnecessary for them, indicating a preference for the usual way of being taught in a more teacher-directed way. The researchers expressed surprise that students in the intermediate level classes were more likely to prefer learning from *receiving* others' strategy explanations whereas those in the lower-level class were more likely to prefer *giving* their strategies to others.

In studying 92 middle and high school mathematics teachers' perceptions of using multiple strategies for teaching algebra, Lynch and Star (2014) found the teachers were more likely to pay attention to cognitive or pedagogical issues rather than the students' affective reactions. Approximately 15% of the teachers made reference to the perception that multiple strategies got more students involved, increased their confidence, and reduced their boredom or frustration. Interestingly, a similar percentage described the *opposite* perception that multiple strategies bored high-ability students and de-motivated struggling students who were confused by trying to understand different methods. Nearly one third of teachers described experiencing resistance from students who preferred simply to learn one single method.

It appears that students' affective reactions to tasks incorporating multiple strategies are diverse; there is more to understand about the factors influencing their reactions and how these might also relate to cognitive and motivational aspects of learning, such as the teaching approaches used, the relevance of the tasks, the set-up of the learning environment, and the students' own dispositions, self-efficacy or ability levels. There were references to a student's level of mathematics performance affecting their response: many teachers perceive higher-ability students as more likely to be bored and lower-ability students as more likely to be confused by learning multiple ways to solve a problem.

Motivational perspectives on learning

A student's disposition, goals and actions influence their participation in learning; their own curiosity and wish to learn may contribute to subsequent educational achievement (Cohen, Raudenbush, & Ball, 2003). Student motivation has been discussed extensively in the literature and is considered a key variable in students' regulation of themselves – their decision to engage or not, how much effort to expend (Middleton & Toluk, 1999; Schoenfeld, 2007), their goal setting, and their choices in long-term academic pursuits (Pintrich & Schunk, 2002). There are different constructs for conceptualising student motivation. Motivation can be described as relating to both intrinsic and extrinsic factors – an individual's disposition *and* the influences on them from their environment (Middleton & Jansen, 2011).

One theoretical perspective describes the dichotomy of an individual's *mastery* or *performance* goal orientation to describe how an individual defines competence (Ames 1992; Pintrick & Schunk, 2002). Dweck (2007) related these to *growth* and *fixed* mindsets. A fixed mindset incorporates the belief that one's qualities are carved in stone – that one has a fixed amount of intelligence, a certain personality, and certain moral character. A growth mindset views these qualities as able to be cultivated or improved through one's effort, and although everyone differs "in their initial talents, attitudes, interests, or temperaments", each person can grow through application and experience (Dweck, 2007, p. 7). *A mastery* goal orientation focuses on improving one's own learning or progress and on task-based outcomes whereas a *performance* goal orientation focuses on comparing oneself with others, such as through test results or competitive situations. Hulleman, Schrager, Bodmann and Harackiewicz (2010) asserted that outcome (or achievement) goals, such as getting a good grade, ought to be classified separately from performance goals since they are neutral in terms of how competence is defined. They can be related to mastery (an "A" grade indicates mastering the content) or to performance (an "A" indicates outperforming others) orientations. Within the performance goal orientation Hulleman and colleagues further distinguished between *appearance* (demonstrating competence), *normative* (performing better than others), and *evaluative,* which is a hybrid of both appearance and normative components (demonstrating competence relative to others).

Another dichotomy considers the direction of motivation – *approach* and *avoidance* – to describe how competence is valenced: the inherent attraction (leading to approach) or aversion (leading to avoidance) of a situation or experience (Elliot, 1999). The resulting two-by-two goals framework using Elliot's original conceptualisations and Hulleman's and colleagues' further distinctions is presented in Figure 1. It has been empirically supported by research substantially for 3 of the 4 motivation types but more recently also with mastery-avoidance (Jang & Liu, 2012).

In a meta-analysis of several studies Elliott (1999) noted that mastery goals were found be linked to persistence, absorption during task engagement, challenge-related affect, self-regulated learning, deep processing, and intrinsic motivation. Examples of mastery-avoidance goals were found in which an individual strives to avoid losing or stagnating one's skills / abilities – of trying to avoid forgetting what one has already learned, and associated with a perfectionist need for achievement (Elliot & Murayama, 2008). Performance-approach goals were found to link to some positive consequences such as persistence, absorption during task engagement, challenge-related affect, and intrinsic motivation. But they were also linked to test anxiety, shallow processing, and reluctance to seek help with tasks. Performance-avoidance goals were found to link to distraction, low absorption during task engagement, threat-related affect, less self-regulated learning, shallow processing, and anxiety about evaluation (Elliot, 1999). A study on student engagement in the middle years using the concept of Dweck's mastery-performance goals dichotomy found that *all* of the higher-achieving students gave evidence of performance goals and that the students with mastery goals were neither in the low or high-achieving groups (Sullivan, Tobias, & McDonough, 2006).

In the literature, there remains ongoing debate about whether or not it is possible for an individual to hold both mastery and performance goal orientations at the same time (Brophy, 2005; Hulleman et al., 2010; Martin, 2013). Inconsistency with definitions and measuring across studies seems to have contributed to the lack of resolution and ongoing controversy (Hulleman et al., 2010). Brophy (2005) also raised the issue that most research using goal theory has involved measuring with experimental induction procedures or pre-defined Likert-scale surveys, which do not allow investigation into the degree to which students *spontaneously* generate these types of goals. He emphasised the need for research into students' expressions in their own words and speculated that there would be less evidence of performance goals compared to results from normative surveys.

The study described in this article sought to explore students' *own* expressions of their reactions to challenging tasks using an open-ended response questionnaire. It elicited their views about learning with multiple strategies to compare with their teachers' perceptions, and used cognitive, affective, and motivational perspectives from the literature to understand more what matters to secondary students for their mathematics learning. The following section provides details of how the study was designed.

RESEARCH DESIGN

Much of the literature on challenging tasks and the use of multiple strategies has focussed on these issues from teachers' perspectives. To consider specifically the voice of *students* (Creswell, 2007; Flutter & Rudduck, 2004; Clough & Nutbrown, 2007) this study was designed to elicit their views in an open-ended rather than normative approach. It sought to examine what students choose to focus on in describing their reactions to a particular challenging task and how their views on multiple strategies might relate to cognitive, affective, and motivational dimensions of learning. It aimed to "address both the pragmatic and highly theoretical issues

Mastery-approach goal orientation	**Performance-approach goal orientation**
Interest and *curiosity*: learning something interesting *Task*: mastering a task *Challenge:* mastering a challenge *Improvement* or *attainment*: Learning as much as possible; improving my knowledge; understanding the content as thoroughly as possible; acquiring new skills	*Appearance*: demonstrating competence / ability *Normative*: performing better than other students *Evaluative*: Demonstrating my ability relative to others in the class (as judged by authority figure such as a teacher)
Mastery-avoidance goal orientation	**Performance-avoidance goal orientation**
Task: Avoiding forgetting what I have already learnt *Improvement* or *attainment*: Avoiding losing my skills / abilities / knowledge; avoiding stagnation or lack of development	*Appearance*: avoiding looking incompetent / 'dumb' *Normative*: Avoiding performing poorly in the class *Evaluative*: Avoiding demonstration of lack of ability relative to others (as judged by authority figure)

Figure. 1. Conceptualising four types of student goals using mastery-performance and approach-avoidance dichotomies (Elliot, 1999; 2008; Hulleman et al., 2010)

simultaneously" to achieve "reflexivity between theory and practice" (Cobb, 2000, p. 308). The study incorporated multiple sources of data (Creswell, 2007) to triangulate different views (Hesse-Biber, 2010) and seek plausible interpretations (Stake, 1995; Wolcott, 2009).

The study involved a cohort of Year 10 high-achieving students whose teachers were participating in a large design-based research project on challenging tasks. The students were from four mathematics classes and had begun attending a select-entry government school that year. Through a testing and interview process, their school had deemed them to be high achieving or to demonstrate an aptitude for mathematics. They were from a diverse range of socio-economic and ethnic backgrounds. Their teachers were invited to participate in the sub-study because of their students' high ability and the research focus on this type of cohort's reactions to challenging tasks incorporating multiple strategies. Previous research had highlighted teachers' perceptions of high-achieving students' reluctance to engage with challenging tasks, evidence of pervasive performance goal orientations, and teachers' perceptions that such students react with boredom to learning multiple strategies. This study sought to explore this further but from the students' perspectives.

Data collection and analysis

Towards the end the academic year, two of the six Year 10 mathematics teachers from the larger project agreed to a joint semi-structured interview to discuss their perceptions of their students' reactions throughout the year to a number of challenging tasks (one hour's duration – Appendix 2). They also raised issues they had encountered when implementing the tasks and responding in class to students. The teachers then suggested a topic for a subsequent challenging task, for which the researcher developed a selection of three quadratic tasks. The teachers chose a particular task and this was used in the sub-study (Appendix 1).

The researcher observed the implementation of the quadratic task across four classes (two large groups each with a pair of teachers in an open-plan area) and collected work samples of the students' written responses to the task. The focus of the lesson observation and analysis of the work samples was on the perceived situational interest in the task, the perceived engagement of the students in tackling it, and their use of multiple strategies to answer the questions (to be reported elsewhere). The two teachers who participated in the earlier interview led the lesson with the four combined classes in two groups, with their co-teachers watching. They each started with a quadratic equation example for the students to sketch; they asked a pair of students to draw their solutions on the board and to explain a way of finding key points on the graph. The students were then asked to work in groups of three or four (their choice) and to record their solutions on the provided A3 sheets. The teachers led a concluding discussion, inviting students to explain their responses to the task, and making connections between the different strategies for answering the questions and the general formulae of quadratic graphs. The students were then invited to complete an anonymous individual open-ended reflection (Appendix 3) and 87 chose to participate (nearly 100% response rate). It was an important tenet of the research that the students were free to respond in any manner they chose without fear of reprisal from their teachers. After the lesson, the researcher and teacher participants debriefed.

The teachers' interview responses and the students' questionnaire responses were analysed using "descriptive and interpretive" approach (O'Toole & Beckett, 2010, p. 43). The teachers' responses were transcribed from an audio recording and coded inductively using line-by-line coding with NVivo 10 qualitative analysis software (Creswell, 2007). The resultant coding framework (Miles & Huberman,

1994) is presented in Appendix 4. The students' written responses were transcribed into Excel spreadsheets for inductive categorisation. The previously presented theoretical framework was used to examine which cognitive, affective, and motivational aspects of learning the students chose to focus on in describing their reactions to challenging tasks and views on learning multiple strategies. The cognitive and affective dimensions of learning described by the students, and evidence of goal-oriented language used in the students' written reflections were examined to explore how high-achieving students view challenging tasks and multiple strategies in terms of their learning, engagement, and motivation.

RESULTS AND DISCUSSION

This section begins with the teachers' perceptions of their students' reactions to challenging tasks earlier in the year before encountering the quadratics task from this study. Four sub-sections then present results from the responses of the students to the reflective questionnaire which explored: their reactions to the quadratics task; their views on challenging tasks in general; their perspectives on learning strategies from a peer; and their views on learning multiple strategies from a teacher. A final sub-section discusses issues the teachers raised about incorporating challenging tasks into their mathematics program with high-achieving students.

The teachers' perceptions of students' previous reactions to challenging tasks

The two Year 10 mathematics teachers who were leading the mathematics program participated in a joint interview to share their experiences of using challenging tasks in the larger project. One was the Head of Mathematics ("Anne") with several years of teaching experience and the other ("Barb") had been teaching at the school for nearly three years and had experience of teaching senior secondary mathematics at another school. Barb explained that in her own earlier efforts to use challenging tasks at a previous school with her specialist mathematics class, the high-achieving students resisted tackling the tasks independently:

> Every single lesson I would run from kid to kid to kid while they had their hands up. And there was no moving forward until I went and helped them. And I put in strategies like see three before me and that kind of stuff and would keep to that and go I know that you've only asked one person, go and ask two more.

She recognised the need for teachers to have strategies to manage the resistance of high-achieving students to challenging tasks:

> I kind of tried to break them away from asking the teacher all the time, but I don't think I had enough strategies to be able to do it consistently. And I think because I'd already recognised that as a problem and particularly a problem with kids who are good at maths – it's that idea where they're scared to try something and they might fail and they're not used to failing and that's a really uncomfortable area for them.

Anne and Barb were asked about their Year 10 students' responses to earlier challenging tasks that year. Anne said, "A lot of them are really uncomfortable for the first time; the really good kids who have been good at maths are really uncomfortable in that space." They highlighted their efforts at the start of the year to explain to the students the purpose of giving them challenges:

> We just explain and explain all the time with the Year 10s, "We don't know where you're at. You don't want to be bored in maths, this is what you tell us, well here you go, here's your chance, show us what you

> know. And it's ok for us to give you something you don't know how to do. You are here to learn, that's your job." (Anne)

At the time of the interview, the students were weeks away from the end of the academic year, and the teachers described the gradual shift in the students' initial reluctance:

> We're just starting to see the results now; they're just starting to come on board and they go to me, "Yeah yeah I know, you've got another problem that I don't think I can do. I know, now I'm going to have a go." (Anne)

They attributed this change to students experiencing "successes from what they're doing"; "they understand *why* it's being done like that" (Anne).

> We've won the kids over in a way and their maths results may not necessarily have improved, but they've enjoyed being in there and they're keen to come to maths. And they know that it's a bit of a journey and they know they've got to work in a different way. (Anne)

Students' reactions to the quadratics task

At the conclusion of the quadratics challenging task lesson, the students were asked how they found the task and their reasons. Their responses were categorised according to cognitive and affective aspects of learning. They are presented in Table 2.

Table 2. Students' descriptions of their reactions to the quadratics task (n = 87)

Type of Response	Percentage of Students (%)	Illustrative Examples
Affective response: Emotion		
- interesting / engaging	27.6	"interesting" because "it was interesting to look at the different forms and connect them together" "interesting" because "we got to able to explore different approaches to quadratics [sic]"
- enjoyable / fun	5.7	"fun" because "it was a group task" "enjoyable" because "it posed a challenge and it wasn't over even when it was done, the teacher was very helpful too"
- confusing	1.1	"confusing" because "I haven't done quadratics in a while and I've forgotten quite a bit"
Affective response: General evaluation		
- okay / alright	13.8	"alright" because "I knew the concepts that we had to use but applying them was a challenge" "okay" because "I understood what was going on"
- good / cool / swell etc.	8.0	"good" because "it was a challenge but it was still doable" "swell" because "we had done similar work but not as an investigation"
Cognitive response: Level of perceived challenge		
- easy / simple	13.8	"easy" because "I understand the concept of where x and y intercepts sit on a graph in an equation" "simple" because "we have generally covered this topic in class"
- difficult	6.9	"difficult" because "only understood certain aspects" "difficult" because "I haven't done it in a while"
- average / moderate	5.7	"moderate" because "the task wasn't a level which was unachievable" "moderately difficult yet achievable" because "I kept confusing concepts"
Cognitive response: Effectiveness for learning		
- useful / beneficial / helpful	10.3	"beneficial" because "it helped to understand the turning point formula better and the discriminant in the quadratic formula" "useful" because "I had lost all memory on quadratics. Helped me remember"
- not useful	3.4	"useless" because "it took too much time and was unnecessary" "not effective" because "I already know this stuff"
Other	2.3	"supercalafraga…" because "for a loss of other words"
No response	1.1	

Overall, over one half of the students described an affective response to the task, with most using positive words such as "interesting" or "engaging". No students described the task as boring. Only one student indicated that they found the task "confusing" because they "[hadn't] done quadratics in a while and [they'd] forgotten quite a bit". Just over one quarter of the students described their perceived level of difficulty of the task. One student who described finding it "difficult" also explained that they "[kept] forgetting how it all works". Another student who found the task "moderately difficult yet achievable" reported that they "kept confusing concepts". For this cohort of high-achieving students who had previously experienced a number of challenging tasks, only a handful reported experiencing confusion or forgetting. Nearly 9% of the students reported that the task helped them revise or remember things they had already learned. Putwaine and Symes (2011) described senior secondary students as likely to develop a *mastery-avoidance* goal orientation in trying not to forget the content at higher levels of schooling. This may relate to assessment procedures such as examinations, which require longer-term retention of knowledge compared to shorter-term topic tests. These students' comments about not wanting to forget and needing to revise may relate to this particular type of goal orientation.

For the 14% of students who reported finding the task easy or simple, most gave a reason related to already knowing the concepts, and a further 7% also commented on understanding the task. For this cohort, one fifth reported *not* experiencing the task as challenging. Another fifth, who mostly described the task as interesting or beneficial, specifically referred to some aspect of the task being *novel* for them – different to usual, something new to be learned, applying their knowledge in a new way, investigating, or exploring. It appears that the students' evaluation of the task's situational interest to ascertain if engagement was worthwhile (Middleton & Toluk, 1999) considered both its novelty (affective aspect) and its relevance to their learning (cognitive aspect). Middleton (2013) asserted that a number of studies have demonstrated that "the development of *interest* promotes more effective *cognitive* processing of academic content and engenders positive *affect*" (italics mine). This finding resonated with this present study's results in that many students described their focus on considering their situational interest in the task from both cognitive and affective perspectives.

From the observation of the quadratics task lesson across the four classes, the researcher noticed that the rhythm of the lesson seemed familiar to the students. Their collaborative group work (sociocultural approach to learning), substantial time attempting the task, and the lack of any widespread resistance or disruption indicated that the expectations of the teachers for challenging tasks were being met by students. The concentration of most of the students appeared to confirm the teachers' perceptions that the students had been "won over" by that stage in the year and were more likely to engage with a challenging task on their own now without premature teacher intervention. It was interesting to observe a continuation of the students' engagement during the final whole-class discussion and their attention to the teachers' drawing together of different strategies and formulae for quadratic functions.

Students' views on challenging tasks

The results for students explaining their reasons for *liking* or *not minding* or *disliking* challenging tasks in general are presented in Table 3.

Approximately 60% of this cohort indicated that they didn't mind challenging tasks and a further 30% reported liking them. For those students who didn't mind them, most of the reasons related to tasks being of value for their learning (cognitive aspect). For those who didn't mind challenging tasks and gave an affect-related

Table 3. Students' descriptions of their views on challenging tasks (n = 87)

Student Comment	Don't Like (%)	Illustrative Example	Don't Mind (%)	Illustrative Example	Like (%)	Illustrative Example	No Response (%)	Total
Affective response								
Positive	-		14.9	"it engages me"	13.8	"they are more fun to explore"	-	28.7
Negative	3.4	"it's not interesting"	-		-		-	3.4
Cognitive response								
Positive	-		37.9	"it makes you think"	12.6	"they help me notice my weak points and also help me revise"	-	50.6
Negative	3.4	"I find it too difficult"	2.3	"it wasn't really challenging"	-		-	5.7
Other / no response	1.1	"I would rather do my unit plan or exam revision"	5.7	"it's okay"	2.3	"it was easy"	2.3	11.5
TOTAL	**8.0**		**60.9**		**28.7**		**2.3**	**100.0**

reason, some referred specifically to their novelty: enjoying a change from the usual learning tasks. One wrote, "it's a change from daily exercises" and another wrote, "they are not too mainstream". For those students who liked challenging tasks, half provided an affective reason related to being engaged, and the other half described a cognitive reason, related to their perceived usefulness of a task. Overall, nearly 7% of the students referred to challenging tasks as helpful for revision and 15% described learning something new or in a new way. Interestingly, nearly 14% described challenging tasks in terms of improving their thinking – "they make you think and aren't just memorising equations", and they "make you think about maths in a different way". A few students described enjoying working together with peers on a challenging task.

For the 8% of students who indicated disliking such tasks, half of their reasons related to finding them too hard. It can be seen that even though the quadratics task was designed to be open-ended and to elicit multiple strategies, there were nonetheless students who found the task too easy and other students who found it too hard. Providing tasks that engage students within reach of their level of understanding and do not dishearten them is a challenge for teachers whatever the achievement level of their students.

Students' views on learning strategies from a peer

Table 4 presents the responses to students' preferences for learning a strategy from a peer (Type 1 in Table 1). Comments that related specifically to the value of knowing multiple ways to solve a problem at the same time (rather than finding another way to use in future) were categorised separately to compare them to the later question on learning multiple strategies from the teacher (Types 2 and 3 in Table 1).

Nearly two thirds of the students reported *liking* to learn a strategy from a peer, with nearly 60% referring to a cognitive aspect: a perceived positive benefit to their learning. A further 30% indicated that they did not mind, again with the majority relating their reason to a positive learning outcome. Overall, nearly 30% of the students reported that they valued learning a *new* or a *different* way to solve a problem from a peer. Just over 10% indicated that they found a peer's way of explaining easier to understand (than a teacher's). One student wrote "I get to see their strategies and also helps me learn new things because they think and speak like teens". Another wrote, "I may not understand and they explain in layman's terms". It is possible that this may be a specific approach to learning multiple strategies, which is effective at secondary levels (rather than primary levels).

Table 4. Students' descriptions of their views on learning strategies from a peer (n = 87)

Student Comments	Don't Like (%)	Illustrative Example	Don't Mind (%)	Illustrative Example	Like (%)	Illustrative Example
Affective response						
Positive	-		3.4	"it is interesting to know how other people think and learn from them"	3.4	"I like hearing other people's input"
Negative	2.3	"I feel belittled"	-		-	
Cognitive response						
Positive	-		12.6	"I may need help"	35.6	"they often make more sense"
Positive & about different strategies	-		8.0	"all methods are possible approaches and some might be easier methods"	20.7	"it shows me different ways to solve it"
Negative	4.6	"sometimes they are wrong"	1.1	"sometimes I don't understand it"	-	
Other response	-		4.6	"it doesn't affect me"	3.4	"it means I get to show them the correct way"
TOTAL	**6.9**		**29.9**		**63.2**	

Approximately 7% of the students indicated a *dislike* for peers explaining strategies to them. A few students described a negative affective response such as feeling belittled or treated with condescension by their peers. This can relate to a performance-avoidance goal orientation, which is associated with concerns about appearance, looking incompetent or 'dumb' to others. A few students referred to a perceived negative cognitive effect on their learning, such as not being taught correctly or not having a clear explanation. A few students didn't trust their peers' knowledge, and another student wrote, "My strategy is the best". Performance-approach goals focus on appearing to be more competent than others, but it is unclear here if the student's comment is simply 'tongue in cheek' or what their reasons might be for thinking their strategy would be the best. No students referred to being bored by learning multiple strategies from a peer, a contradictory finding to those studies exploring teachers' perceptions of high-achieving students.

Students' views on learning multiple strategies from their teacher

The results for students' preferences for learning more than one way to solve a problem from their teacher are presented in Table 5. Specific responses that related to wanting to be able to understand *multiple* strategies (rather than being able to choose one strategy from among those taught) were categorised separately.

Nearly 60% of the students indicated that they *liked* learning more than one strategy from the teacher and another 35% that they *did not mind*, with the vast majority giving cognitive reasons related to perceived effectiveness for their learning. Nearly 30% of the students referred to the value of being shown more than one way to solve a problem because they could choose a strategy for future use since it was *easier* or *quicker* or more *understandable* for them. Their reasons resonate with a recent study in which secondary mathematics teachers perceived the teaching of multiple strategies as a useful way of 'covering all the bases' to increase the likelihood of each student understanding one strategy that they could then competently use (Lynch & Star, 2014).

Slightly more students made a specific comment about a better or quicker strategy as such, but surprisingly, a similar proportion described wanting to know *more than one way* to tackle a problem *at a time*, for example:

Table 5. Students' descriptions of their views on learning strategies from the teacher (n = 87)

Student Comments	Don't Like (%)	Illustrative Example	Don't Mind (%)	Illustrative Example	Like (%)	Illustrative Example
Affective response						
Positive	-		-		1.1	"there are some ways which I don't like doing, so an option is always fun"
Negative	3.4	"I get confused which one to use"	1.1	"sometimes at first it can be confusing"	-	
Cognitive response						
Positive			12.6	"it's sometimes helpful"	17.2	"it improves the way of thinking"
Positive & about choosing a better / quicker strategy			4.6	"one way could be much easier than the other"	24.1	"we can decipher a way to see which one is suited to us"
Positive & about learning multiple strategies			9.2	"in case one method isn't working out I could use another, or I could use different methods to compare answers"	16.1	"it gives me options and I get to choose which method is suitable for certain questions"
Negative	1.1	"I have not yet fully understood / remembered the first part"	2.3	"it is extended learning but can be unnecessary"	-	
Other response	2.3	"in tests we will probably have to show all ways to solve a problem"	4.6	"I know the teacher is right and knows what they are talking about"	-	
TOTAL	**6.9**		**34.5**		**58.6**	

- "It gives me options and I get to choose which method is suitable for certain questions."
- "I want to know more solutions."
- "Different ways means more understanding of relationships between formulas."
- "It allows students to use more than one approach."
- "It is good to know more than one path."

This reason for learning multiple strategies has not been evidenced in studies researching this approach from teachers' perspectives. Elementary teachers refer to students inventing their own strategies and then learning to find a *more efficient* strategy (e.g., Stein et al., 2008). Secondary teachers refer to hoping students find *one strategy* amongst a selection that they can understand (e.g., Lynch & Star, 2014). It appears that for high-achieving students like these, knowing *multiple strategies at the same time* is a different motivation from those described by teachers.

The 7% of students who *disliked* learning more than one strategy referred to a sense of confusion or to being concerned about remembering them, which gives some evidence of mastery-avoidance goals. Only one student expressed learning more than one way to solve the problem as "unnecessary". Earlier studies found this view amongst lower-achieving students; it appears that this may be different to the views of high-achieving students.

To compare the students' views on learning strategies from a peer or from a teacher, a cross-tabulation of the previous two tables was made, and is presented in Table 6.

Even though a similar proportion of students (approximately 60%) indicated liking having a peer explain their strategy and having a teacher show more than one way to solve a problem, the cross-tabulation demonstrates that these are not the same students. Nearly 44% of the students indicated *liking both*, and every

Table 6. Cross-tabulation of students' views on learning strategies from a peer and from the teacher (n = 87)

		Having the teacher show us more than one way to solve a problem			
Having a peer explain their strategy for solving a problem to me		**DON'T LIKE**	**DON'T MIND**	**LIKE**	**TOTAL**
	DON'T LIKE	1%	3%	2%	7%
	DON'T MIND	-	17%	13%	30%
	LIKE	6%	14%	44%	63%
	TOTAL	7%	34%	59%	100%

combination of preference is represented with the exception of not minding peer explanations and disliking teacher demonstration. A Pearson product-moment co-efficient (when the categories are ordered numerically – 'don't like' as 1, 'don't mind' as 2 and 'like' as 3) of 0.2 also adds to the finding that for this cohort, the students' preferences for these two ways of learning were not correlated. It appears that these students have reasons for preferring a peer's or a teacher's strategies which do not necessarily overlap in both approaches.

The student who did not like *either* way explained that peers can be wrong and that they are confused when being shown multiple strategies by the teacher. Of the other two students who disliked peer explanations but did like their teacher showing multiple strategies, one felt belittled by peers but found the teacher helpful. The other wanted to solve problems individually, and liked to be given options by the teacher.

These results indicate overall that although there are variations in the students' preferences for learning mathematics in ways that engage them, learning with challenging tasks and learning multiple strategies are viewed by them as engaging if they are interesting and also relevant to their learning. The reasons given by the students related to cognitive and affective aspects, and their use of language provided evidence more often of a mastery than a performance goal orientation.

The teachers reflect on incorporating challenging tasks in their Year 10 program

In addition to explaining to students the purpose of challenging tasks and the value of not always knowing how to do something, the teachers also found that they needed "to work out when is the best time to intervene" since "some of [the students] are so used to getting intervention straight away and they never get into that uncomfortable space" (Anne). They suggested that the students might over time 'rise to the challenge' if the teachers learned to hold back from modifying or over-explaining the task:

> If you give them a task like this, but then step in the minute that they struggle, then they get to learn that they don't need to struggle and so they're like, "If we just sit back for another two minutes they're going to show us how to do it on the board." So we made it really clear I think through the other tasks that we've done that we're not just going to show them on the board and if it takes longer than we thought, then it takes longer than we thought. (Barb)

They addressed the issue of teachers having to "[let] go of the control" when using challenging tasks – "I don't know what's going to happen in this lesson and that freaks me out" (Barb). Their strategy for managing teacher anxiety was to timetable some of the lessons together with the other Year 10 classes and to communicate with each other during the task to discuss what was unfolding and decide collaboratively how and when to respond.

The teachers also highlighted the need for teachers to have a strong knowledge of the mathematics, particularly to be able to manage multiple strategies:

> Sometimes you can't follow what the kids are doing. You've got to actually be able to engage with every different kid's strategy and make sure that they're mathematically on the right track. And that can be very, very hard to do right. (Anne)

They thought that the way to develop their knowledge was to plan tasks together to anticipate different strategies – "If teachers have some sort of structure around it and they know what they're looking for, then it might be a little bit less daunting, so it's all in the planning of that" (Anne).

IMPLICATIONS AND CONCLUSION

Previous research has highlighted that teachers may be reluctant to use challenging tasks because of their perception that students would resist (Anthony, 1996). They reported believing that learning more than one way to solve a problem bores high-achieving students and that less capable students become confused. Theoretical perspectives connect students' *situational interest* in a task with their decision to engage or not (Middleton & Toluk, 1999). An interesting task is more likely to promote effective cognitive processing and positive affect (Middleton, 2013). Research on the motivation of students has found that high-achieving students are likely to hold performance goals, related to appearing or performing better than others (Sullivan, et al. 2006), rather than mastery goals, related to mastering a challenge or learning as much as possible. Senior secondary students have shown an increase in mastery-avoidance goals, related to trying not to forget what has been learnt (Putwain & Symes, 2011).

This study focused on the perceptions of high-achieving Year 10 students after their experience of a year-long teaching intervention incorporating challenging tasks as part of a design-based research project. Data were sought (anonymously) on: the students' views of a particular task immediately after the lesson; their views on challenging tasks in general; and their preferences for learning multiple strategies from peers and from the teacher. This was compared to the perceptions of their teachers.

Having experienced a number of challenging tasks throughout the year, the majority of students in the study expressed a positive response to them, giving reasons that related to their affect – their interest or engagement or enjoyment – or to a cognitive aspect: their perception of benefit to their learning. Their responses resonate with different theoretical perspectives on student motivation. In terms of an adaptive theory of motivation (Middleton & Toluk, 1999), the students had prior experience of challenging tasks and evaluated the particular quadratics task to ascertain its situational interest. The findings suggest that this cohort considered the novelty of the task and its relevance to their learning; assessing the task as both interesting *and* relevant meant that they were more likely to engage with it than resist. A focus on learning something interesting and mastering a challenge are related to a mastery goal orientation. Teachers may be able to increase the likelihood of students like these engaging with challenging tasks by choosing unusual problems that are also relevant to the content students believe they need in a particular course of study.

In their responses, the students overwhelmingly made reference to aspects showing a mastery goal orientation rather than performance. Their cognitive reasons for liking a challenge or multiple strategies related to learning something new or interesting, rather than to achieving high grades or improving performance. Unlike previous research, these high-achieving students described wanting to learn new concepts, to understand the content, to think actively, and to be challenged.

There were very few spontaneous references to performance goals, which Brophy (2005) speculated would be the case when not using pre-set Likert-scale survey instruments for data collection. Yet these students were approaching the last two years of schooling in which competitive performance on external examinations play a significant role in determining future eligibility for university entrance. It is therefore surprising that the students did not reference this in their responses. There was some mention of not wanting to forget and appreciating the value relevant challenging tasks for revision purposes, which resonates with research on the likely development of mastery-avoidance goals by senior secondary students (Putwain & Symes, 2011).

An external environment is seen as influencing an individual's adoption of goals independent of other intrinsic motivational features of that individual (Elliot, 1999). It is suspected that the school's explicit culture of valuing progress above performance, and the teachers' use of strategies to encourage task persistence, may have played a role in influencing these students' mastery-oriented responses, which were different from expected. Contexts promoting the possibility of success are more likely to activate approach goals whereas those promoting threat are more likely to activate avoidance goals. He cautioned, however, that goals emerging from environmental effects are likely to be weaker and less stable than those related to individual dispositions. Middleton et al. (2013) found that students display different motivational patterns with different teachers. Dweck (2007) asserted that students pick up on what teachers value and act on their messages. Middleton (2013) asserted that teachers play a role in changing a student's motivational pattern by inducing them to "buy in". In this study, the school environment and the teachers' use of strategies for encouraging persistence on challenging tasks resonate with these perspectives on motivation. Yet one limitation of the study was that the students' motivational goals were not examined before the implementation of the teaching intervention to see if and how they might be influenced by environmental factors related to the teachers' strategies and the choices of tasks. It would also be valuable to explore the goals of these students in other subject areas and longitudinally to see how they might vary across different disciplines and over time after having experienced this environment and a mastery-oriented approach to learning in Year 10. Although a majority of the student responses in this study highlighted their interest in learning effectively (approach goals), there were still references to not forgetting or to needing revision (avoidance goals), which seems to indicate that assessment pressures may still be perceived by some individuals as threatening even in this environment. A cohort of younger or older students may exhibit different proportions of approach and avoidance goals. And of course, individual dispositions and other factors cannot be discounted.

Previous research found that teachers perceived that their students pressured them to over-explain or simplify a challenging task (e.g., Anthony, 1996; Sullivan, et al. 2009). In this study, and after the students had experienced a number of challenging tasks, this exertion of pressure was not noticeable during the quadratics task lesson (as perceived by the researcher). The teachers in their interview had highlighted their awareness of this issue and their explicit use of teaching approaches to resist the urge to modify the task or give in to student pressure. They explained to students the purpose of a challenging task for their learning and then deliberately provided time for the students to grapple with the task without stepping in. Some of the students did, however, express dissatisfaction afterwards in their reflections that they found the task too hard or too easy for them. Finding novel and relevant tasks to suit a range of levels of understanding, even for groups streamed according to achievement level, remains an ongoing issue for teachers.

With regard to learning more than one way to solve a problem, nearly two thirds of the students liked having a peer explain their strategy, and two thirds liked being shown more than one way by the teacher. Approximately 44% liked both of these approaches. Some students liked learning strategies from a peer because their language was easier to understand. This approach to multiple strategies may be one that is specifically useful at secondary levels. Students' reasons related overwhelmingly to the *cognitive* aspect of perceived benefit to their learning rather than to their affect, unlike their views on challenging tasks. For those students who specifically referred to learning more than one way to solve a problem, similar proportions of students described wanting to find one strategy that they found easier / quicker / understandable, or wanting to know *multiple ways at the same time*. This latter preference is at odds with the findings of recent research on secondary teachers' pedagogy (Lynch & Star, 2014). The teachers were more likely to show different ways of solving a problem for the express purpose of making sure each student could find *one* strategy that worked for them, rather than to promote understanding of *multiple* strategies at the same time. It seems that there is more to understand about if and how secondary teachers might incorporate multiple strategies in their teaching to help students learn, through comparing them and assessing their efficiency in different situations, rather than simply hoping that one strategy might at least 'stick'. The students in this study showed that they valued knowing multiple strategies *at the same time*.

The findings of other studies – that teachers perceive less capable students as being confused by learning more than one way to solve a problem – could not be investigated in this study given its focus on high-achieving students. Yet a handful of students did mention their confusion with both challenging tasks and with learning multiple strategies. There is more to understand about the approaches to using multiple strategies at secondary levels and with different cohorts of students. Overall this particular cohort of high-achieving Year 10 students, with many evidencing mastery goals, viewed challenging tasks and the opportunity to learn multiple strategies as engaging their interest affectively and effective for their learning cognitively.

REFERENCES

Ames, C. (1992). Classrooms: Goals, structures, and student motivation. *Journal of Educational Psychology, 84*, 261-271.

Anthony, G. (1996). *Classroom instructional factors affecting mathematics students' strategic learning behaviours*. Paper presented at the 19th Annual Conference of the Mathematics Education Research Group of Australasia (MERGA), University of Melbourne.

Australian Curriculum Assessment and Reporting Authority. (2009, January, 2011). The Australian curriculum: Mathematics. Retrieved October 1, 2011, from http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10

Bingolbali, E. (2011). Multiple solutions to problems in mathematics teaching : do teachers really value them? *Australian Journal of Teacher Education, 36*(1), 18-31. doi: 10.14221/ajte.2011v36n1.2

Brophy, J. (2005). Goal theorists should move on from performance goals. *Educational Psychologist, 40*(3), 167-176. doi: 10.1207/s15326985ep4003_3

Clough, P., & Nutbrown, C. (2007). *A student's guide to methodology* (2nd ed.). London: Sage.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 307-333). Mahwah, NJ: Lawrence Erlbaum Associates.

Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25*(2), 119-142. doi:10.3102/01623737025002119

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage.

Davis, P. J., Hersh, R., & Marchisotto, E. A. (1995). *The mathematical experience* (Study ed.). Boston: Birkhäuser.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*(2), 131-147.

Dweck, C. S. (2007). Boosting achievement with messages that motivate. *Education Canada, 47*(2), 6-10.

Dweck, C. S. (2010). Mind-Sets and Equitable Education. *Principal Leadership, 10*(5), 26-29.

Elliot, A. J. (1999). Approach and avoidance motivation and achievement goals. *Educational Psychologist, 34*(3), 169-189. doi: 10.1207/s15326985ep3403_3

Elliot, A. J. (2008). Approach and avoidance motivation. In A. J. Elliot (Ed.), *Handbook of approach and avoidance motivation* (Vol. 1, pp. 3-14): Taylor & Francis.

Elliot, A. J., & Murayama, K. (2008). On the measurement of achievement goals: Critique, illustration, and application. *Journal of Educational Psychology, 100*(3), 613-628. doi: 10.1037/0022-0663.100.3.613

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A Longitudinal Study of Learning to Use Children's Thinking in Mathematics Instruction. *Journal for Research in Mathematics Education, 27*(4), 403-434. doi: 10.2307/749875

Flutter, J., & Rudduck, J. (2004). *Consulting pupils: What's in it for schools?* London: Routledge Falmer.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. C. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education, 30*(2), 148-170. doi: 10.2307/749608

Fujii, T. (2014). Understanding the concept of variable through whole-class discussions: the community of inquiry from a Japanese perspective. In F. K. S. Leung, K. Park, D. Holton & D. Clarke (Eds.), *Algebra teaching around the world* (pp. 129-148). Rotterdam: Sense Publishers.

Guberman, R., & Leikin, R. (2013). Interesting and difficult mathematical problems: Changing teachers' views by employing multiple-solution tasks. *Journal of Mathematics Teacher Education, 16*(1), 33-56. doi: 10.1007/s10857-012-9210-7

Hesse-Biber, S. N. (2010). *Mixed methods research: Merging theory with practice*. New York: Guilford Press.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom Mathematics teaching on students' learning. In F. K. Lester Jr. (Ed.), *Second handbook of research on Mathematics teaching and learning* (Vol. 1, pp. 371-404). Charlotte, NC: National Council of Teachers of Mathematics, Information Age Publishing.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116. doi: 10.2307/30034933

Hulleman, C. S., Schrager, S. M., Bodmann, S. M., & Harackiewicz, J. M. (2010). A meta-analytic review of achievement goal measures: Different labels for the same constructs or different constructs with similar labels? *Psychological Bulletin, 136*(3), 422-449. doi: 10.1037/a0018947

Jang, L., & Liu, W. (2012). 2 × 2 Achievement goals and achievement emotions: a cluster analysis of students' motivation. *European Journal of Psychology of Education, 27*(1), 59-76. doi: 10.1007/s10212-011-0066-5

Jitendra, A., Star, J. R., Starosta, K., Leh, J. M., Sood, S., Caskie, G., . . . Mack, T. R. (2009). Improving seventh grade students' learning of ratio and proportion: the role of schema-based instruction. *Contemporary Educational Psychology, 34*, 250-264. doi: 10.1016/j.cedpsych.2009.06.001

Leikin, R., & Levav-Waynberg, A. (2007). Exploring mathematics teacher knowledge to explain the gap between theory-based recommendations and school practice in the use of connecting tasks. *Educational Studies in Mathematics, 66*(3), 349-371. doi: 10.2307/27822710

Leikin, R., Levav-Waynberg, A., & Mednikov, L. (2006). Implementation of multiple solution connecting tasks: Do students' attitudes support teachers' reluctance? *Focus on Learning Problems in Mathematics, 28*(1), 1-22.

Lynch, K., & Star, J. R. (2014). Teachers' views about multiple strategies in middle and high school mathematics. *Mathematical Thinking and Learning, 16*(2), 85-108. doi: 10.1080/10986065.2014.889501

Marsh, H. W. (2006). *Self-concept theory, measurement and research into practice: The role of self-concept in educational psychology.* Paper presented at the 25th Vernon-Wall Lecture, Annual Meeting of the Education Section of the British Psychological Society, Durham University.

Martin, Andrew J. (2013). Goal orientation. In J. Hattie & E. M. Anderman (Eds.), *International guide to student achievement* (pp. 353-355). New York, NY: Routledge.

Megowan-Romanowicz, M. C., Middleton, J. A., Ganesh, T., & Joanou, J. (2013). Norms for participation in a middle school mathematics classroom and its effect on student motivation. *Middle Grades Research Journal, 8(1)*, 51-76.

Middleton, J. A. (1999). Curricular influences on the motivational beliefs and practice of two middle school mathematics teachers: A follow-up study. *Journal for Research in Mathematics Education, 30*(3), 349.

Middleton, J. A. (2013). Introduction/editorial: the problem of motivation in the middle grades. *Middle Grades Research Journal, 8*(1), xi-xiii.

Middleton, J., & Jansen, A. (2011). *Motivation matters and interest counts: Fostering engagement in mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Middleton, J. A., & Toluk, Z. (1999). First steps in the development of an adaptive theory of motivation. *Educational Psychologist, 34*(2), 99-112. doi: 10.1207/s15326985ep3402_3

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

O'Toole, J., & Beckett, D. (2010). *Educational research: Creative thinking & doing*. Melbourne: Oxford University Press.

Ohtani, M. (2014). Construction zone for the understanding of simultaneous equations: An analysis of one Japanese teachers' strategy of reflecting on a task in a lesson sequence. In F. K. S. Leung, K. Park, D. Holton & D. Clarke (Eds.), *Algebra teaching around the world* (pp. 113-128). Rotterdam: Sense Publishers.

Pintrich, C. H., & Schunk, D. (2002). *Motivations in education: Theory, research, and applications* (2nd ed.). Upper Saddle River, NJ: Prentice Hall.

Putwain, D. W., & Symes, W. (2011). Teachers' use of fear appeals in the Mathematics classroom: Worrying or motivating students? *British Journal of Educational Psychology, 81*(3), 456-474. doi: 10.1348/2044-8279.002005

Rittle-Johnson, B., & Star, J. R. (2007). Does Comparing Solution Methods Facilitate Conceptual and Procedural Knowledge? An Experimental Study on Learning to Solve Equations. *Journal of Educational Psychology, 99*(3), 561-574. doi: 10.1037/0022-0663.99.3.561

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2009). The importance of prior knowledge when comparing examples: Influences on conceptual and procedural knowledge of equation solving. *Journal of Educational Psychology, 101*(4), 836-852. doi: 10.1037/a0016026

Schiefele, Ulrich , & Csikszentmihalyi, Mihaly (1995). Motivation and ability as factors in Mathematics experience and achievement. *Journal for Research in Mathematics Education, 26*(2), 163-181.

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. T. F. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *The Journal of Mathematical Behavior, 24*(3–4), 287-301. doi: http://dx.doi.org/10.1016/j.jmathb.2005.09.009

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage.

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction, 18*(6), 565-579. doi: http://dx.doi.org/10.1016/j.learninstruc.2007.09.018

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating Productive Mathematical Discussions: Five Practices for Helping Teachers Move Beyond Show and Tell. *Mathematical Thinking and Learning, 10*(4), 313-340. doi: 10.1080/10986060802229675

Stein, M. K., & Lane, S. (1996). Instructional Tasks and the Development of Student Capacity to Think and Reason: An Analysis of the Relationship between Teaching and Learning in a Reform Mathematics Project. *Educational Research and Evaluation, 2*(1), 50-80. doi: 10.1080/1380361960020103

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2014). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*, 1-18. doi: 10.1007/s10857-014-9279-2

Sullivan, P., Clarke, D., Clarke, B., & O'Shea, H. (2009). Exploring the relationship between teacher actions and student learning. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 185-193). Thessaloniki.

Sullivan, P., & Mornane, A. (2014). Exploring teachers' use of, and students' reactions to, challenging mathematics tasks. *Mathematics Education Research Journal, 26*(2), 193-213. doi: 10.1007/s13394-013-0089-0

Sullivan, P., Tobias, S., & McDonough, A. (2006). Perhaps the decision of some students not to engage in learning mathematics in school is deliberate. *Educational Studies in Mathematics, 62*(1), 81-99. doi: 10.1007/s10649-006-1348-8

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education, 10*(4-6), 217-237. doi: 10.1007/s10857-007-9038-8

Tzur, R. (2008). A researcher perplexity: Why do mathematical tasks undergo metamorphosis in teacher hands? In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepúlveda (Eds.), *Proceedings of the Joint Meeting of PME 32 and PME-NA* (Vol. 1, pp. 139-147). Morelia, México: International Group for the Psychology of Mathematics Education.

Wolcott, H. F. (2009). *Writing up qualitative research* (3rd ed.). Thousand Oaks, CA: Sage.

Woodward, J., Beckmann, S., Driscoll, M., Franke, M. L., Herzig, P., Jitendra, A., . . . Ogbuehi, P. (2012). *Improving mathematical problem solving in Grades 4 through 8: A practice guide (NCEE 2012-4055)*. Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

Zazkis, R., & Leikin, R. (2007). Generating examples: From pedagogical tool to a research tool. *For the Learning of Mathematics, 27*(2), 15-21. doi: 10.2307/40248566

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APPENDIX 1

Can you find some equations of parabolas that:

a) Cut across the x-axis twice?
b) Cut across the x-axis once?
c) Don't cross the x-axis at all?

What do you notice about each of the different groups of parabolas you have found?
Extra challenge:
Can you use your previous answers to find the equations of horizontal lines that cut across the parabola $y = x^2 - 2x$ once, twice, or not at all?

Appendix 2 – Sample questions from teachers' interview schedule

What year levels do you teach this year? Streamed?
How long have you been teaching at the school? In general?
What types of challenging tasks have you trialled with your class? Who wrote the task?
What was important to you in the design of the task? What did you want it to achieve in terms of the students' learning? engagement?
Can you describe how you set up the lesson? How was this different to your usual lessons?
What did you notice about the lesson? The students' different responses? Did anything surprise or puzzle you?
What was it about the task that some students found engaging / disengaging? Challenging? Uncomfortable?
Did you experience any "push-back" from anyone subsequent to the lesson? The students? Parents?
What do you think might make it difficult for teachers to use this sort of teaching?
Do you think the students learn better?
For what topics in Year 10 do you think the use of challenging tasks would be effective?
Have you been able to compare notes with the other teachers about their experiences?
How do you currently feel about using challenging mathematical tasks with this year level?
How would you change your approach to using challenging mathematical tasks in future?
What do you plan to do from here?
Additional comments?

Appendix 3 – Student post-task reflective questionnaire

1. I found this task ________________ because ____________________________________
2. I (choose option) *like / don't mind / don't like* doing challenging maths problems like this one because ____ ____
3. I am most engaged in my maths learning when I get to __________________________
4. I (choose option) *like / don't mind / don't like* having a peer explain their strategy for solving a problem to me because ____
5. I (choose option) *like / don't mind / don't like* having the teacher show us more than one way to solve a problem because ____
   Additional comments?

Appendix 4 – Teachers' interview coding hierarchy

Code	Number of references
School mathematics program for Year 10	6
Previous challenging tasks trialled	
Learning intentions	3
Mathematics content	3
Students' responses	5
Teachers' responses	2
Teaching strategies used	4
Issues identified	
Challenges for teachers in using challenging tasks	
Kickback	11
Knowing when to intervene	2
Lack of control	4
Understanding students' multiple strategies	1
Students' attitudes, mindset	10
Students' learning	4
Teachers' attitude, mindset	9
Traditional teaching approaches	9
Suggestions	7