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Sequenced Integration and the Identification of a Problem-Solving Approach Through a Learning Process

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Preservice teachers (N=27) in two sections of a sequenced, methodological and process integrated mathematics/science course solved a levers problem with three similar learning processes and a problem-solving approach, and identified a problem-solving approach through one different learning process. Similar learning processes used included: conjecture and test, reason, and experiment and collect data. Although the problem was solved by similar processes: 26 out of the 27 preservice teachers categorized the problem as one of mathematics because of its association with formulas, equations, and numbers. This learning process, which is not shared with science, signals a difference in the disciplines. This difference may be associated with sequenced integration, a form of integration which allows problem-solving in depth and enriches an understanding of epistemology. The implication for this study is that the current movement towards total, enhanced, and parallel integration may not allow students to strongly enrich aspects of mathematics learning.

Keywords: problem-solving, integrated education, mathematics and science education, epistemic understanding, learning process

INTRODUCTION

Standards documents in the United States [US] such as the *Next Generation Science Standards* (NGSS Lead States, 2013) and the *Common Core State Standards* (NGACBA; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) emphasize the importance of problem-solving approaches, learning processes, and epistemological understanding for the enrichment of student learning in mathematics and science. Since mathematics and science have similar problem-solving approaches (Rutherford, & Ahlgren, 1990), learning processes (Bossé, Lee, Swinson, & Falconer, 2010), and epistemologies (Lederman & Niess, 1998), it has been proposed that integrated courses focused on these similarities can enrich learning (National Academy of Engineering and National Research Council; NAENRC, 2014). Although integration has strong

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philosophical support (see Roebuck & Warden, 1998; Rutherford & Ahlgren, 1990), it is not known how to best design integrated courses (NAENRC, 2014). One reason for this problem is that integration takes on multiple forms (Hurley, 2001) and types (Miller, Metheny & Davison, 1997), and some forms of integration can impede the problem-solving processes (Berland & Busch, 2012).

Unfortunately, there is little research on the association of the forms and types of integration with the learning of problem-solving, learning processes, and epistemologies. This lack of research may be due to the recent emphasis of high stakes testing in the US which may occlude the integration of mathematics and science (Berlin & White, 2012) and the omissions of theories which guide integration in the literature (NAENRC, 2014). The purpose of this study is to investigate how preservice teachers (PSTs) in a sequenced, methodological and process integrated course (a) solved a total integration levers simulator problem, (b) explained how they solved the problem, (c) categorized the problem as one of mathematics or science, and (d) explained the reasoning for the categorization. A final purpose was to determine whether sequenced integration did or did not impede the problem-solving processes of disciplines.

State of the literature

- Multiple forms and types of integration exist which can incorporate problem-solving approaches, learning processes, and epistemologies.
- Although integration is often seen as a panacea for the enrichment of learning, it is not known which forms and types of integration are most beneficial to the learning of problem-solving, learning processes, and epistemologies.
- It seems that forms of integration that temporally separate the disciplines, like sequenced, may allow PSTs to experience both similarities and differences between mathematics and science, and allow epistemological goals to drive an uninterrupted problem-solving process by thoroughly investigating concepts.

Contribution of this paper to the literature

- Preservice teachers in two sections of a sequenced, methodological and process integrated mathematics/science course solved a levers problem with three similar learning processes and a problem-solving approach, and identified a problem-solving approach through one different learning process.
- The identification of the one different learning process, which is not shared with science, signals a difference in the disciplines and is evidence of preservice teacher learning.
- The identification of the difference between disciplines may be associated with a sequenced integration, a form of integration which allows problem-solving in depth and enriches an understanding of epistemology.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Problem-solving approaches are used by teachers to help students learn mathematics and science content, processes, and epistemological knowledge, and the *NGSS* (Achieve, Inc., 2013) and *Common Core State Standards* (NGACBA, 2010) emphasize the use of problem-solving approaches in the classroom. Research shows that teachers who learn how to problem-solve with problem-solving are more likely to use the approaches in their classrooms (Geier et al., 2008). Experimental studies with control groups have shown that teachers who have been taught how to problem-solve with problem-solving have enriched student understanding to a greater degree than teachers who have not been taught with problem-solving (Carpenter, Feneman, Peterson, Chiang, & Loef, 1989; Saxe, Gearheart, & Nasir, 2001). Therefore, it is believed that the problem-solving approaches should be used in teacher education programs so that teachers will understand the approaches and use them with their students.

Problem-solving approaches in mathematics and science are based upon constructivism (Schoenfeld, 1992). Constructivism, which is the learning theory and philosophy in which this study was investigated, postulates that students use prior knowledge and learning processes to solve problems and learn content (von Glasersfeld, 2005). Learning processes are used to construct new knowledge on prior knowledge, strengthen prior knowledge, organize prior knowledge in a more sophisticated manner, or replace an aspect of the prior knowledge with new knowledge (Cakir, 2008). The learning that occurs through problem-solving can be modeled with the processes that exist on either side of a spectrum and named assimilation and accommodation. Assimilation takes place when a student encounters a phenomenon which agrees with the student's prior knowledge. This phenomenon strengthens the structure of prior knowledge. However, connections are not made to other prior knowledge, new knowledge does not emerge, and little learning takes place. On the other side of the spectrum is accommodation. Accommodation occurs when a new phenomenon does not coincide with prior knowledge. This phenomenon creates a state of disequilibrium and the student either integrates the new knowledge into the prior knowledge or rejects the phenomenon (Wadsworth, 2003). If the student integrates the new knowledge through accommodation, a degree of learning greater than assimilation takes place.

Learning through accommodation is most likely to take place in an enriched learning environment which reflects the problems one would encounter in the real world (NRC, 2007). These problems are often solved with the assistance of peers, tools, and an instructor who scaffolds (Hmelo-Silver, Duncan, & Chinn, 2007). The problems are often presented to students with neither the strategies to solve the problem nor the solution to the problem. After students determine a strategy to solve the problem, they must present and justify their strategies and solutions to the class, and reflect on others' strategies and solutions (NCTM, 2014). Problem-solving approaches in mathematics and science vary greatly from traditional mathematics and science instructional approaches in which students are given the solution to the problem with the problem, work independently, and solve slightly different forms of the problem.

The problem-solving approach that teachers of science ask their students to employ is called inquiry, and the approach that teachers of mathematics ask their students to employ is also called problem-solving. Since inquiry and problem-solving are used in different ways in the classroom, they are conceptualized in different ways across the literature. The most detailed and operational description of inquiry was created by Minner et al. (2010) and is represented by Table 1. This table lists three aspects of inquiry instruction: (a) presence of science content; (b) student engagement; and (c) student responsibility for learning, student active thinking, and student motivation within components of instruction. The five components of inquiry instruction include: question, design, data, conclusion, and communication. The literature on mathematics education does not have an equivalent, well-defined, operationalized model; however, the *Common Core State Standards* (NGACBA, 2010) and *Principles and Standards for School Mathematics* (NCTM, 2000) emphasize a similar description of problem-solving. For example, both documents list the following as important aspects of problem-solving: (a) presence of mathematical content; (b) student engagement; and (c) student responsibility for learning, student active thinking, and student motivation. The components of problem-solving instruction may also include: question, design, data, conclusion, and communication.

In the elementary science classroom, inquiry is often placed into a model called the 5E. The name comes from the components of the model in which students participate, and includes engagement, exploration, explanation, elaboration, and evaluation (Bybee et al., 2006). Mathematics does not lend itself as easily to a model like the 5E, so the approach is often taught with NCTM processes standards (see NCTM, 2000). Bossé, Lee, Swinson, and Falconer (2010) compared the five NCTM processes standards with the 5E model and found that the disciplines shared 41 of 51 descriptors. These 41similar descriptors were further reduced to the 14 learning processes depicted in the middle column in Figure 1. These are learning processes

Table 1. Inquiry science instruction conceptual framework

Presence of **Science Content**	• Science as Inquiry • Life Science • Physical Science • Earth and Space Science
Type or **Student Engagement**	• Students manipulate materials • Students watch scientific phenomena • Students watch a demonstration of scientific phenomena • Students watch a demonstration that is NOT of scientific phenomena • Students use secondary sources (e.g., reading material, the Internet, discussion, lecture, others' data)

Components of Instruction	**Elements of the Inquiry Domain**		
	Instruction emphasizes **Student Responsibility for Learning** when it demonstrates the expectation that students will:	Instruction emphasizes **Student Active Thinking** when it demonstrates the expectation that students will:	Instruction emphasizes **Student Motivation** when:
Question	Decide which questions to investigate; seek clarification of the investigation question(s).	Generate investigation question(s); use prior knowledge to inform the question(s); consider or predict possible outcomes of the question; Explore the reasons question(s) are being asked to determine if they are appropriate for scientific investigation; refine questions so that they can be investigated; discuss questions based on previous study or data collected.	**It demonstrates the expectation that students will**: display/ express interest, involvement, curiosity, enthusiasm, perseverance, eagerness, focus, concentration, pride (all affective)
Design	Identify when and where they need help understanding the design; ensure that they (or the class/group/partner) grasps the design and how to implement it; decide what investigation design to use: ensure that the design addresses the research question.	Use prior knowledge to inform the design; determine if the design is an appropriate match for the question including variables and procedures; debate the merits or different investigation designs and whether it is "doable" and will result in needed data; consider where and how issues of bias may need to be Instruction emphasizes Student Motivation when: addressed; generate investigation designs.	
Data	Decide the data organization strategy; decide what data collection strategy to use and/or how to adapt it; identify if they or others need help collecting or organizing data; seek out clarification and advice when it is needed.	Alter and refine their approach to gathering. recording, or structuring the data based on information they acquire as they proceed.	
Conclusion	Decide what strategies to use to summarize, interpret or explain the data; identify when they or others need help in summarizing, interpreting or explaining: and. seek out other relevant information to assist in drawing conclusions	Ensure that their conclusions are supported by their data; apply prior knowledge to summarize, interpret, or explain the data; construct conclusions; consider conclusions' reasonableness and credibility; identify applications or their findings to other situations and/or contexts; offer explanations for variations in the findings among the class and/or within their working groups; generate new questions that arise out of their explanations.	
Communication	Decide how to structure their communication seek advice and suggestions from others about how/what to communicate; provide feedback to others about their communication.	Engage in sound discussion and debate; demonstrate the logic they used to draw conclusions and interpretations; articulate the reasonableness and credibility of others' work; discuss appropriate communication mechanisms including language, visual aids, technology, etc., articulate the merits and limitations of their work.	

Note: Adapted from "Inquiry Science Instruction Conceptual Framework," by D. D. Minner, A. J. Levy, and J. Century, 2010, Journal of Research in Science Teaching, 47, p. 479. Copyright 2009 by Wiley Periodicals, Inc.

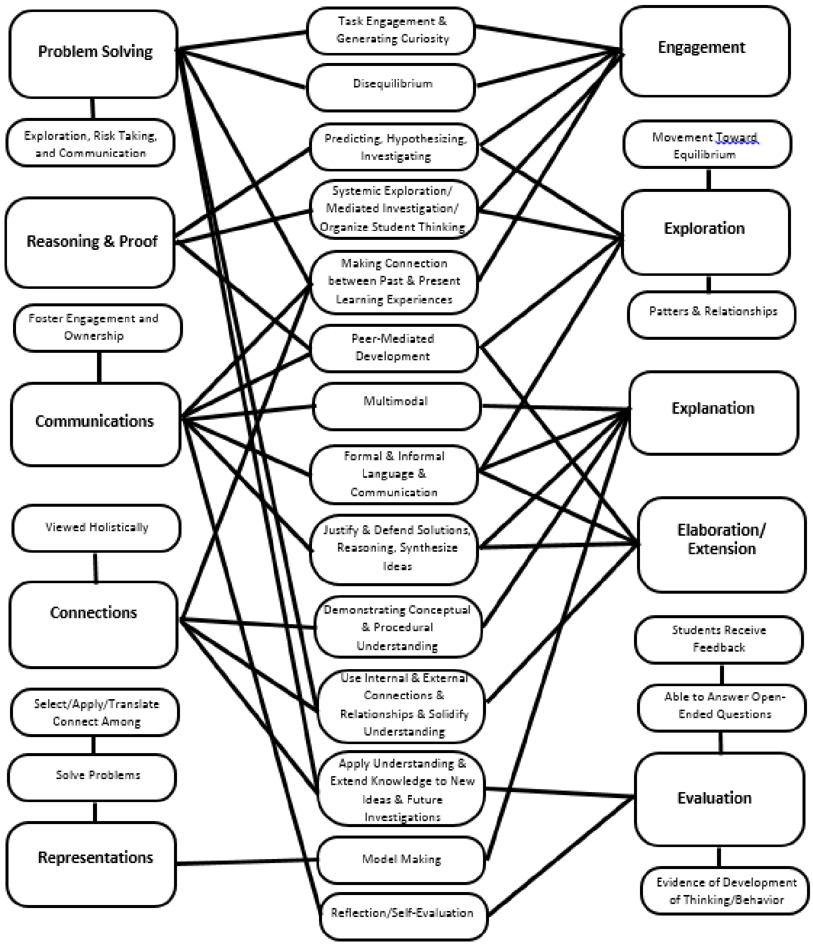

Figure 1. Connections among learning processes from mathematics and science education.
Adapted from "The NCTM Process Standards and the Five Es of Science: Connecting Mathematics and Science" by Bossé, Lee, Swinson, and Falconer, 2010, School Science and Mathematics, 110(5) p. 269. Copyright 2010 by School Science and Mathematics.

shared by both disciplines. For example, Communication, a NCTM mathematics learning process, and Elaboration/Extension, a 5E model component, both share a descriptor which asks students to justify solutions (i.e., Justify & Defend Solutions, Reasoning, Synthesize Ideas).

Multiple forms and types of integration exist which can incorporate problem-solving approaches, learning processes, and epistemologies. The multiple forms or degrees of integration include (a) sequenced, (b) parallel, (c) partial, (d) enhanced, and (e) total (Hurley, 2001). Sequenced occurs when the disciplines are planned together, share some form of connection, but are taught separately. Parallel occurs when the disciplines are taught and planned concurrently through parallel concepts. Partial occurs when the concepts are taught partially together and partially separate. Enhanced is when one discipline dominates the others, and total integration completely blurs the lines between the disciplines to the point where a discipline is not apparent (Hurley, 2001). The five types of integration include (a) discipline specific, (b) content specific, (c) process, (d) methodological, and (e) thematic (Miller, Metheny & Davison, 1997). Discipline specific is centered on two or more branches of one discipline, content specific is creating a lesson or activity which uses an objective from mathematics and science, process is centered on the processes of mathematics and science, methodological is centered on the problem-solving approaches of mathematics and science, and thematic is based on a themes which can be investigated within the disciplines.

Although integration is often seen as a panacea for the enrichment of learning, it is not known which forms and types of integration are most beneficial to the learning of problem-solving, learning processes, and epistemologies. The little literature which exists provides some insight. Koirala and Bowman (2003) studied a team-taught, middle-level, integrated course which was centered on constructivist philosophy, processes, and learning cycles, but not specifically on inquiry and problem-solving. All five types of integration were used and the form of integration appeared to be parallel. PSTs' understanding of integration was increased, but they became frustrated when integration became difficult. The authors found it challenging to explain to PSTs that mathematics and science epistemologies do not have to match because the disciplines ask different questions. This disequilibrium, in turn, helped enrich PSTs' understanding in the epistemological similarities and differences of the disciplines. Berland and Busch (2012) studied 15 middle and high school inservice teachers who participated in a six-week summer professional development endeavor in the teaching of engineering. The form of integration can be best described as enhanced integration, as engineering was the dominate discipline and mathematics and science were secondary. The type of integration seemed to be methodological since similar problem-solving and epistemic commitments of engineering, science, and mathematics were the focus. The researchers studied how teachers, who had expertise in mathematics and science, applied their knowledge in engineering decision-making. It was discovered that teachers used engineering problem-solving, learning processes, and epistemology to solve the problems. Although mathematics and science concepts were applied in solving the engineering problems, the disciplines were rarely investigated in depth as problem-solving approaches were disrupted. Author (2013) studied a sequenced, methodical and process integrated course centered on problem-solving approaches, learning processes, and epistemology. PSTs participated in an inquiry lesson on biological classification and a problem-solving lesson on polyhedra. PSTs successfully reconciled an epistemological issue by determining why it was possible to derive a mathematical formula for similarly-grouped polyhedra and not similarly-grouped organisms. This study showed that PSTs could identify and explain the reasoning for a slight difference between the disciplines. This learning may be due to the sequenced form of integration which allowed the problem-solving approaches to progress without disruption from competing epistemic commitments.

Based on the limited literature review it seems that forms of integration that temporally separate the disciplines, like sequenced, may allow PSTs to experience both similarities and differences between the disciplines, and allow epistemological goals to drive an uninterrupted problem-solving process by thoroughly investigate mathematics and science concepts. Therefore, a course was created which separated the disciplines with sequenced integration, and connected the disciplines by methodological and process integration. These types and form of integration meant that PST were taught science for the first half of the course and mathematics for the second half, and entire course was centered on the similarities and differences between problem-solving approaches, learning processes, and epistemologies of mathematics and science.

During the course, PSTs had multiple experiences participating in and teaching problem-solving lessons. At the end of the course, PSTs were asked to solve and categorize a total integration problem which could be solved with either inquiry or problem-solving. It was important to investigate how PSTs solved the total integrated problem in order to determine which similar and different learning processes were used, and why they categorized the problem as mathematics or science. These questions are important because they illustrate (a) if and how PSTs see the differences between mathematics and science, (b) if these differences are based on different learning processes between the disciplines, and (c) what similar learning processes are used to solve the problem, and (d) if sequenced integration is associated with the PST responses.

RESEARCH QUESTIONS

1. How do PSTs solve a problem which could be solved by different problem-solving approaches and similar learning processes?
2. After solving the problem, do PSTs categorize the problem as one of mathematics or science?
3. What reason do PSTs give for the categorization of the problem as one of mathematics or science?

METHODOLOGY

A constructivist research paradigm was used to answer the research questions because no theory based on empirical evidence had been posited which could capture the complexity of the PSTs' responses through quantitative means. Constructivism, which uses a relativistic and pluralistic view of reality (Guba & Lincoln, 1994), was chosen because the study's data and analysis is believed to be variable and transformable depending on one's perspective. The constructivist research paradigm used was grounded theory because it could answer the research questions by identifying and explaining the social processes. The constant comparison method, a method that is often used in ground theory, was used to answer research questions one and three. The constant comparison method was used because it can answer the research questions by comparing data with data, inducing themes, and creating categories.

Participants

Two sections of a sequenced, methodological and process integrated science and mathematics education course at a northeastern, United States masters-level college were analyzed. PSTs (N=27) were students in a dual undergraduate elementary/special education major and were required to take an integrated course titled "Teaching Science and Mathematics in the Elementary School." The PSTs were sophomores and juniors between the ages of 19 and 21; 26 of the 27 PSTs were female; all had taken at least one college-level mathematics course; and five had taken at least one college-level science course. PSTs were also enrolled in a complementary practicum course which met once a week in a local, low socioeconomic status, elementary school.

Procedure

The length of the course was 15 weeks with the first seven weeks of the course devoted to science education and the second seven weeks to mathematics education. Classes were held three days a week (Tuesday, Wednesday, and Friday) for 50 minutes, and both sections were taught by the same instructor. PSTs were asked to participate in 11 problem-solving lessons throughout the course. This included six inquiry 5E lessons for science, four problem-solving lessons for mathematics, and the total integration levers simulator problem. PSTs participated in inquiry 5E lessons on Tuesdays during Weeks one to four and six to seven, and problem-solving lessons for mathematics on Tuesdays during Weeks ten to 13. The purposes of these lessons were for PSTs to: (a) experience and use problem-solving approaches and learning processes; (b) enrich their science and mathematics content knowledge; (c) determine how to create similar, developmentally appropriate lessons for elementary students; and (d) understand the epistemologies of mathematics and science.

The problems were created by the instructor who took into account the PSTs' collective prior knowledge, learning processes, and experiences. The instructor presented the problems to the PSTs without the solutions, and the PSTs engaged in a productive struggle to solve the problems. The problems were challenging and meant to create some form of cognitive dissonance: This prompted the instructor to use scaffolding when warranted. In order to solve the problems, the PSTs reasoned, worked in groups and with tools, and presented and justified their strategies and solutions to the class. Table 1, the inquiry science instruction conceptual framework, can be used to describe the inquiry problems (Minner et al., 2010). The framework can also be used to describe problem-solving since it is closely-related to inquiry. All 11 lessons had science or mathematics content, some type of student engagement, and an emphasis on student motivation. Eight of the 11 lessons included all components of instruction - with the expectation of questioning - for student responsibility for learning and student active thinking. The other three lessons only emphasized conclusion and communication, as components of instruction, for student responsibility for learning and student active thinking. The Appendix lists the components of instruction and summaries for all lessons.

Since the course was also process integrated, it was designed for PSTs to experience all 51 learning processes descriptors, the 41 similarities, the 10 differences, and the 14 reduced descriptors during the problem-solving lessons. However, these processes were not documented during the course and some may have been omitted or modified. PSTs were also asked to create and teach problem-solving lessons to students at a local elementary school on a weekly basis. PSTs were placed in pairs or triads, given concepts to teach by the classroom teachers, and asked to create their lessons. They created ten lessons in all: five inquiry 5E science lessons, three mathematics problem-solving lessons, and two mathematics direct instruction lessons.

The 11 problem-solving lessons taught to PSTs during the 14 weeks follow. Due to restraints related to length, each summary is centered on the problem which PSTs had to solve. For example, Week 3 describes a lesson on owl pellets. This inquiry 5E lesson had five parts, but since the exploration potion of the 5E is most closely associated with problem-solving, only this portion of the lesson was described. The summary of the problem-solving lessons is as follows: 1. Dry Ice, Week 1: PSTs were given a piece of dry ice, a metal utensil, and a beaker of warm water. They were asked to perform the following and to record observations and inferences for each task (a) place dry ice on table and observe it, (b) push down on the dry ice with a metal utensil, and (c) drop the dry ice into the container filled with warm water. After PSTs finished the three tasks, they were asked to answer the following question in pairs: Please use your data and prior knowledge to explain why you believe the dry ice "behaved" as it did for each experiment. All three of your observations can be explained with one big idea. After PSTs solved the problem, they were asked to present and justify their reasoning and solution to the class. 2. Circuits, Week 2: The instructor flicked the classroom lights on and off multiple times and asked the PSTs to explain the phenomenon and draw a schematic. The PSTs were placed in pairs and given a bulb, two wires, and a battery, and asked to light the bulb. The PSTs were then asked to (a) explain how they got the bulb to light and to draw a schematic illustrating the circuit, and (b) describe the relationship between the two schematics (adapted from Harvard-Smithsonian Center for Astrophysics, 1997). After PSTs solved the problem, they were asked to present and justify their reasoning and solution to the class. 3. Owl Pellet, Week 3: PSTs were placed in pairs and were given a magnifying glass, probes, and a mystery object (owl pellet wrapped in aluminum foil). They were asked to investigate the object by dissecting it and to try to identify it. After PSTs solved the problem, they were asked to present and justify their reasoning and solution to the class. 4. Preserved Organisms, Week 4: PSTs were presented with a three-set Venn diagram worksheet labeled annelids, cnidarians, and chordates. Preserved organisms in plastic jars were placed on three separate tables in the classroom. Four annelids were placed on one table, four cnidarians on another table, and five chordates on a third table. PSTs were asked to find—using observation, inference, reasoning, and prior knowledge—the similarities within the organisms on each table and the differences between the groups of organisms at each table. These characteristics were to be written in the appropriate six spaces on the three-set Venn diagram (see Author, 2013). After PSTs solved the problem, they were asked to present and justify their reasoning and solution to the class. 5. Simulators, Week 5: Due to a holiday, the week was abbreviated and no inquiry 5E lesson was presented. In place of an inquiry lesson, PSTs were asked to investigate elementary level simulators at http://phet.colorado.edu/en/simulations/category/by-level/elementary-school. They then presented the simulators and how they could use the tool in their practicum class to others. 6. Electromagnets, Week 6: PSTs were placed in pairs and given a steel nail, battery, wire, and a box of steel paper clips. They were told the following: The iron in your nail is composed of billions of electrons that point in random directions. The nail becomes magnetized when all or most of the electrons point in the same direction (become aligned). You can make an electromagnet by aligning the randomized electrons in your nail with an electric current. Electric currents that run through circuits have the ability to align the randomized electrons when they are close enough to the nail. PSTs were then asked to create a circuit and, using the information above and their prior knowledge on circuits, pick up two paper clips with the nail. After PSTs solved the problem, they were asked to present and justify their reasoning and solution to the class. 7. Forces and Motion, Week 7: PSTs were placed in pairs, given laptops, and asked to go to the following link: http://phet.colorado.edu/en/simulation/forces-and-motion. PSTs were then told the following: Four forces act upon an object which is pushed. If a crate is pushed, this force is called applied force. But there are three other forces acting on the crate…gravity, friction, and the support force of the ground. The crate moves when there is a sum force. Create a mathematical formula which approximately describes the relationship between the applied force (AF) and the object mass (OM). For example, if I told you that the object mass of the crate was 50kg, how would you find the value of the applied force if you could not use the simulator? Your formula should contain at least the following symbols: OM, AF, =. After PSTs solved the problem, they were asked to present and justify their reasoning and solution to the class. 8. Least Common Numerator, Week 8: PSTs were given a direct instruction lesson on a novel concept, the least common numerator (adapted from NCTM, 1991). 9. Traditional Algorithm for Division, Week 9: PSTs were given a direction instruction lesson on how to teach the traditional algorithm for division to children. This was modeled with base-ten materials. 10. Algebra and Tug-of-War, Week 10: PSTs were given the following problem and asked to solve it in pairs: Who will win the third round of this tug-of-war? Round one: On one side are four acrobats, each of equal strength. On the other side are five neighborhood grandmas, each of equal strength. The result is dead even. Round two: On one side is Jamal, a dog. Jamal is pitted against two of the grandmas and one acrobat. It's a draw. Round three: Jamal and three of the grandmas are on one side, and the four acrobats are on the other (adapted from Burns, 1996). PSTs could use manipulatives and tools (e.g., counters, polygon blocks, paper) to solve the problem. After they solved the problem, they were asked to present and justify their strategy and solution to the class. 11. Rates and Paper-Shredding, Week 11: PSTs were given the following problem and asked to solve it in triads: Ron's Recycle Shop started when Ron bought a used paper-shredding machine. Business was good, so Ron bought a new paper-shredding machine. The old machine could shred a truckload of paper in four hours. The new machine could shred the same truckload of paper in only two hours. How long will it take to shred a truckload of paper if Ron runs both machines at the same time (adapted from Van de Walle, Karp, & Bay-Williams, p. 16, 2013)? PSTs could use manipulatives and tools (e.g., counters, colored tiles, scissors, paper) to solve the problem. After they solved the problem, they were asked to present and justify their strategy and solution to the class. 12. Probability and Spinners, Week 12: PSTs were given the following problem and asked to solve it in triads: Three students are spinning to "get purple" with two spinners, either by spinning first red and then blue or first blue and then red. They may choose to spin each spinner once or one of the spinners twice. Mary chooses to spin twice on spinner A; John chooses to spin twice on spinner B; and Susan chooses to spin first on spinner A and then on spinner B. Who has the best chance of getting a red and a blue (adapted from Van de Walle et al., p. 18, 2013)? PSTs could use manipulatives and tools (e.g., spinners, colored tiles, paper) to solve the problem. After they solved the problem, they were asked to present and justify their strategy and solution to the class. 13. Euler Formula and Polyhedra, Week 13: PSTs were given individual laptops and asked to explore the following simulator: http://illuminations.nctm.org/ActivityDetail.aspx?ID=70 (NCTM, 2012). PSTs were then asked to: (a) work in pairs to find the relationship between the number of faces, vertices, and edges for the five polyhedra; and (b) create a formula by symbolically reducing/abstracting the relationship between faces, vertices, and edges. The instructor also informed the PSTs that their formula should include, at the very least, the symbols V, E, F, =. (see Author, 2013). PSTs could use manipulatives and tools (e.g., applet, paper) to solve the problem. After they solved the problem, they were asked to present and justify their strategy and solution to the class.

During the 14th week of the course, PSTs participated in a levers simulator lesson which was centered on a simulator titled Balancing Act (PhET, 2011). The problem was a total integration problem which blurred the lines between disciplines and could be solved by either problem-solving approach. At the start of the lesson, PSTs were presented with Figure 2: a screen capture of the simulator which showed a fire extinguisher and a garbage can balanced on a lever. The fire extinguisher weighed 5kg and was 2m left of the fulcrum and the trash can weighed

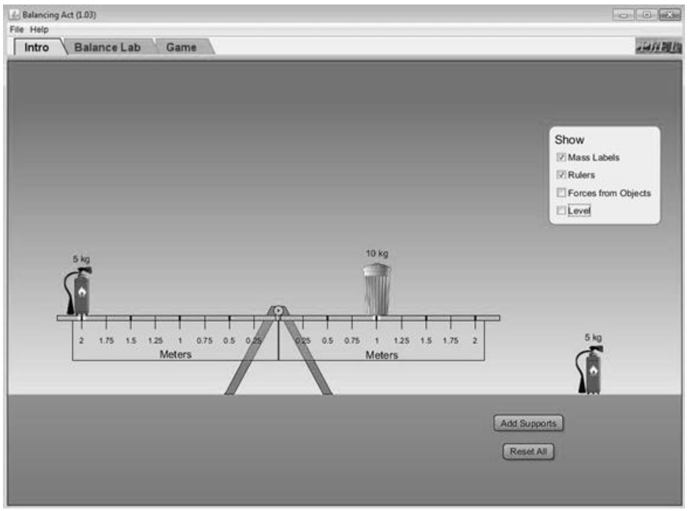

Figure 2. Screen capture of PhET Balancing Act

PhET: Balancing Act by University of Colorado, 2011, http://phet.colorado.edu/en/simulation/balancing-act. Copyright 2013 by University of Colorado.

10kg and was 1m right of the fulcrum. The PSTs were not told if the problem was a science or mathematics problem and were asked to solve it using the approach of their choice. PSTs were then asked to create a formula or equation which could predict how these items could be balanced if moved. For example, if the fire extinguisher were moved to 1m, where would one move the trash can to balance the lever? What if the fire extinguisher were moved to 1.5m? PSTs were told that one can solve the problem by simply looking at the screen capture, experimenting with the simulator, or using both.

If PSTs wanted to use the simulator, they would have to take a laptop from a laptop cart in the room. After the instructions, the PSTs were given approximately 30 minutes to work in groups and complete the task. During this time, some PSTs began to solve the problem on a sheet of paper, while others got a laptop and experimented with the simulator. Meanwhile, the instructor moved among the groups, probed the PSTs' ideas, and asked for explanations to the reasoning while not evaluating answers. Typical responses to probing included "I experimented with the simulator until I noticed a pattern (Student 1, personal communication, April 2013)," "I remember doing a problem like this in high school, and I checked my idea with the screen capture (Student 2, personal communication, April 2013)," and "I had no idea how to solve the problem so I played with the simulator (Student 3, personal communication, April 2013)." All 27 PSTs derived a formula which could correctly solve the problem.

Materials and data collection

As each PST completed the task, the instructor passed out a worksheet with the following questions: (a) How did you solve the problem? Did you use the simulator or only paper?; and (b) You have participated in inquiry-based 5E activities which dealt with dry ice, lighting bulbs, owl pellets, preserved organisms, and electromagnets. You also participated in problem-solving mathematics activities which dealt with acrobats and algebra, paper-shredding and rate, polyhedra applets and formula, and spinners and probability. Based on these activities, do you think today's activity was more of a mathematics or science problem (please only pick one)? Why do you believe this? After 30 minutes, the PSTs were asked to place their formula and accompanying work on a document camera which projected to a Smart Board. PSTs were asked to be prepared to present their formula and reasoning, and to justify their rationale to anyone who disagreed or did not understand.

Data analysis

Research questions one and three were answered by using the constant comparison method and research question two was answered by simply tallying responses. The method started with coding responses on the PSTs' worksheets by attaching codes, or labels, to pieces of text that were relevant to a particular theme or idea. Then passages of text from the worksheets were grouped into patterns according to the codes and subcodes (Miles & Huberman, 1994), and a focus coding table was created. After looking over the worksheet responses along with the focus coding table, memos were written to study and understand the PSTs' responses. All of the themes are expanded upon in the results and discussion portion of this study.

RESULTS

The answer to research question one is that PSTs solved the problem with conjecture and test 48% (13/27), reason 44% (12/27), and experiment and collect data 41% (11/27). Table 2 provides a description of PSTs' responses to research question one. The answer to research question two is 26 out of 27 PSTs categorized

Table 2. Memos

Conjecture and test
Conjecture and test was a theme that emerged in response to how PSTs solved the problem. This theme appeared in 13 out of 27 responses (48%). PSTs stated that they either derived the formula from the screen capture or after having a conjecture and experimenting with the simulator. After PSTs had a conjecture, they tested it. PSTs' responses included: "I knew I had an idea of how the formula was going to work….[and] I knew I was [heading] in the right direction. I used the applet, applied my equation to three problems [and] knew that it was right" (Student 11, personal communication, April 2013). "I applied my theory to the applet, and it proved to be [correct]." (Student 12, personal communication, April 2013). "After I made my prediction, I tested my formula with the applet" (Student 13, personal communication, April 2012). "[W]e predicted that since the weight of the fire extinguisher is 5kg and the weight of the trash can is 10kg, the trash can is = fire extinguisher/2. We pulled up the applet and tested our equation… " (Student 13, personal communication, April 2013).
Reason
Another theme that emerged in response to how PSTs solved the problem was reason. This theme appeared in 12 out of 27 responses (44%). PSTs used reasoning to explain their insight toward problem-solving and as a way to formalize and defend their thoughts to others. PSTs' responses included: "I knew that the fire extinguisher is ½ the weight of the trash can and the trash can is ½ the distance of the fire extinguisher. This means that if you move the fire extinguisher to 1 [meter], the trash can would have to move to .5 [meters] for the two objects to balance. And if the fire extinguisher was moved to 1.5 [meters], the trash can would have to be moved to .75 [meters], ½ of 1.5 [meters]" (Student 14, personal communication, April 2013). "The equation…[is]…$d_1=2xd_2$. This equation makes sense because the lighter object must have a greater distance to balance an object of greater weight with a closer distance" (Student 15, personal communication, April 2012). "I used the applet to solve for the distance of the 10kg [trash can] when the 5kg [fire extinguisher] was at 1 meter. I then moved the 5kg [fire extinguisher] to .5 meter and used the applet to find the distance for 10kg [trash can] again. I then wrote the equation out and replaced one of them with X. I knew that mass one times distance one was equal to mass two times distance two because they had been balanced on the applet" (Student 17, personal communication, April 2013).
Experiment and collect data
The last theme that emerged in response to how PSTs solved the problem was experimenting and collecting data. Some PSTs used the simulator to collect data and inductively derive a formula, while others deduced a conjecture from the screen capture and created scenarios with drawings. This theme appeared in 11 out of 27 responses (41%). PSTs' responses included: "To solve the problem, we experimented with the computer simulation to test all the possibilities" (Student 18, personal communication, April 2013). "I used the simulation to figure out the problem. I…[created multiple balanced scenarios]…and then made a chart showing where each object should be placed in relation to each other to achieve balance" (Student 19, personal communication, April 2013). "I solved the problem by creating two columns and putting down which masses they both balanced at. Once I did this, I observed all the numbers and looked at the relationship of the numbers" (Student 20, personal communication, April 2012). "I did not use the applet…Instead, I used paper and created a number line with jumps...[so I could]…manipulate the numbers in viewing relationships. I also prefer to work on paper because I can write down thoughts right after solving a relationship. I used the number line to figure out that the fire extinguisher or distance is = to 2x the distance of the trash can." (Student 21, personal communication, April 2013).

Note: PSTs' responses to research question one.

the problem as one of mathematics. Only one PST stated that it was a science problem. The answer to research question three reports that 89% of the PSTs categorized the total integration levers simulator problem as one of mathematics because the problem was associated with formulas, equations, and numbers. Table 3 provides a description of PSTs' responses to research question three.

To conjecture and test, reason, and experiment and collect data are aspects of inquiry and problem-solving, and are shared learning processes found in the middle column of Figure 1. The third shared learning process in the middle column is Predicting, Hypothesizing, and Investigating. In science, hypotheses are tentative statements which serve as an explanation for some phenomenon in the natural world. Hypotheses or aspects of hypotheses can be systematically tested. Hypotheses do not become theories, but they can contribute to or shape theories. In mathematics, similar tentative statements are called conjectures, and less often hypotheses. Through rigorous deductive reasoning, conjectures can be proven to form theorems about the mathematics world. The main difference between scientific hypotheses and mathematical conjectures is that observed evidence in the natural world is necessary for the former but not the latter. PSTs and their students should be able to make and test mathematical conjectures and scientific hypotheses (NGSS Lead States, 2013; NGACBA, 2010). To some degree, all 11 problem-solving lessons in the course asked PSTs to make and test conjectures or hypotheses.

Table 3. Memos

Formulas, equations, numbers
A theme that emerged most often was Formulas/Equations/Numbers. This theme appeared in 24 out of 27 responses (89%). These terms were used almost interchangeably to describe the problem as one of mathematics. PSTs' responses included: "The problem dealt with finding the relationships between various numbers and a formula [to represent] it" (Student 4, personal communication, April 2013). "We used numbers to balance out an equation" (Student 5, personal communication, April 2013). "…[I]t involves coming up with an universal equation and using numbers that applies [sic] to all situations…" (Student 6, personal communication, April 2012). "Since we had to find an equation and calculate our answers, we were using mathematics tools" (Student 7, personal communication, April 2013). "We created a formula that could be applied to multiple variables and still be accurate every time…" (Student 8, personal communication, April 2013). "[W]e had to create a formula using numbers that came from data" (Student 9, personal communication, April 2013). "I think this is more of a mathematics problem because it involves numbers" (Student 10, personal communication, April 2013).

Note: PSTs' responses to research question three.

Predicting, Hypothesizing, and Investigating are also slightly different in each discipline and can be found in either column of Figure 1. NCTM Reasoning and Proof process standards, which are found in the first column, emphasize the importance of conjecture in mathematics. PSTs and their students should be asked to make and investigate conjectures and use conjectures toward proofs. In the total integration levers simulator problem, nearly half of all students made conjectures and moved toward proving their conjectures. This was done by continuously testing and reevaluating conjectures throughout the problem-solving process. Although PSTs did not formally prove the formula they used in the levers problem, many were using the necessary mathematical reasoning to prove it. PSTs and their students must do the same in science. Hypothesis, which is part of Exploration and found in the third column in Figure 1, is part of the inquiry 5E model. Most of the PSTs did not have a scientific hypothesis for the levers problems because they didn't know the theory necessary to solve the problem. However, the next time they are asked to solve a similar problem, they will have a strong understanding of how to balance a lever and will be able to propose a hypothesis to solve the problem.

Reason appears in the shared column in Figure 1 as Justify & Defend Solutions, Reasoning, Synthesize Ideas. Reason is used in science and mathematics to construct and formalize ideas, to express insight into problem-solving, and to take steps toward testing ideas in defendable ways. In mathematics, PSTs and their students should be able to use reason to develop and evaluate conjectures, and ultimately prove these arguments when applicable. In science, PSTs and their students should use reason to develop and evaluate hypotheses, and either support or refute their hypotheses. In the levers problem, PSTs used reason to correctly solve the problem and to explain and justify their formulas on paper. When they wrote their response to the problem, they wrote it in a way which defended their strategy and solution. This same reasoning was used when presenting their strategy and solution to their classmates on the document camera. Presentation allowed them to create clear and organized arguments; this was especially helpful when others either disagreed or could not understand the presenter's reasoning. Reason was used in all 11 problem-solving activities in the course, and the justification of reasoning always occurred when PSTs presented their solutions and strategies to others.

Experiment and collect data are closely associated with Predicting, Hypothesizing, Investigating; which is found in the center column of Figure 1. PSTs and their students should understand how to systemically experiment and collect data to investigate and solve problems. Experiment and collect data is more closely associated with science than mathematics because science often uses experimentation to gather information from and about the natural world. Data that is collected through experimentation can be analyzed to become evidence which either supports or refutes a hypothesis, theory, or related mental schema. Although science is more closely associated with experimentation and data collection, mathematics, and more often than not applied mathematics, is also concerned with data collection. For example, work in probability theory is often based on experimental work which yields data.

In the levers problem, PSTs inductively and abductively solved the problem by using the simulator to experiment and collect data. Some PSTs used the simulator to collect data by creating multiple scenarios and observing where the lever balanced. This data was collected and a pattern emerged which helped one derive the formula. Others solved the problem abductively by using the simulator to test their conjectures. Interestingly, some PSTs deduced a conjecture from the screen capture and created scenarios with drawings. Even with only the use of drawings, they experimented on paper and collected data to either support their conjecture or derive a formula. To some degree, all 11 problem-solving lessons listed in the course asked PSTs to experiment and collect data.

Twenty-four out of 27 (89%) PSTs categorized the total integration levers simulator problem as one of mathematics because the problem was associated with formulas, equations, and numbers. As to be expected, this difference between the disciplines is not found in the middle column in Figure 1 because it is a difference. Formulas, equations, and numbers are most closely associated with Representations - Solve Problems which is found in the first column. Representations like formulas, equations, numbers, and other symbols are often used as tools in mathematical problem-solving (NCTM, 2000). These representations are usually flexible, reflect the problem-solving process, and can be used to communicate and justify information about the problem to others (Greeno & Hall, 1997). Although science may use forms of representation like models, the abstracted nature of mathematics requires representation for the mathematics problem-solving process. In the levers problem, PSTs used formulas, equations, or numbers as a representation of how the lever balanced when objects were moved. For example, Table 3 reports that one PST used $d_1=2xd_2$ as a model to represent the behavior of the lever (Student 15, personal communication, April 2012). This formula can explain the behavior of the lever if one knows the fire extinguisher's (d_1) and trash can's (d_2) distances from the fulcrum. In theory, this representation can predict the lever's behavior if the objects are very far from the fulcrum. Representations were used as a tool to solve mathematical problems in the other four mathematics problem-solving lessons.

DISCUSSION

PSTs solved the levers problem with three similar learning processes and a problem-solving approach, and identified the problem-solving approach through one different learning process. The results of the study show that sequenced integration in conjunction with methodological and process integration allows PSTs to: (a) enrich their understanding of epistemology by experiencing the similarities and differences between mathematics and science processes, and (b) enrich their science and mathematics understanding by using epistemological goals to drive an uninterrupted problem-solving process. This study informs integration theory because the integrative nature of the course may have influenced PSTs to thoroughly investigate, solve, and identify the levers problem as mathematics, and identify the different learning process as formulas, equations, or numbers. The ability to accomplish these tasks is evidence of learning. If the course was centered on other forms of integration (e.g., total, enhanced, or parallel), PSTs may not have been able to name the different learning process and correctly identify the problem with the associated problem-solving approach. This is because PSTs would have had no experiences in comparing the differences between mathematics and science, the two problem-solving approaches, or the 10 different learning processes. If total, enhanced, or parallel integration were exclusively used in the course, PSTs would have experienced only similarities: This includes one problem-solving approach and, at most, 14 similar learning processes.

This study also informs learning theory because PSTs were able to identify the different learning process between mathematics and science. The ability to identify this process is evidence of learning because it represents a difference between the epistemology of mathematics and science. Using constructivist learning theory to interpret the results, it can be hypothesized that the PSTs compared and contrasted their prior knowledge of the science and mathematics problems and approaches that they encountered in the courses. This reflection allowed them to organize their knowledge in a more sophisticated manner and to think about the similarities and differences between the disciplines and their approaches. Next, PSTs may have compared the levers problem to the mathematics problems that they encountered in the course. Since the levers problem was similar to their prior knowledge of mathematics problems, the problem was assimilated. The PSTs may then have compared the levers problem to their prior knowledge of the science problems. Since more differences were encountered between the levers problem and the science problems, disequilibrium occurred. After further reflection, PSTs detected a different learning process which was common to the levers problem and mathematics problems, but not the science problems. This new learning emerged through accommodation as the PSTs tried to emerge the levers problem into their prior knowledge of science problems.

This study is timely because there appears to be a recent movement towards total, enhanced, and parallel integration in science, technology, engineering, and mathematics [STEM]. These forms of integration are centered on problems which are solved in the context of a dominant discipline or with one epistemic commitment. In the recent report, *STEM Integration in K-12 Education: Status, Prospects, and an Agenda for Research* (NAENRC, 2014), there is an emphasis on these forms of integration and other kinds of STEM connections for integration. The section on STEM connections contains the following:

> Regarding the nature of connection, integrated STEM education may bring together concepts from more than one discipline (e.g., mathematics and science, or science, technology, and engineering); it may connect a concept from one subject to a practice of another, such as applying properties of geometric shapes (mathematics) to engineering design; or it may combine two practices, such as science inquiry (e.g., doing an experiment) and engineering design (in which data from a science experiment can be applied). (NAENRC, 2014, p. 42)

These examples of integration are important because they maximize the number of connections or similarities between the disciplines, and therefore, are more likely than sequenced integration to enrich certain types of knowledge. However, the report fails to recognize that when concepts are brought together from more than one discipline or a concept from one subject is applied to the practice of another, the unique epistemology of the dominate discipline guides the problem-solving process (Berland & Busch , 2012). This means that the secondary discipline's problem-solving process is interrupted, rarely investigated in depth, and may yield in an inchoate understanding of the concept (Lehrer & Schauble, 2006).

Mathematics is often a secondary discipline in solving STEM problems because its epistemological and ontological flexibility allows it to be used as a tool. Using mathematics as tool, or mainly or always as a secondary discipline, works against the vision of the US reform movement which calls for an appreciation of mathematical epistemology (NCTM, 2000). Ironically, mathematics's flexibility in solving STEM problems does not allow students to see the interconnectedness and internal consistency of the discipline, and mathematical understanding may suffer. This may be the reason that mathematics has been reported to have fewer benefits compared to other STEM disciplines during integration (see Hartzler, 2000). This also may be the reason why mathematics achievements scores are lower for total, enhanced, and parallel integration compared to sequenced. For example, a meta-analysis of 31 integration studies in K-16 education found the following differences in mathematics achievement by forms of integration: sequenced (*ES*=.85), parallel (*ES*=-11), partial (*ES*=.13), enhanced (*ES*=.17), and total (*ES*=.20) (Hurley, 2001).

A practical example may best illustrate the issue with total, enhanced, and parallel integration. It would be very difficult to integrate the following fourth grade Common Core standard with the other STEM disciplines while using mathematics as the dominate discipline: "Determine whether a given whole number in the range 1–100 is prime or composite" (NGACBA, 2010, p.29). The reason for this is mathematical problem-solving, the kind that leads to an understanding of interconnectedness and internal consistency, is not possible when guided by science, technology, and engineering questions within their more objective ontologies. In this example, prime and composite numbers must be investigated with logic in a platonistic ontology. It would not be possible to use, say, science as a secondary discipline because science uses observations from the natural world to answer its questions.

There are two major limitations in the study. The first is that there were few participants, and it may be difficult to generalize the finding to a larger population. The second is that the problem-solving approaches and learning processes used throughout the course may not have influenced PSTs' responses to the levers problem. The PSTs may have responded the same way to the levers problem if it were presented at the start of the course. Further research may help address these and other limitations. Other possible further research questions include the following: Did the problem-solving approaches and learning processes influence PSTs to use similar approaches and processes in the problem-solving lessons taught to their elementary students? Can PSTs identify which learning processes from Figure 1 were used to solve the levers problem; in other words, do they know that they used conjecture and test, reason, and experiment and collect data? Do PSTs understand the similarities and differences between scientific inquiry and mathematical problem-solving, or do they see the learning processes as a sum of their parts? And, lastly, how do the connections and integration espoused by NAENRC report (2014), compared to sequenced integration, enrich mathematics epistemology in PSTs?

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