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Student Difficulties in Mathematizing Word Problems in Algebra

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To investigate student difficulties in solving word problems in algebra, we carried out a teaching experiment involving 51 Indonesian students (12/13 year-old) who used a digital mathematics environment. The findings were backed up by an interview study, in which eighteen students (13/14 year-old) were involved. The perspective of mathematization, i.e., the activity to transform a problem into a symbolic mathematical problem, and to reorganize the mathematical system, was used to identify student difficulties on the topic of linear equations in one variable. The results show that formulating a mathematical model—evidenced by errors in formulating equations, schemas or diagrams—is the main difficulty. This highlights the importance of mathematization as a crucial process in the learning and teaching of algebra.

Keywords: algebra education, digital mathematics environment, linear equations in one variable, mathematization, word problems

INTRODUCTION

Solving word problems is among the main difficulties in algebra for many secondary school students all over the world (see, for instance, Bush & Karp, 2013; Carpraro & Joffrion, 2006; MacGregor & Stacey, 1998; Van Amerom, 2003). In Indonesia, student difficulties with solving word problems were revealed in the Trends in International Mathematics and Science Study (TIMSS) in 2007; for instance, only eight percent of the Indonesian participants were able to solve the word problem shown in Figure 1. This result was significantly below the international average of 18 percent (Mullis et al., 2008). Similar results can be found for other word problems.

To help Indonesian students to overcome these low performances in solving word problems in algebra, we wonder whether digital tools might be of value. Over the last decade, ICT-use has become widespread in mathematics education (e.g., Bokhove & Drijvers, 2010; Barkatsas, Kasimatis, & Gialamas, 2009), and research on the integration of ICT in algebra education suggests a positive influence on student

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achievement in general (Li & Ma, 2010), and in solving word problems in particular (Ghosh, 2012).

In an earlier interview study, mathematization, that is, the activity of organizing any kind of reality with mathematical means (Freudenthal, 1991; Treffers, 1987), was identified as one of the obstacles that students experience in initial algebra (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). To better understand the nature of the difficulties with solving word problems while using digital tools, we use the lens of mathematization.

To further investigate student difficulties in solving word problems from the perspective of mathematization, we set up a teaching experiment that included technology-rich lessons on solving word problems on the topic of linear equations in one variable. Here we report on this teaching experiment. Below, we first describe a theoretical background, including a brief description of difficulties in initial algebra, and the theory of mathematization. Next, the research question and methods are addressed. The results section elaborates student difficulties observed in the teaching experiment in the light of the mathematization perspective. These findings are triangulated with earlier interview data (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). Finally, we reflect upon the results in the conclusions and discussion section.

State of the literature

- Difficulties in initial algebra include applying arithmetical operations, understanding the notion of variables and algebraic expressions, understanding the different meanings of the equal sign, and mathematization.
- Mathematization is distinguished into horizontal and vertical aspects: the former concerns a transformational process from a problem to a mathematical model, and the latter concerns solving the problem expressed in the model and interpretations of the solution.
- The mathematization process has a cyclic character: understanding the problem, formulating a mathematical model from the problem, solving the problem expressed in the model, and interpreting the solution in terms of the original problem.

Contribution of this paper to the literature

- The study contributes to an understanding of student difficulties in solving word problems leading to linear equations in one variable from the perspective of mathematization.
- The study provides a concrete example of how to put into practice the lens of mathematization for identifying student difficulties in solving word problems.
- The ICT-rich approach adopted in this study does not affect the findings, compared to the results found in the interview study; this suggests a general phenomenon.

THEORETICAL BACKGROUND

Difficulties in initial algebra learning

The term "difficulties" in this section's title refers to obstacles that cause errors or mistakes made by students when dealing with algebra problems. By "initial algebra" we mean formal algebra topics—such as arithmetical operations on algebraic expressions, and linear equations and inequalities in one variable—which are in the curriculum for 12-14 year-old students in Indonesia as in many other countries.

From the existing research literature and from an interview study, we earlier identified the following five categories of difficulties in initial algebra (Jupri, Drijvers & Van den Heuvel-Panhuizen, 2014):

- The category of applying arithmetical operations in numerical and algebraic expressions (abbreviated as ARITH) includes difficulties in adding or subtracting similar algebraic terms (e.g., Herscovics & Linchevski, 1994; Linchevski, 1995); also difficulties in using associative, commutative, distributive, and inverses properties; and in applying priority rules of arithmetical operations (e.g., Booth, 1988; Bush & Karp, 2013; Warren, 2003).
- The category of understanding the notion of variable (VAR) concerns difficulties to distinguish a literal symbol as a variable that can play the role of a placeholder, a generalized number, an unknown, or a varying quantity (Booth, 1988; Bush & Karp, 2013; Herscovics & Linchevski, 1994).

Joe knows that a pen costs 1 zed more than a pencil. His friend bought 2 pens and 3 pencils for 17 zeds. How many zeds will Joe need to buy 1 pen and 2 pencils?

Show your work.

Figure 1. TIMSS 2007 algebra word problem (Mullis et al., 2008)

- The category of understanding algebraic expressions (AE) encompasses the parsing obstacle, the expected answer obstacle, the lack of closure obstacle, and the gestalt view of algebraic expressions (Arcavi, 1994; Thomas & Tall, 1991). The parsing obstacle in this case refers to understanding the order in which the algebraic expressions must be processed, the expected answer obstacle concerns the expectation to get a numeric result rather than an algebraic expression, and the lack of closure obstacle refers to discomfort in handling algebraic expressions that cannot be simplified any further.
- The category of understanding the different meanings of the equal sign (EQS) concerns difficulties in dealing with the equal sign, as an equal sign in arithmetic usually invites a calculation, while it is a sign of equivalence in algebra (Bush & Karp, 2013; Herscovics & Linchevski, 1994; Kieran, 1981).
- Finally, the category of mathematization (MATH) concerns the difficulty to translate back and forth between the world of the problem situation and the world of mathematics, and in the process of moving within the symbolic world (Treffers, 1987; Van den Heuvel-Panhuizen, 2003).

The first four categories are elaborated in Jupri, Drijvers, and Van den Heuvel-Panhuizen (2014). To shed new light on student difficulties in dealing with word problems, the present paper focuses on the fifth category of mathematization.

Mathematization

The notion of mathematization originates from the theory of Realistic Mathematics Education (RME). It refers to the activity of organizing and studying any kind of reality with mathematical means, that is, translating a realistic problem into the symbolic mathematical world, and vice versa, as well as reorganizing and (re)constructing within the world of mathematics. 'Reality' can either refer to real life, to fantasy world, or to mathematical situations as far as they are meaningful and imaginable to the student, for example because their essential elements have been previously experienced and understood by the student (Freudenthal, 1991; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2000; Van den Heuvel-Panhuizen & Drijvers, 2013).

Within mathematization, horizontal and vertical mathematization are distinguished (Treffers, 1987; Van den Heuvel-Panhuizen, 2003). Horizontal mathematization refers to the activity of transferring a realistic problem to a symbolic mathematical problem through observation, experimentation, and inductive reasoning (Treffers, 1987). Activities that characterize horizontal mathematization include, for instance, identifying the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, and discovering relations (De Lange, 1987). Solving word problems—including the problems that combine both symbolic expressions and natural language—appeals to horizontal mathematization.

Vertical mathematization refers to the activity of reorganizing and (re)constructing within the world of symbols which includes solving the problem, generalization of the solution and further formalization (Treffers, 1987). Activities that characterize vertical mathematization include, for instance, manipulating and refining mathematical models, using different models, combining and integrating models, and generalizing (De Lange, 1987). Freudenthal (1991) points out that vertical mathematization includes both mechanical—in the sense of automatized procedures—and comprehensive aspects of reorganizing and (re)constructing within the world of symbols: "... symbols are shaped, reshaped, and manipulated mechanically, comprehendingly, reflectingly; this is vertical mathematization." (Freudenthal, 1991, p. 41-42).

In all phases of mathematical activity, the two types of mathematization complement each other (De Lange, 1987). Figure 2 depicts the global idea of horizontal and vertical mathematization activity. De Lange (1987) elaborates on the interplay between horizontal and vertical mathematization activity. He states that the process of mathematization undertaken by students in the learning processes is personal and may take different routes depending on the students' perception of the realistic situation, their skills, and their problem solving abilities. Figure 3 depicts the different routes of possible mathematization processes. Rather than expecting all students to travel the same route from A to B, the routes may be different and may not end up in the same point. These may include many horizontal steps and few vertical ones, or vice versa.

According to De Lange (2006) the process of mathematization as it is carried out by the student has a cyclic character (see Figure 4). First, given a meaningful problem situated in reality, the student who acts as a problem solver starts the process by *understanding the problem* and identifying the relevant mathematical concepts within it (1). Next, based on the identified mathematical concepts, the problem solver trims away the irrelevant elements that exist in reality by *formulating the problem into a mathematical model* (2). Third, *the mathematical problem included in the model* is solved and the student *reflects* on the solution process (3). Finally, the student is able to interpret the mathematical solution in terms of the original, realistic situation (4).

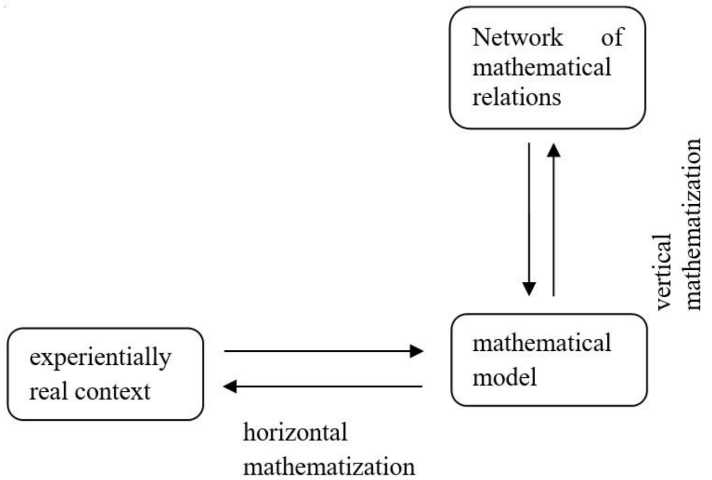


Figure 2. Horizontal and vertical mathematization (based on Drijvers, 2003, p. 54)

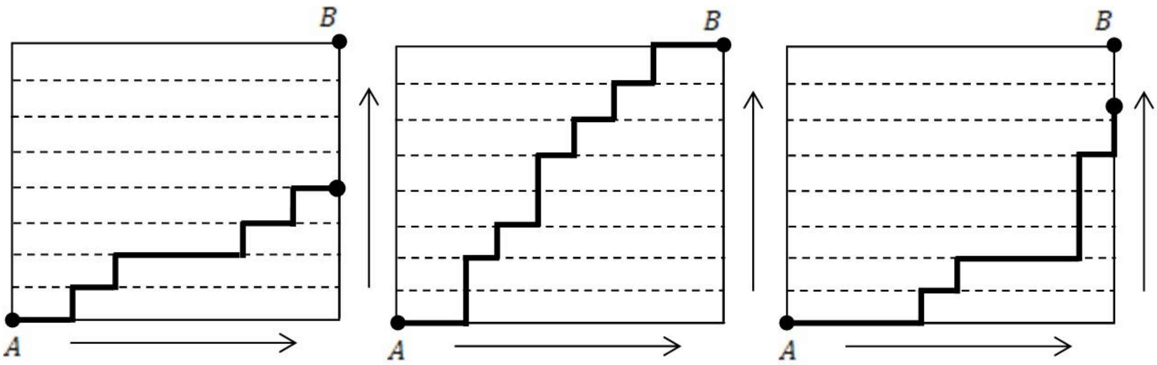


Figure 3. Different routes of mathematization (De Lange, 1987, p. 45)

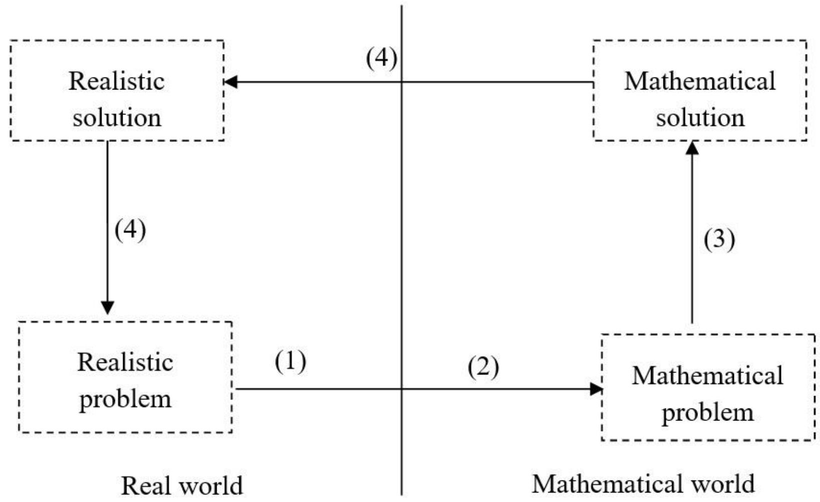


Figure 4. The mathematization cycle (based on De Lange, 2006, p.17)

A rectangle has length and width $(3x-4)$ cm and $(x+1)$ cm, respectively. If the perimeter of the rectangle is 34 cm, find the area of the rectangle.

Figure 5. A problem to illustrate a mathematization cycle

The first two steps transform a realistic problem into a symbolic mathematical problem, and as such concern horizontal mathematization. The third step takes place within the symbolic mathematical world, and therefore characterizes vertical mathematization. Step four, the interpretation of the mathematical solution in terms of the realistic solution again concerns horizontal mathematization. If the construal of the realistic solution in terms of the original realistic problem includes verifying all conditions in the problem, generalizing the solution procedure and recognizing a possible application of this procedure in other similar problems, then vertical mathematization is involved.

To illustrate the cyclic character of the mathematization process, we consider the problem shown in Figure 5, which was taken from the interview study (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). Even if the length and the width are already expressed in symbolic form in this task, this problem involves the horizontal mathematization of setting up mathematical expressions for the perimeter and the area:

- *Given a problem situated in a reality, the mathematization process is started by understanding the problem to identify the relevant mathematical concepts within the problem (1)*. As a problem solver, a student should realize that the reality involved in the task is a mathematical reality in the domain of geometry. We consider that the object involved, namely the rectangle, is imaginable in the student's mind as it has been present since primary school. The relevant mathematical information in the task includes the length, the width, and the perimeter of a rectangle.
- *Formulating the problem into a mathematical model (2).* Based on the identified mathematical concepts, the student should be able to transform the given information by formulating, for instance, the following mathematical model: $2[(3x-4)+(x+1)]=34$ and $A=(3x-4)(x+1)$, where A is the area of the rectangle. This action has transformed the problem into a mathematical problem.

- *Mathematical problem solving* and *a reflection* on the solution process *(3).* For the student who is a novice in algebra, a mathematical model in the form of an equation is still new, and as such the solution process is still not a routine procedure. Therefore, to solve the equation $2[(3x-4)+(x+1)]=34$, the student should be able to see the pattern of it and to manipulate with the algebraic expression for planning an efficient strategy. For instance, the student should decide whether to first divide 34 by 2, next simplify the equation into $4x-3=17$, and eventually get $x=5$; or to first apply a distributive property to get $2(3x-4)+2(x+1)=34$, to multiply and then simplify, and to finally obtain $x=5$. Substituting this value for x into $A=(3x-4)(x+1)$, the student will get $A=66$. As a reflection on the solution process, the student can check whether it is correct by for instance substituting the value of $x=5$ into the equation $2[(3x-4)+(x+1)]=34$, and see if the equivalence is maintained; or by scrutinizing each step of the solution process.
- *Interpretation (4).* The student is able to interpret the solution $A=66$, in terms of the realistic solution, i.e., as the area of the rectangle. Next, to understand this in terms of the realistic problem, the student can verify all conditions given in the problem using the obtained mathematical results. By substituting $x=5$ into $(3x-4)$ cm and $(x+1)$ cm, the student will find 11 cm and 6 cm as the length and the width of the rectangle, respectively. In this way, the student can confirm that the perimeter of the rectangle is indeed $2(11+6)=34$ cm and that the given conditions are met.

RESEARCH QUESTION

We consider that identifying and understanding student difficulties in solving word problems from a mathematization perspective can lead to a better insight into students' learning of algebra. Therefore, we focus on the following research question:

> What are student difficulties in mathematizing word problems in the domain of linear equations in one variable?

In line with the literature (e.g., Carpraro & Joffrion, 2006; Clement, 1982; Stacey & MacGregor, 2000), by "word problems" in algebra we mean algebra tasks that are at least partially represented in natural language. Solving these tasks—which may include graphs, images, tables, geometric figures, or mathematical symbols—involves transformation into mathematical models, such as equations or inequalities, if algebraic methods will be used in the problems solving. As such, these tasks appeal for horizontal mathematization.

METHODS

To answer the research question, a teaching experiment was carried out because we would like to study student learning rather than just capture student thinking at one specific moment, as was done in the interview study. In addition, results from a teaching experiment include exemplary teaching materials and teaching practices that can inform teachers. We included digital tools in the teaching experiment, as we expect this can support students' mathematization processes, while offering an explorative and expressive environments for doing mathematics (e.g., Drijvers & Doorman, 1996; Drijvers, 2000; Drijvers, Boon, & Van Reeuwijk; 2010). Finally, we re-analyzed part of the older interview data for two reasons: (i) Mathematization, which is the main lens in this study, was one of categories of difficulties that emerged in the interviews. Therefore, after analyzing the teaching experiment data, we looked back at the interview data to investigate in retrospective whether the teaching experiment findings match with the interview results; and (ii) even if these two studies have different settings, i.e., different students and teaching approaches, we claim that mathematization difficulties are so general that they should be recognizable in both data sets.

Below, we describe the methods of the teaching experiment study and provide some information on the interview study, the data of which were used for a triangulation.

The design of the teaching experiment

The learning arrangement that we designed consisted of student activities including digital tasks within applets embedded in a digital environment; intermediate formative paper-and-pencil assessment tasks; a final written test; and a teacher guide.

As digital environment, the Digital Mathematics Environment (DME) was used. The DME is a web-based electronic learning environment which offers: (i) interactive mathematical tools for algebra, geometry, and other domains; (ii) a design of open online tasks and appropriate immediate feedback; (iii) conventional mathematical notations and techniques; (iv) access to the environment at any time and place, as long as technological conditions, especially the availability of internet connection and web-browser, are met; and (v) a storage of student work (Boon, 2006; Drijvers et al., 2013). The DME applets Algebra Arrows and Cover-up Strategy were included in the designed arrangement.

Algebra Arrows is an applet which offers the possibility to construct and use chains of operations on numbers and formulas, and provides automatic calculations. Initially, it is designed to support the construction of input-output chains of operations as a model of a dependency relationship in the function concept (Doorman et al., 2012). In this study, the applet was used as a support for solving word problems.

Figure 6 shows how a word problem is solved with the Algebra Arrows applet. 1) The applet provides a word problem to solve, a window for the solution process, input-output boxes *(kotak masukan-keluaran)* which can be dragged and connected with operation boxes in the solution window, and a white box with an unknown (x) in the solution window. 2) A student translates the word problem word-by-word—i.e., translating words or phrases into mathematical operations (through dragging and connecting operation boxes with the input box)—into a mathematical expression. 3) By clicking the Table button, the table appears below the expression $\frac{3x-5}{5}$ and automatically provides some of its values. Considering the problem, the appropriate value of the expression is 5, which means the equation representing the problem is $\frac{3x-5}{5} = 5$. Finally, 4) through applying a reverse-strategy, the student solves the equation to find the value of $x = 10$.

The Cover-up Strategy applet provides an environment to set up equations based on the given word problems, and allows for solving equations in one variable of the form $f(x) = c$. The equation solving process is carried out by subsequently selecting a part of the expression in an equation with the mouse and finding its value. For example, Figure 7 shows a scenario for solving a word problem with the Cover-up Strategy applet. In step 1, a student is expected to formulate an equation based on the given word problem. As he made a mistake, the applet gives feedback, namely a crossed mark in red signifying an incorrect action. If the student formulated a correct equation, namely $\frac{y+2}{3} = 1$ as shown in step 2, the applet provides a tick mark in yellow signifying a correct action. In step 3, the student highlights the expression $y + 2$ and the applet provides $y + 2 = \cdots$ in the next line. In step 4, the student fills in 3, and the applet gives a yellow tick mark signifying a correct response. This scenario proceeds until step 6 and ends up at $y = 1$ as the solution of the equation (signified by the green tick mark and the final feedback: “The equation is solved correctly!”).

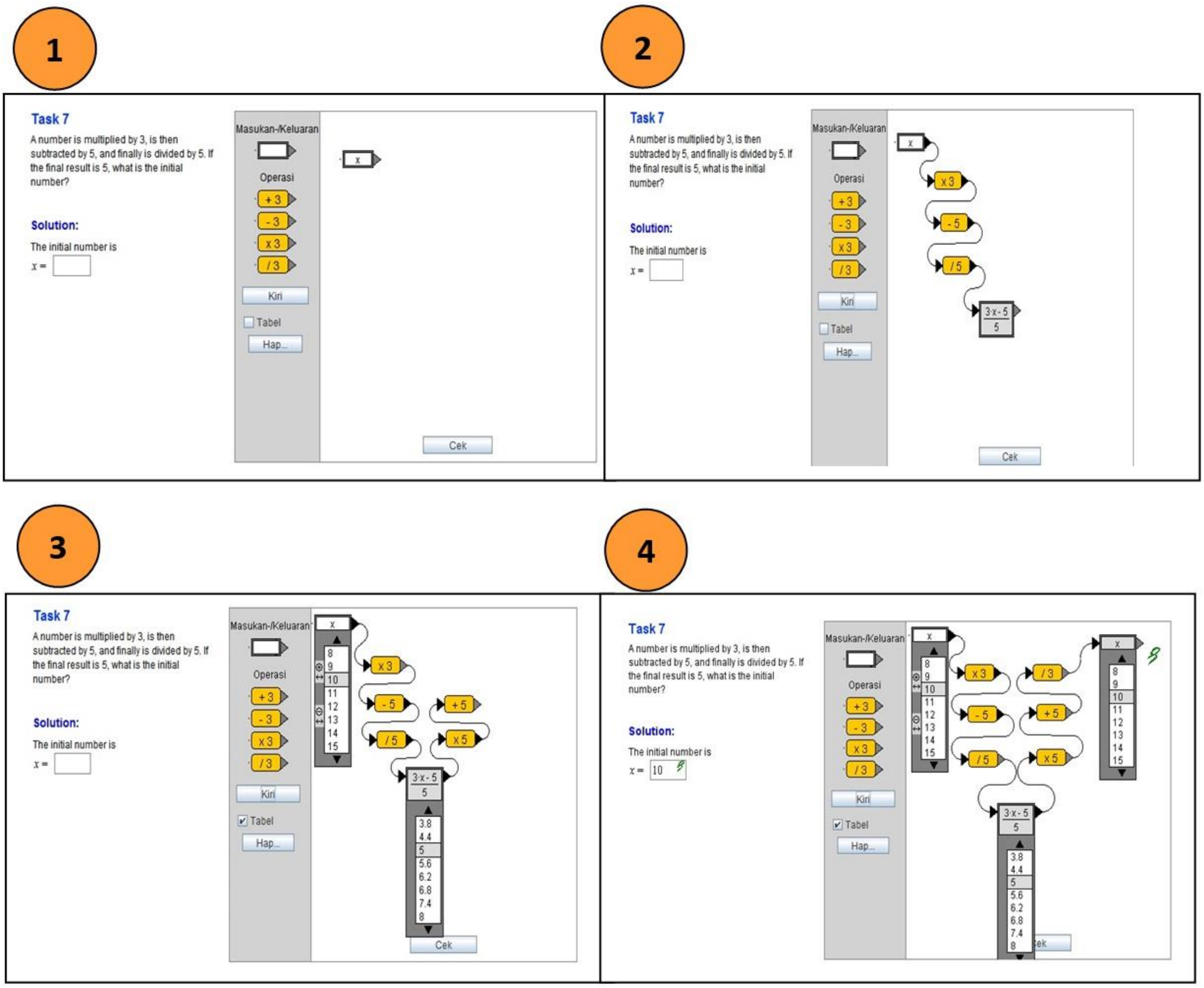


Figure 6. A scenario for solving a word problem using the Algebra Arrows applet

Nine Indonesian master students in mathematics education tested preliminary versions of the activities with the above two applets. Their inputs were incorporated in order to improve the activities presented to the students involved.

A teacher guide was designed for five lessons. Lessons 1 and 2 were enriched with the Algebra Arrows applet and respectively focused on word problems and symbolic equations. Lessons 3 and 4 included the use of the Cover-up Strategy applet and subsequently focused on symbolic equations and word problems. Lesson 5 consisted of a final written test covering the topics of the four previous lessons. The experiment took 80-minutes for each lesson. The learning sequence in each of the first-four lessons consisted of three parts: paper-and-pencil activity, digital activity, and paper-and-pencil assessment as well as reflection. The paper-and-pencil activity included posing problems and classroom discussion. The digital activity consisted of a demonstration of an applet, student group digital work and discussion. During the digital activity, the teacher or the observer gave help to groups of students when necessary, including guiding students during the learning process. In the end-of-the-lesson formative assessment, students were requested to do paper-and-pencil tasks individually. The tasks were designed based on the tasks used in the DME session and initially referred to Indonesian mathematics textbooks. Finally, the teacher guided students to reflect upon the lesson.

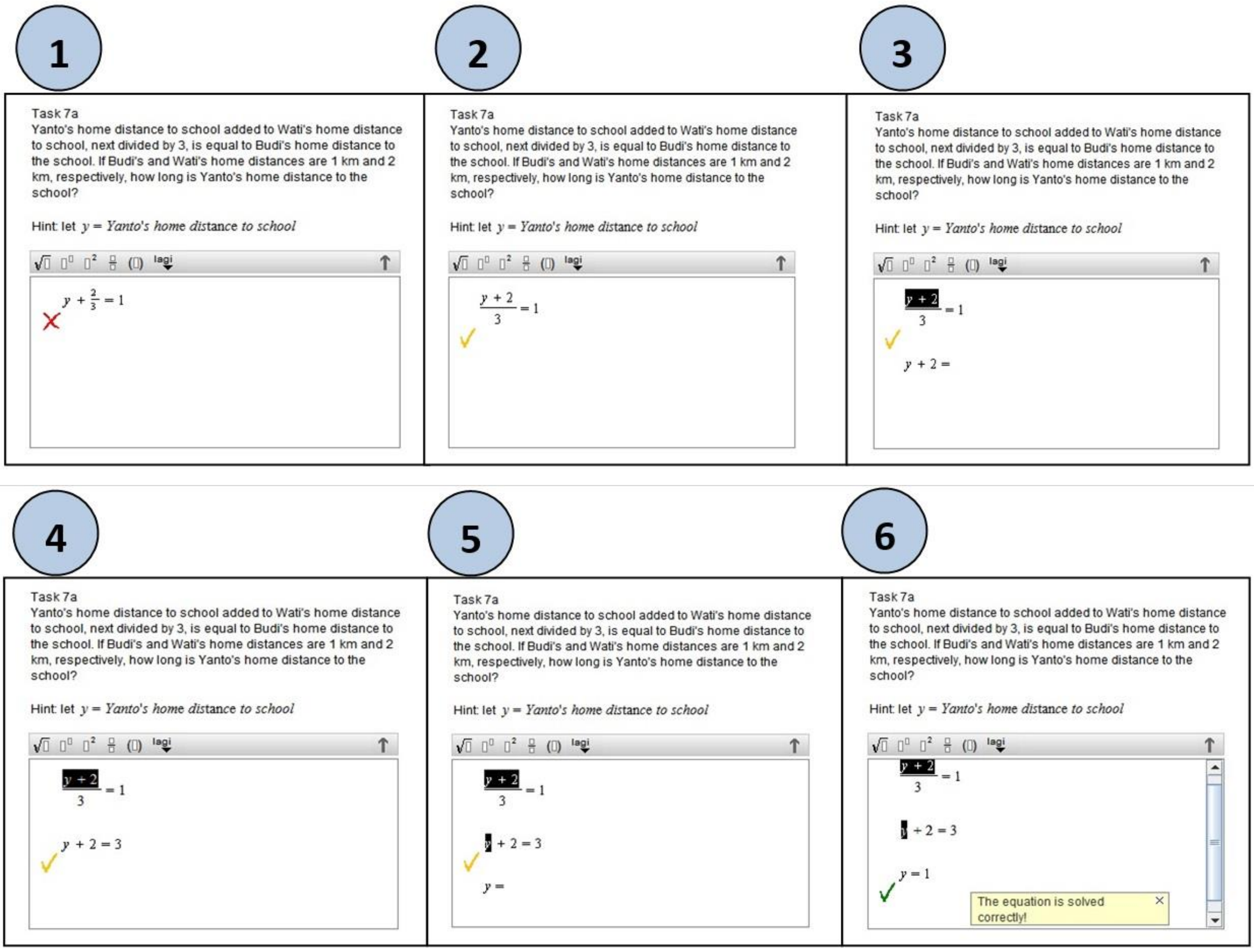


Figure 7. A scenario for solving a word problem using the Cover-up Strategy applet

Data collection

The teaching experiments were conducted in two schools in Indonesia. One complete class with 41 grade seven students (12-13 year-old) was chosen from the first school, and ten grade seven students (12-13 year-old) participated from the second school. The ten students who were selected by their mathematics teacher to participate in this study included high, medium and low achievers in a balanced manner. Data that were collected from each school consisted of video registrations of four teaching sessions (including paper-and-pencil-board activity, group digital work, and classroom discussion), student written work from each assessment and from the final written test, and field notes. During the periods of group work with the applets, the video registration focused on two groups of students. As the teacher usually did not take care of these video groups too much, the researcher-observer to some extent guided these two groups if needed, as to give all students the same treatment.

Data analysis

The analysis of the data from the teaching experiment was carried out in three phases. In the first phase, a preliminary analysis of video registration of student digital group work as well as on individual written work was carried out with software for a qualitative analysis (Atlas.ti). This analysis included marking and transcribing crucial moments in paper-and-pencil activity and in classroom discussions as well as in student digital group work; examining and assigning difficulties on written student work (including a written final test) for each single task—which serves as a case of analysis. In total there are 394 cases of data. To confirm the analysis of the written student work, transcripts from observations during the learning activities with the digital technology were used. Thus, the results of analysis integrate the quantitative data from the intermediate formative assessments and the qualitative analysis of the video data from students' activities in the digital mathematics environment.

The second phase of analysis consisted of an in-depth analysis of student difficulties from the perspective of mathematization (see section 2.2). We classified student difficulties identified in the first phase into four subcategories. First, difficulties in understanding words, phrases, or sentences, and ignoring parts of the problem were classified in the subcategory of *understanding the problem*. Second, difficulties in formulating equations, schemas, or diagrams were classified under the subcategory of *formulating mathematical models*. Third, mistakes made in the solution process were grouped into the subcategory of symbolic *mathematical problem solving*. We argued that types of difficulties in applying arithmetical operations (ARITH), in understanding the notion of variable (VAR), in understanding algebraic expressions (AE), and in understanding the different meanings of the equal sign (EQS) can be included in the third subcategory because they normally occur during the solution processes. Fourth, difficulties in checking the solution process were included in the subcategory of *reflection*. To check the inter-rater reliability, a second coder—an external research assistant not included in this study—analyzed 20% of the cases after being given an explanation and the coding manual for data analysis. With a Cohen's Kappa of 0.91, the agreement between the first author and the second coder was found to be almost perfect (Landis & Koch, 1977).

To check the findings of the teaching experiment, the third phase of analysis concerns triangulation with the interview data from an earlier study and of word problems on linear equations in one variable in that study in particular (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2014). The interview study involved eighteen Indonesian students who finished grade seven (13/14 year-old). These students were asked to solve a set of five algebra tasks (two of which are on word problems) with paper and pencil individually for thirty minutes. Next, interviews were conducted, during which the students were encouraged to explain their reasoning in their written work. The interview data had been analyzed before, and an additional analysis of the two algebra word problems involved in the interviews was carried out using a similar mathematization framework and coding schemes focusing on mathematization.

RESULTS

This section presents the results of the teaching experiment which were backed up with the findings from the interview study. The main results of the teaching experiment include individual written student work after the use of the applets and if necessary are confirmed by observations of student group work in the DME sessions. The written final test findings are used to corroborate the results of this analysis. To confirm these findings, we revisit findings from earlier student interviews.

Student mathematization difficulties revealed in the Algebra Arrows lesson

A total of 49 students participated in the lesson which focused on solving word problems with the Algebra Arrows applet. Table 1 summarizes the results of these

Table 1. Results from data analysis of the Algebra Arrows lesson (N = 49)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
1. You have a number. The number is subtracted by 7, next the result is divided by 5. If the final result is 11, what was the starting number?	43(88)		Horizontal math:
		misunderstand or ignore words, phrases, sentences (10)	Understand the problem
		errors in formulating equations, schemas, diagrams (10)	Formulate math model
			Vertical math:
		mistakes in solution processes: • ARITH: calculation errors (2) • EQS: notational errors (25)	Math problem solving
		checking the solution process (2)	Reflection
2. A number is multiplied by 2, the result is then subtracted by 4, and finally is divided by 5. If the final result is 3, what was the starting number?	31(63)		Horizontal math:
		misunderstand or ignore words, phrases, sentences (4)	Understand the problem
		errors in formulating equations, schemas, diagrams (4)	Formulate math model
			Vertical math:
		mistakes in solution processes: • ARITH: calculation errors (12) inverse errors (10). • EQS: notational errors (35)	Math problem solving
		checking the solution process (25)	Reflection
3. A number is multiplied by 2, next the result is added to 50, and finally is divided by 5. If the final result is 5, what was the starting number?	14(29)		Horizontal math:
		misunderstand or ignore words, phrases, sentences (25)	Understand the problem
		errors in formulating equations, schemas, diagrams (12)	Formulate math model
			Vertical math:
		mistakes in solution processes: • ARITH: priority rules (10), calculation errors (25), inverse errors (4). • EQS: notational errors (35).	Math problem solving
		Checking the solution process (45)	Reflection

students for the three tasks they worked on with paper and pencil at the end of the lesson. Columns 1-4 subsequently present: tasks, number of students who solved the tasks correctly (#C), types of difficulties revealed, and mathematization subcategories which might explain the difficulties. Corresponding percentages, relative to the total number of participating students, are provided for columns 2 and 3.

Task 3 seems to be difficult for most of the students. Of the 49, fourteen students (29%) solved this task correctly. Even if task 3 has the same structure as task 2—namely, the mathematical models of these two tasks are similar—it seems that task 3 is more difficult. This could be caused by the fact that task 3 requires students to work with negative numbers which is often demanding for them.

The difficulties in written student work include mistakes in understanding words, phrases or sentences; in formulating equations, schemas, or diagrams; in the solution processes; and in checking the solution processes. In particular, inverses, priority rules and calculation errors (ARITH category), and notational errors in the use of the equal sign (EQS category) occurred during the solution processes.

The data for each task revealed that difficulties in the solution processes were the most frequent. From a mathematization perspective, this means that students encountered difficulty in vertical mathematization and in the subcategory of mathematical problem solving in particular. We predict that this is caused by the following factors: (i) the context of the tasks, namely number, is familiar to the students; (ii) the structure of the tasks is operational in the sense that it provides an opportunity to translate the tasks word-by-word directly into mathematical models. Therefore, to some extent, the students did not encounter serious difficulty in understanding these problems, and in formulating corresponding mathematical models.

To illustrate these findings, we present two representative examples of written student work on task 3. Figure 8 (left screen) shows an example of written student work containing difficulties in understanding a phrase, and in the solution process. First, rather than to translate the phrase "*sebuah bilangan* (a number)" into an unknown, x for instance, the student translated it as an integer number "one". As a consequence, she translated the problem into an incorrect mathematical model: $1 \times 2 + 50 : 5 = 5$. In this case, if the student had understood the phrase "*sebuah bilangan*" correctly, she would probably have got a correct model. Second, if we assume that the model is correct, then the student did an incorrect calculation, namely $52 : 5 = 25$ instead of $52 : 5 = 10\frac{2}{5}$. From the perspective of mathematization, the first difficulty concerns horizontal mathematization and understanding the problem in particular. The second difficulty concerns vertical mathematization and the subcategory of mathematical problem solving—namely, lack of proficiency in arithmetical calculation—in particular.

Figure 8 (right screen) shows an example of written student work which contains mistakes in the solution process (subcategory of mathematical problem solving) and an indication of not checking the solution (subcategory of reflection). It seems that the student understood the problem and was able to translate it into a correct mathematical model. However, she made two mistakes in the solution process. First, she made an additive inverse error: instead of subtracting 50 from 25 and next dividing by 2, she added 50 to 25. The second mistake concerns an improper use of the equal sign: the student wrote down $5 \times 5 = 25 + 50 = 75 : 2 = 37.5$, which is incorrect since, for instance, 5×5 is not equal to $25 + 50$. Furthermore, after getting 37.5, she seemed not to check this by substituting it into the model. This indicates that she forgot to check the solution process. In the light of mathematization, the student encountered difficulty in vertical mathematization and the mathematical problem solving and the reflection subcategories in particular. The frequent difficulties in the solution processes, which were also observed in digital group work, seem to be a direct consequence of automatic calculation provided by the Algebra Arrows applet during the learning process. As a consequence, when students were working on word problems on paper, they were not used to doing the solution processes, in particular the calculation, by themselves.

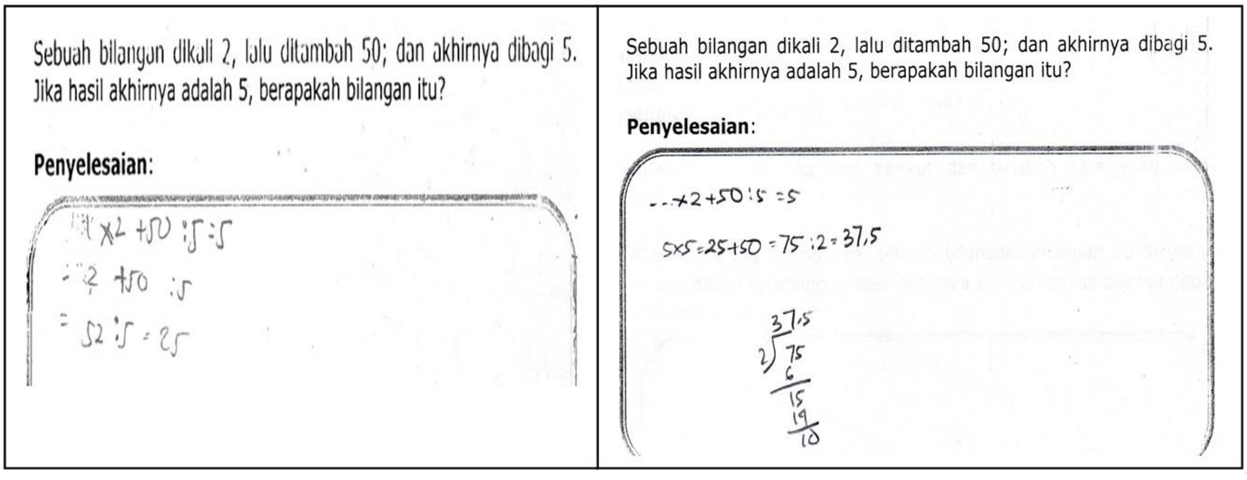


Figure 8. Representative examples of written student work on task 3

Aside from the above findings, our data on student group digital work shows that five out of eight groups (25 students) failed to deal with word problems, in which the context concerns real life and is not merely on number. The following observation excerpt on the group digital work corroborates this finding.

A group of five students was doing the following task:

> Tom is 7 years older than Safira. If Safira is 4 years old, how old is Tom?

> After reading the task, the group was puzzled.
> Student 1: [Reads the task out loud].
> Student 2: It seems 7 – 4, does not it? [She suggests Student 1 to do her idea].
> Student 1: [She represents 7 – 4 using the applet].
> Student 2: Wait! It must be 7 + 4 – 4.
> Student 1: [She represents 7 + 4 – 4 on the computer].
> Student 3: What number should be clicked below the expression $x + 7$ [to get a direct value of x]?
> Student 1: Seven.
> Student 3: Why is x [Tom's age] zero? [See Figure 9]. [Even if they succeeded eventually after getting a guidance, this group took time to ponder the task].

During the observation, one of the students could solve the task mentally, i.e., without using the applet. The obstacle for this group of students was that they could not represent the word problem into a mathematical expression using the Algebra Arrows properly. This could be because the structure of the task that could not easily be translated into a mathematical expression. In short, the above excerpt suggests that students seemed to encounter difficulty in understanding the problem and as such implies difficulty in formulating a mathematical model from the problem. This means that the main difficulties encountered by students when dealing with word problems—in which the context relates to real life—concern horizontal mathematization.

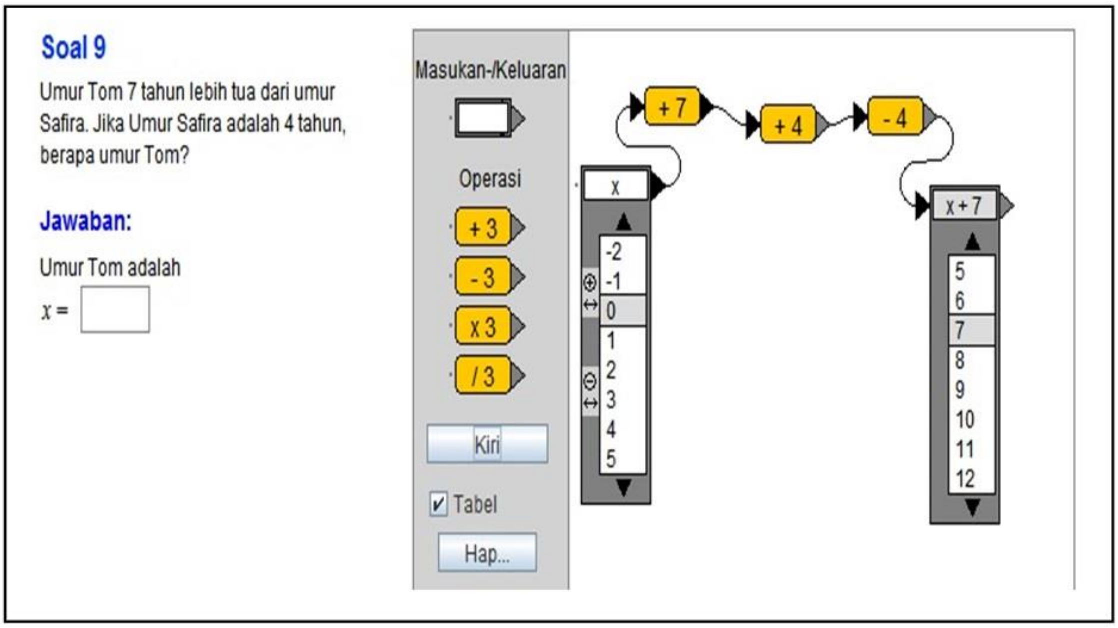


Figure 9. A student's work on the Algebra Arrows applet

Table 2. Results from data analysis of the Cover-up Strategy lesson (N = 51)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
4. Two times a number is subtracted by 4, next divided by 5, and finally added by 2. If the final result is 10, find the number.	21(41)		Horizontal math:
		- misunderstand or ignore words, phrases, sentences (33)	Understand the problem
		- errors in formulating equations, schemas, diagrams (26)	Formulate math model
			Vertical math:
		- mistakes in solution processes: • ARITH: inverses errors (4), priority rules (2) calculation errors (2) • EQS: notational errors (24)	Math problem solving
		- checking the solution process (35)	Reflection
5. Adin's height is divided by 3, next the result is added to Budin's height. If the final result is equal to 180 cm and Budin's height is 130 cm; find Adin's height.	32(63)		Horizontal math:
		- misunderstand or ignore words, phrases, sentences (16)	Understand the problem
		- errors in formulating equations, schemas, diagrams (16)	Formulate math model
			Vertical math:
		- mistakes in solution processes: • ARITH: inverses (10), calculation errors (6) • EQS: notational errors (12)	Math problem solving
		- checking the solution process (16)	Reflection
6. The difference of the distances from Yanto's and Wati's homes to their school divided by 2 is equal to twice the distance from Budi's home to the school. If Budi's and Wati's home distances are 1 km and 2 km, respectively, find the distance between Yanto's home and the school.	6(12)		Horizontal math:
		- misunderstand or ignore words, phrases, sentences (84)	Understand the problem
		- errors in formulating equations, schemas, diagrams (82)	Formulate math model
			Vertical math:
		- mistakes in solution processes: • ARITH: inverses (2), calculation errors (12) • EQS: notational errors (14)	Math problem solving
		- checking the solution process (8)	Reflection

Student mathematization difficulties revealed in the Cover-up Strategy lesson

A total of 51 students participated in the lesson which focused on solving word problems with the Cover-up Strategy applet. Table 2 which has the same headings as Table 1 summarizes the results of these students for the three tasks they worked on with paper and pencil at the end of the lesson.

Task 6 seems to be difficult for most of the students. Of the 51, six students (12%) solved this task correctly. Even if task 4 is a typical problem of the Algebra Arrows lesson, still this task is more difficult than task 5. This could be because the structure of task 4 is more complex than task 5, and as such it is difficult to translate and to solve.

All categories and the corresponding subcategories of difficulties that emerged in this lesson were the same as the findings of the Algebra Arrows lesson. Although difficulties in the solution processes were still frequent, other difficulties occurred more often. The two most frequent subcategories of difficulties that emerged in each task were: (i) understanding words, phrases, sentences; and (ii) formulating equations, schemas, or diagrams. From the perspective of mathematization, the first subcategory concerns understanding the problems, and the second subcategory concerns formulating mathematical models. In other words, the emergence of these two subcategories of difficulties signifies difficulties in horizontal mathematization.

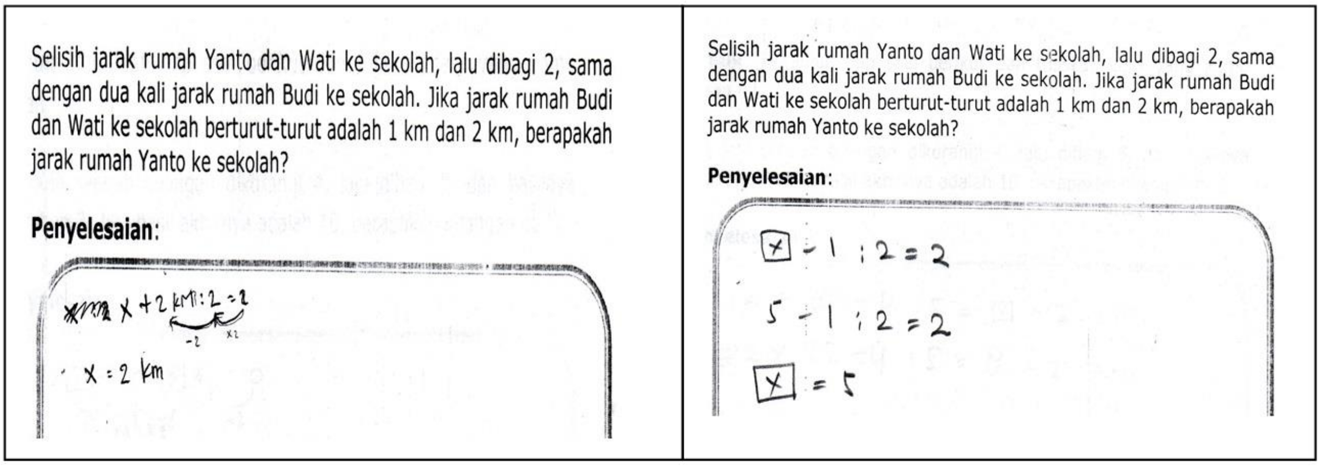


Figure 10. Representative examples of written student work on task 6

We clarify these findings by two representative examples of written student work on task 6. Figure 10 (left screen) shows student work containing the difficulty in understanding a phrase which causes a mistake in formulating an equation. Rather than translating the phrase, “The difference of the distances from Yanto’s and Wati’s homes to their school,” into, for example, $x - 2$ (in which x and 2 represent respectively Yanto’s and Wati’s homes’ distances to their school), the student translated it into $x + 2$. This led to an incorrect equation. Figure 10 (right screen) shows a similar mistake. Even if the student seems to understand the aforementioned phrase, he assigned 1 rather than 2 as the distance of Wati’s home to the school. This then also led to an incorrect equation. From a mathematization perspective, both examples illustrate difficulties in horizontal mathematization and in understanding the problem and formulating mathematical models in particular.

The difficulties in understanding problems and in formulating mathematical models were also observed in digital group works. The following observation illustrates this.

A group of students was working on the following task:

> “Wenny’s and Yudi’s ages together are 27. If Yudi is 9 years younger than Wenny, how old is Wenny? Hint: Let w =Wenny’s age.”
>
> After reading the problem, one student typed an equation on the computer, namely $w + 9 + w = 27$. The applet provided direct feedback that the equation was incorrect. Next, the observer suggested the group to reread the problem.
>
> Student A: So, it must be subtracted! [He erases the incorrect equation and types $w - 9 - w = 27$. Student B presses the enter button to check, but it is still incorrect. See Figure 11.]
> Student B: Why is it still wrong?
> [This group seems to think again. With the help of the observer, the group succeeds eventually.]

This observation suggests that the difficulty in formulating a mathematical model (equation) is caused by students’ limited understanding of the problem.

Student mathematization difficulties revealed in the final written test

The results of the final written test were used to confirm the findings of the Algebra Arrows and Cover-up activities. A total of 47 students participated in the final written test. Table 3 which has the same headings as Tables 1 and 2 summarizes the results

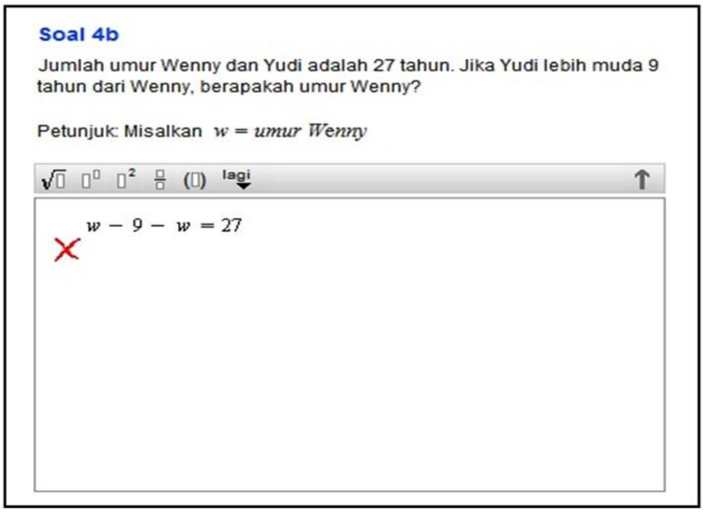


Figure 11. A student mistake on the Cover-up Strategy applet

Table 3. Results from data analysis of final written test (N =47)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
7. You have a number. The number is subtracted by 2, the result is then multiplied by 7, and finally 4 is added. If the final result is 25, what was the starting number?	41(87)	- misunderstand or ignore words, phrases, sentences (2) - errors in formulating equations, schemas, diagrams (2) - mistakes in solution processes: • ARITH: inverses errors (2), priority rules (4) • EQS: notational errors (23) - checking the solution process (11)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection
8. The sum of distances of Tom's and Jerry's homes to the city center divided by 9 is equal to the distance of three times Udin's home to the city center. If Udin's and Jerry's home distances are 1 km and 7 km, respectively, find the distance between Tom's home and the city center.	13(28)	- misunderstand or ignore words, phrases, sentences (68) - errors in formulating equations, schemas, diagrams (68) - mistakes in solution processes: • ARITH: priority rules (4), calculation errors (15) • EQS: notational errors (9) - checking the solution process (0)	Horizontal math: Understand the problem Formulate math model Vertical math: Math problem solving Reflection

of these students on the two word problems they worked on with paper and pencil in this test. It shows that students performed well on task 7 (87% correct results), but encountered difficulties in dealing with task 8 (28% correct results).

In general, the difficulties in the final written test were the same as found in the Algebra Arrows and Cover-up Strategy activities. Mistakes in the solution processes (mathematical problem solving subcategory) and in checking solutions (reflection subcategory) were two most frequent difficulties on task 7—which is a typical task addressed in the Algebra Arrows activity. These results confirmed the findings of the Algebra Arrows activity, namely most of the students encountered difficulties in vertical mathematization and in mathematical problem solving and reflection in particular.

Difficulties in understanding words, phrases, or sentences (*understanding the problem*); and in formulating equations, schemas, or diagrams (*formulating mathematical models*) were the two most frequent difficulties on task 8. In other words, most of the students encountered difficulties in horizontal mathematization. These results also confirmed the findings of the Cover-up Strategy lesson because task 8 is a typical problem addressed in this lesson.

Backing up the findings with data from student interviews

Table 4 which has the same headings as Tables 1, 2 and 3 summarizes the results of interviews for the two word problems worked by students. Both tasks seem to be difficult for most of the students. Of the eighteen, one student (6%) solved task A and seven students (39%) solved task B correctly.

In general, the difficulties revealed in the interviews results match the teaching experiment findings. Even if mistakes in the solution processes occurred quite often, we observed that mistakes in formulating equations, schemas or diagrams were the most frequent. From a mathematization perspective, these findings show that difficulties in the horizontal mathematization and formulating mathematical models subcategory in particular were the most important obstacles revealed in the interviews.

Concerning task A and in relation to the difficulties in formulating mathematical models, we observed that fourteen students (78%) used incorrect arithmetical methods—which include incorrect arithmetical models—to solve the task. Such arithmetical methods include, for instance, dividing 30,000 by 2, next adding and

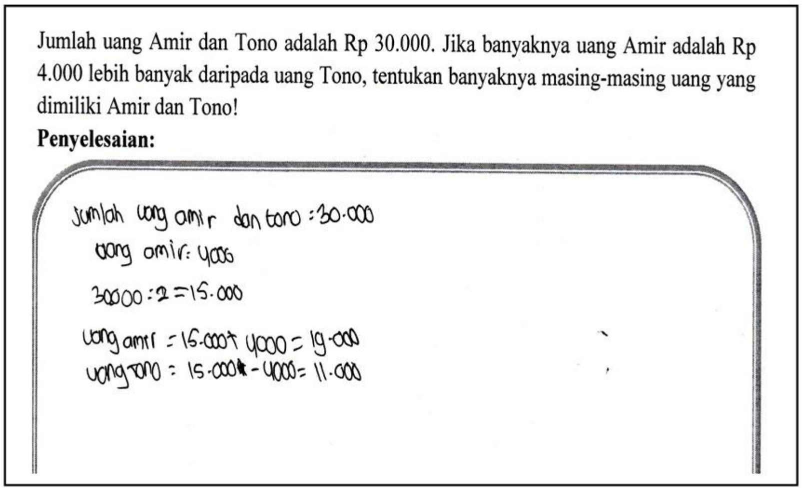
Table 4. Results from data analysis of the interviews (N = 18)

Word problems to solve	#C (%)	Difficulties (%)	Mathematization category
A. Amir and Tono together have Rp 30,000. If Amir's amount of money is Rp 4,000 more than Tono's, find each of their amounts	1 (6)		Horizontal math:
		- misunderstand or ignore words, phrases, sentences (22)	Understand the problem
		- errors in formulating equations, schemas, diagrams (83)	Formulate math model
			Vertical math:
		- mistakes in solution processes: • ARITH: associative error (6) • EQS: notational errors (28) • VAR: Unknown (11)	Math problem solving
		- checking the solution process (44)	Reflection
B. A rectangle has length and width $(3x-4)$ cm and $(x+1)$ cm, respectively. If the perimeter of the rectangle is 34 cm, find the area of the rectangle.	7 (39)		Horizontal math:
		- misunderstand or ignore words, phrases, sentences (6)	Understand the problem
		- errors in formulating equations, schemas, diagrams (50)	Formulate math model
			Vertical math:
		- mistakes in solution processes: • ARITH: distributive (17), calculation errors (22), inverse errors (6). • EQS: notational errors (6). • AE: lack of closure (22) Expected answer (17) Parsing obstacle (6) Gestalt view (22)	Math problem solving
		- Checking the solution process (39)	Reflection

subtracting 4,000 to 15,000 to find the amounts of Amir's and Tono's money, respectively. Figure 12 shows an example of such incorrect arithmetical methods.

We observe that the difficulty in formulating mathematical models for task B, which was also frequent, seems to be caused by students' lacking understanding and abilities to connect mathematical concepts from different mathematical strands, such as connecting algebra and geometry. The following interview excerpt provides evidence for this.

> I (interviewer): Please can you read the problem? [As the solution space of the student is blank, the interviewer asks the student to understand the problem.]
> S (student): [Reads the problem out loud.]
> I: Do you know what a rectangle is? [To check whether the student understood what he reads].
> S: [Draws a rectangle on his paper.]
> I: Which are the length and the width of the rectangle?
> S: [Points the length and the width of the rectangle correctly.]
> I: In the problem you read, what is the length [of the rectangle]?
> S: $3x - 4$.
> I: What is the width of the rectangle?
> S: $x + 1$. [He writes down $3x - 4$ and $x + 1$ beside the length and the width of the rectangle, respectively. See Figure 13.]
> I: What is the value of the perimeter given in the problem?


Jumlah uang Amir dan Tono adalah Rp 30.000. Jika banyaknya uang Amir adalah Rp 4.000 lebih banyak daripada uang Tono, tentukan banyaknya masing-masing uang yang dimiliki Amir dan Tono!

Penyelesaian:

jumlah uang amir dan tono : 30.000
uang amir: 4000
30000 : 2 = 15.000
uang amir : 15.000 + 4000 = 19.000
uang tono : 15.000 - 4000 = 11.000

Figure 12. A student's work with an incorrect arithmetical method on task A

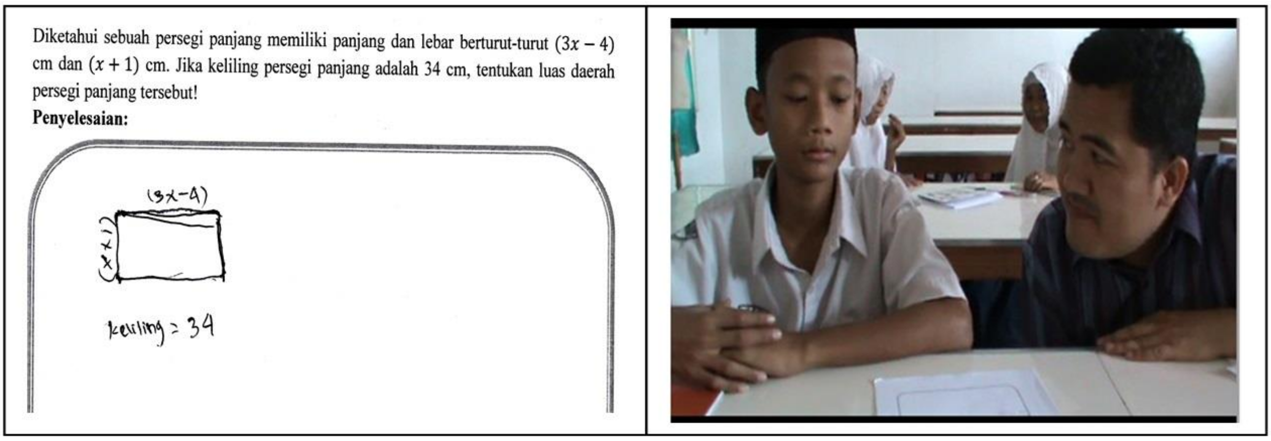


Figure 13. A student interview showing an inability to formulate a math model

S: 34 cm.
I: Do you know the perimeter [of this rectangle]?
S: [Keeps silent. But then he points to the length and the width of the rectangle and seems to round the rectangle.]
I: Okay, can you write [the formula for] the perimeter of this rectangle?
S: [He writes "Keliling = 34" on his paper. Keliling means the perimeter.]
I: So, based on your explanation before, can you write an equation representing the perimeter?
S: I do not know [to write it].

This excerpt shows that even if the student understood the problem, he could not formulate an equation because he was unable to represent the concept of the perimeter of the rectangle with an algebraic expression. The lack of closure obstacle might explain this inability to deal with algebraic expressions.

CONCLUSIONS AND DISCUSSION

The research question addressed in this paper concerns identifying student difficulties in solving word problems in the topic of linear equations in one variable using a mathematization perspective. The results described in the previous section lead to the following conclusions. First, the main difficulties in students' written work after the Algebra Arrows lesson concern the solution processes and to a lesser extent, checking solutions. These findings suggest that the main obstacle concerns vertical mathematization and the mathematical problem solving and reflection subcategories in particular. Second, the main difficulties shown in the students' written work at the end of the Cover-up Strategy lesson concern understanding words, phrases or sentences; and formulating equations, schemas or diagrams. These findings suggest a lack of ability in horizontal mathematization, and understanding problems and formulating mathematical models in particular. Third, the findings from both lessons are confirmed by the results of the final written test: the difficulties in vertical mathematization emerge in student work on typical tasks of the Algebra Arrows activity, whereas the difficulties in horizontal mathematization appear in student work on typical tasks of the Cover-up activity.

How do we explain these differences? Factors that may explain the Algebra Arrows activity findings include: (i) the context for most of the tasks involved in this activity, namely number, is familiar to the students; (ii) the structure of the tasks is relatively operational in the sense that it gives an opportunity to translate them word-by-word into mathematical models; and (iii) the automatic calculation provided by the Algebra Arrows applet avoids calculation errors. As a consequence, students did not encounter serious difficulty in understanding problems and in formulating mathematical models, but found more obstacles in the paper-and-pencil solution processes and reflection—as they are not used to do calculations by themselves. These two mathematization subcategories characterize difficulties in vertical mathematization. However, our observation on student digital group work in the Algebra Arrows lesson suggests that the main obstacle encountered by students when dealing with word problems—in which the context concerns real life and not merely number—concerns understanding the problems and formulating mathematical models which characterize difficulties in horizontal mathematization. Factors that may explain the Cover-up activity findings include: (i) the contexts of tasks are various and are closer to real life than in the Algebra Arrows lesson; (ii) the structure of the tasks is complex and as such is difficult to translate directly into a mathematical model. As a consequence, students encounter obstacles in understanding problems and in formulating mathematical models. These two mathematization subcategories characterize difficulties in horizontal mathematization. All together, we conclude that the main difficulties encountered by students who deal with word problems concern transforming problems into mathematical models, i.e., in horizontal mathematization.

The data from the interviews confirm the above findings. Even if difficulties in the solution processes—which can be included in the subcategory of mathematical problem solving—appeared quite often, the most frequent difficulties revealed in the interviews concern formulating equations, schemas or diagrams. In the light of mathematization, these findings indicate that the main obstacle concerns horizontal mathematization and formulating mathematical models in particular.

As a discussion of these results, we might explain student difficulties in formulating mathematical models as an effect of the prevailing conventional teaching approach in Indonesia, in which students tend to do more routine bare algebra tasks than algebra word problems (e.g. Sembiring, Hadi & Dolk, 2008; Zulkardi, 2002). As a result of this tradition, students may not acquire adequate mathematization skills. Furthermore, this teaching tradition often relies heavily on textbooks (Sembiring, Hadi & Dolk, 2008). Future research on analyzing Indonesian textbooks on the topic of algebra might be fruitful to investigate if adequate resources are available for developing mathematization skills.

Concerning the effect of the ICT-rich approach, we conjecture that student difficulties in understanding problems, in formulating mathematical models, and to a lesser extent in symbolic mathematical problem solving are at least partially caused by a lack of a transfer between the digital and paper-and-pencil environments. On the one hand, students learn to deal with word problems with applets, in which immediate feedback and to some extent automatic calculations are available during the learning process; on the other hand, students are tested to do word problems with paper and pencil without feedback and automatic calculations. These are apparently two different conditions. For future research on developing better mathematization skills, it seems to be useful that feedback and automatic calculations in the applets gradually fade out in the applets and are varied in a systematic, didactical manner (Bokhove & Drijvers, 2012).

Finally, to improve the design of the learning arrangement, we retain the following ideas:

- To reduce student difficulties in transforming words, phrases, or sentences into mathematical expressions—such as translating, "the difference of distances between A and B; the sum of distances of P and Q; etc"—we suggest to give students more translation practices on this. In this way, they will become familiar with translating such phrases into appropriate mathematical expressions.
- As a didactical idea, to develop a better problem solving skills dealing with word problems, we suggest to using four subcategories of mathematization as a problem solving strategy in the learning and teaching processes. In this way, student difficulties can be observed more easily and teachers can give appropriate help to students during the learning processes.
- To extend skills in solving word problems on the topic of equations in one variable, without being exhaustive we suggest to widen the scope of mathematical models of the problems: not only of the form $f(x) = c$ as addressed in the present research, but also of the forms $f(x) = g(x)$ and $f(x) = \frac{c}{g(x)}$. Doing so will provide a more comprehensive insight into student conceptual understanding of and difficulties with word problems.

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