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# The effect of the activity-based approach on grade 11 learners' performance in solving two-dimensional trigonometric problems

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**Abstract**
The study explored the effect of the activity-based approach (ABA) on grade 11 learners' performance in solving two-dimensional (2D) trigonometric problems. A sequential explanatory design was used, starting with the quantitative, followed by the qualitative method. The experimental group (EG) and the control group (CG) were facilitated using ABA, while CG was taught using traditional teaching approach. A 2D trigonometry test was used as a pre- and post-test to measure learners' performance. EG's experiences of using ABA were captured through focus group interviews. The study findings reveal significant differences between EG and CG post-tests [t(100)=.3. 95; p=.05] with Cohen d=.79. Also, ABA did not segregate between boys and girls. Despite ABA being time-consuming, learners in EG developed collaborative skills, trigonometry vocabulary, the ability to solve trigonometric problems, and learning autonomy, which improved their performance. The study recommends ABA to improve learners' performance.

**Keywords:** activity-based approach, enjoyed, autonomy, traditional teaching approach, learners' performance

## INTRODUCTION

Trigonometry is a topic in mathematics that improves various cognitive skills. It deals with side lengths and angles of triangles relationships in real-world activities (Serpe & Frassia, 2021; Wijaya et al., 2020a). If this specialist branch of geometry is not well facilitated, it could lead to learners' misconceptions and poor mathematics performance. Facilitation differs from teaching. Facilitation refers to learner-centered approach through which learners learn by sharing and collaborating with each other (Regmi, 2012) while teaching means to control what is to be learned by conveying "a message of control over students rather than student empowerment" (Estes, 2004, p. 146). Although in the South African curriculum and assessment policy statement (CAPS), grade 11 trigonometry topic accounts for 50 out of 150 marks (30%) of the final mathematics paper two (DBE; 2011), learners perform poorly (DBE, 2020; Wijaya et al., 2020b). Therefore, there is a need to research trigonometry teaching and learning in South Africa.

Researchers in trigonometry identified various factors that contribute to poor performance. These factors arise from the abstract nature of the topic, its apparent fragmentation, the many terminologies, and several complex mathematical concepts to be mastered (Simons & Wibawa, 2021). Other factors include learners' failure to use scientific calculators (Kagenyi, 2016) and learners' inability to link trigonometry with real-life and other topics such as algebra and geometry when solving trigonometric problems (Mosese & Ogbonnaya, 2021; Ngcobo et al., 2019). As such, teachers need to use learner-centered approaches, real-life examples, and practical applications of trigonometry concepts during the learning process (Spangenberg, 2021).

According to Usman et al. (2020), the activity-based approach (ABA) is one of the learner-centered approaches. Effective teachers use ABA, to challenge learners to do activities to enhance their autonomy

This work stems from the first author's PhD thesis. The second and the third authors were supervisors of the thesis.



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**Contribution to the literature**

- The research reports that the activity-based approach improves learners' skills in solving two-dimensional trigonometric (2D) problems and enhances learners' performance.
- The activity-based approach helps learners to acquire mathematical process skills such as representation, connection, communication, reasoning, and problem solving.
- The learners' experiences of learning trigonometry using the activity-based approach show that learners enjoyed learning through ABA which also improved their learning autonomy.

(Barai, 2018). ABA enhances collaborative skills needed in the future workplace (Kaur & Sankhian, 2017; Usman et al., 2020) and fosters critical thinking and mathematical process skills (Deringol et al., 2021; Pokhrel, 2018). ABA has been widely used in science but not in mathematics. Noreen and Rana (2019), as well as Celik (2018) examined the impact of ABA on learners' performance in mathematics, while Emaikwu (2012) investigated the effect of ABA in teaching trigonometry. Emaikwu (2012) focused on all the sections of trigonometry using multiple-choice questions and not open-ended questions. In this study, ABA was used when learners were engaged in open-ended activities during the learning process, which is a gap in trigonometry teaching. The study explored the effect of ABA on learners' performance in solving 2D trigonometry problems.

This study addressed the following three hypotheses:

**$H_{01}$** : There are no differences in experimental group (EG) and control group (CG) pre-test scores.

**$H_{02}$** : There are no differences in EG and CG post-test scores.

**$H_{03}$** : There are no differences in EG boys' and girls' post-test performance.

## THEORETICAL FRAMEWORK

The theory of didactical situations (TDS) (Brousseau, 1997) was adopted to guide data collection and analysis processes. TDS assisted in developing the study's conceptual framework's SEEDS phases. In this paper we used the SEEDS phases to guide the teaching and learning for the EG. According to Brousseau (1997, p. 15), "doing mathematics does not consist only of receiving, learning, and sending correct, relevant (appropriate) mathematical messages". TDS consist of two phases: the didactical and the adidactical phases.

During the didactical phase, learners and teachers get into a didactical contract, which embeds lists of expectations between the teacher and the learner (Warfield, 2006). In this contract, the teacher's role is to choose and prepare active, relevant, thought-provoking activities that will stimulate learning. The first author prepared 2D contextual trigonometric activities that aroused learners' interactions with one another and the content (Brousseau, 1997), helping them to take responsibility of their learning. These chosen problems were used as a scaffold to transfer learners into an adidactical phase providing the most independent and fruitful interaction between the learners.

Adidactical phase happens when a didactical contract between the teacher and the learners is broken to give learners a chance to be actively involved in their learning independently or with peers. In the adidactical phase, the teacher allows learners to accept and take responsibility for their learning when solving the given problem. Learners build their knowledge using different strategies devised by them or from previous lessons. Grade 11 learners were divided into groups and worked together in the 2D trigonometric activities. In this phase, social interactions between teacher and learners and between learners and learners are considered. In TDS adidactical phase, there are four basic situations. First is the situation of action where learners read the given problem, analyze it, and draw information from it. Second is the situation of formulation, where learners form concepts. The third is the situation of validation which involves learners verifying concepts. Fourth is the institutionalization situation where concepts, symbols and knowledge are used (Radford, 2008).

The SEEDS consists of five phases, stimulation, elucidation, execution, discussion, and summary. During the stimulation phase, learners and teacher interacted with each other, whereas during the elucidation phase, the learner explained to the teacher and the whole class what they understood about the activity. Both the stimulation and the elucidation phases are in the didactical phase of the TDS, where learners and the teacher interact and make a contract. During the execution phase, the contract was broken, and learners worked together independent of the teacher, and this phase is adidactical in TDS. The execution phase, which happened in situation of action, prompted the learners to act. Thereafter, learners reread the activity, followed by them acting out the given scenario in their groups when formulating ideas to solve the given activity. During the discussion phase, the learner and the teacher interacted again. This is didactical-adidactical phase. The two interact in the validation situation, during which the learner independently generated the essay to contribute to the discussion while the teacher scaffolded the discussion. During the summary phase, which is the situation of institutionalization, the learner reflects on the solution process, and the teacher scaffolds as the

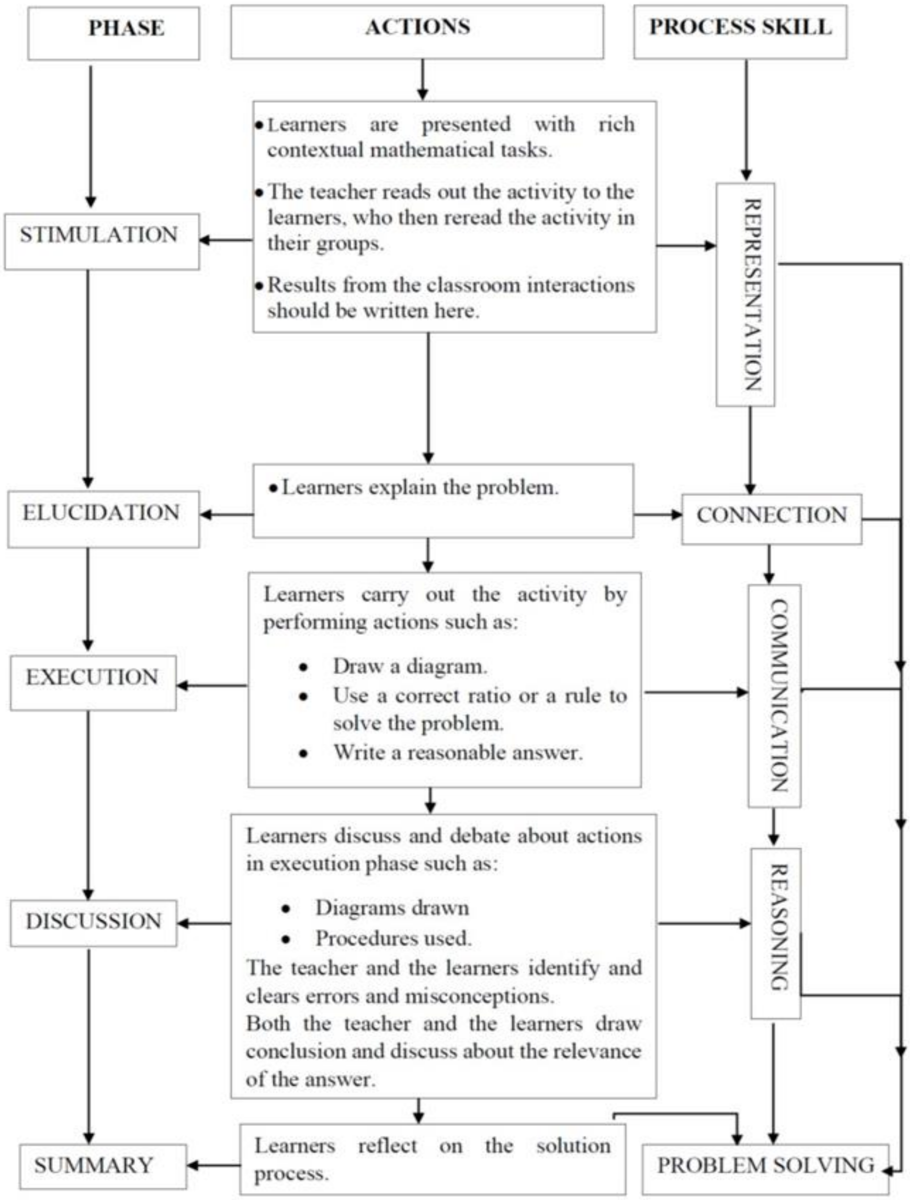


**Figure 1.** Phases of the SEEDS

class wraps up. Therefore, this phase is didactical-adidactical in TDS. **Figure 1** summarizes the SEEDS phases, actions to be looked for during classroom interactions and their connections to mathematical process skills.

TDS is depicted in all the five phases of SEEDS. More importantly, the two aspects of TDS, didactical and adidactical, are not mutually exclusive, and are two aspects on the same continuum. So, when we say the phase is didactical, it means actions in that phase are more didactical than adidactical and vice versa. The more and the less didactical or adidactical concept is similar to learner-centered and teacher-centered concepts. There is no time when the teacher will be alone in class situations or the learner alone in the classroom. The two interact at varying degrees at each point during the lesson. Finally, the five-phase unfolded in three stages, as follows:

1. didactical, where the teacher dominates,
2. didactical-adidactical, where the powers are equally shared between the teacher and the learner, and
3. adidactical, where learners work independently and share knowledge with other learners.

## METHODOLOGY

To get more insight into the complexities of teaching and learning trigonometry, quantitative and qualitative approaches were used (Creswell, 2009; Tashakkori & Teddlie, 2003). A sequential explanatory design was used, accommodating the quantitative phase, followed by the qualitative phase (Johnson & Onwuegbuzie, 2004).

Qualitative data was used to enhance quantitative results, thus triangulating the data (Creswell, 2014). The quantitative phase used a non-equivalent quasi-experimental pre- and post-test design (Campbell & Stanley, 1963). The design was chosen because it allows for the CG and EG to be randomly assigned to intact classes without disrupting the classroom set-up (Cohen et al., 2007). A phenomenological design was used in the qualitative phase, to determine learners' experiences of learning trigonometry in an ABA environment.

## Sample

A simple random sampling was used (Cohen et al., 2007) to select two secondary schools in one circuit of Limpopo Province in South Africa. This sampling procedure was used as it poses a minimum threat of bias in selection of participants (Cohen et al., 2007). Numbers were assigned to all 10 secondary schools and placed in a bowl, and two numbers were drawn from the bowl. The selected schools are ranked quintile two (Q2) and are all part of government feeding scheme. According to DBE (2017), quintiles 1, 2, and 3 are poor and declared no-fee-paying schools.

One school was randomly designated as the CG and the other the EG. During the quantitative phase, the EG consisted of 45 learners (17 boys and 28 girls), whereas the CG consisted of 57 (26 boys and 31 girls). Learners in the EG were instructed to divide themselves into seven groups (G1-G7) of five to seven members each. They were requested to ensure that there is a minimum of two boys and two girls in each group. Ultimately, four groups consisted of seven members, two groups of six members, and the seventh group of five members. The groups' composition was as follows: group 1 (two boys, three girls), group 2 (two boys, four girls), group 3 (three boys, three girls), group 4 (two boys, five girls), group 5 (three boys, four girls), group 6 (three boys, four girls) and group 7 (two boys, five girls). Learners within the groups were labelled, for example, L2G1, L3G3, L3G5, L4G1, L6G6, and L4G3. L in the label stands for the learner, whereas G stands for the group. For example, L2G1 stands for learner 2 in group 1. L3G3–learner 3 group 3. The CG was not divided into groups. Learners were seated the way they used to in their classroom when attending other lessons.

During the qualitative phase, purposive sampling was used to select all the EG learners as they were exposed to the intervention. However, only 37 of the 45 learners participated in the phenomenological part because eight did not consent to participate in the focus group discussion interviews (FGDI). For the FGDI, learners were divided into groups A-F according to their performance in the post-test. One focus group consisted of five members, and three groups of six members each and two groups of seven members each. According to Krueger and Casey (2001), a FGDI of five to seven is big enough to capture the required information without disruptions.

## Data Collection

For the quantitative phase, data were collected through a pre- and post-test in the two secondary schools. For the qualitative phase, data were collected through FGDI from six groups (Krueger & Casey, 2001), and the classroom interactions using the audio-video recording to document EG learners' experiences of ABA. Lessons in the EG were facilitated using ABA whereas in the CG the traditional teaching approach (TTA) was used. In the TTA class, all lessons followed the same pattern: the teacher explained the concept to be learnt to learners, then gave them an example and an exercise. The first author, a secondary school mathematics teacher for 19 years and a mathematics education lecturer for five years, facilitated lessons in the EG and taught the CG to control the confounding variable of teacher ability (Suter, 2011).

### *Instruments*

A pre-test, which also served as a post-test was used to collect quantitative data. In the post-test, the same questions given in the pre-test with a different arrangement were given to the learners. The subtopics used in the facilitation of the EG, and the teaching of the CG were trigonometric ratios, angle of elevation/inclination and depression/declination and area, sine, and cosine rule. The content validity index (CVI) of each item was calculated using Polit and Beck's (2006) formula:

$$CVI = \frac{no.of\ items\ judged\ as\ right\ by\ all\ judges}{total\ number\ of\ items}.$$

The CVI for 10 items was one indicating 100% agreement. Thus, all 10 items were relevant for the study (Zamanzadeh et al., 2015).

To establish reliability a Cronbach's alpha ($\alpha$) formula (Bland & Altman, 1997) was used:

$$\alpha = \frac{N.\bar{c}}{\bar{v}+(N-1)\bar{c}}.$$

The test items were correlated, and $\alpha$ was found to be 0.80, indicating that the test was reliable and appropriate for the study (Fraenkel et al., 2012).

During the intervention for the EG, the first author facilitated all the lessons on 2D trigonometric problems following the SEEDS phases, which provided the ABA learning environment. The EG school had two grade 11 classes, and each class had both mathematics and mathematical literacy learners in them. The learners come together in one class during the mathematics period. According to the South African CAPS, mathematics in grades 10-12 is divided into two papers (paper 1 and paper 2). Paper 1 includes number patterns, sequences, series, functions, finance, growth and decay, algebra, differential calculus, and probability. Paper 2 includes statistics, trigonometry, analytical geometry, Euclidean geometry, and measurement. In the EG school, paper 2 was facilitated for three days a week

(Monday, Tuesday, and Friday). Therefore, in this study, the EG and the CG mathematics lessons were facilitated for those three days in a week (Monday, Tuesday, and Friday) so that learning time would be equal. The researcher facilitated lessons in both schools during those mentioned days.

Activities were prepared, photocopied, and given to learners. All the activities to be utilized were developed using the following criteria:

1. Is the item in the activity contextual?
2. Does the question require learners to apply their prior knowledge and mathematical process skills?
3. When drawing a diagram for the given scenario, will it be a 2D diagram?

For the intervention lessons, learners were given eight activities over four weeks. All the activities were word problems which needed learners to first draw the diagram of the given problem, then apply trigonometric ratios or rules to solve them. The problems were categorized into four cognitive demand levels as in the CAPS document (DBE, 2011). The four cognitive demand levels are knowledge, routine procedures, complex procedures, and problem solving. These are comparable to how Stein and Smith's (1998) categorized cognitive demand levels. Like Stein and Smith's (1998) memorization category, tasks classified under knowledge require learners to reproduce the previously learnt formulas and definitions, while routine procedures are those without connections. The complex procedures category requires learners to use procedures with connections, whereas problem solving category needs learners to "do mathematics" (Stein & Smith, 1998). Knowledge and routine procedures are categorized at the lower-order levels whereas complex procedures and problem solving are at higher-order level. **Table 1** summarizes for the activities' questions and their cognitive level.

**Table 2** summarizes the total percentages for all cognitive demand levels.

In this paper we only focused on activity 1 item 1 to illustrate how ABA unfolded in the EG class. Activity 1 item 1 was as follows:

> A ladder leaning against the side of a house forms an angle of 65$^o$ with the ground. The foot of the ladder is eight meters from the building. Find the length of the ladder to the nearest meter.

The first author designed FGDI questions guided by how learners responded to the post-test. For the face validity of the questions, two experts at the University checked the questions, and necessary changes were made following their comments before data collection.

**Table 1.** Activities given to learners

| Week | Activity | Questions | Cognitive level (DBE, 2012) |
|---|---|---|---|
| 1<br>Lessons were about activities involving trigonometric ratios & angles of elevation & depression | 1 | 1. A ladder leaning against the side of a house forms an angle of 65$^o$ with the ground. The foot of the ladder is eight meters from the building. Find the length of the ladder to the nearest meter.<br>2. A guy wire is attached to a 100-foot tower that is perpendicular to the ground. The wire makes an angle of 55$^o$ with the ground. What is the length of the wire? | 1. Routine/procedures without connection<br>2. Routine/procedures without connection |
| | 2 | 1. If your distance from the foot of the cellphone tower is *20 m* and the angle of elevation from where you are standing to the top of the cellphone tower is 40$^o$, find the height of the tower.<br>2. From the top of the shopping center, the angle of depression to the foot of the civic center is 25$^o$. If the shopping center is *150 m* tall, find the distance between the two centers. | 1. Routine/procedures without connection<br>2. Routine/procedures without connection |
| 2<br>Lessons were about activities that involve area rule | 3 | 1. Mphahlele mall will be built on a piece of land that is in the shape of a parallelogram. The shorter side of the land measures 1.5 km long and the longer side is 2.5 km. One of the angles is 75$^o$. The planners decide to put a road through the shorter diagonal. Find area of the piece of land.<br>2. Mr. Sebake is planting a rose-bed in his garden. It is to be in the shape of an equilateral triangle of side 2$m$. What area of lawn will he need to remove to plant his rose-bed? | 1. Complex/procedures with connections<br>2. Complex/procedures with connections |
| | 4 | 1. The bonnet of a car is held open by a metal rod at an angle of 57$^o$. If the bonnet is *101 cm* long and the distance from the base of the bonnet to the front of the car is *98 cm*. Represent the information in a diagram and calculate the area of the drawn diagram.<br>2. Phiri, the right-hand side striker of Baroka FC, kicked the ball to Chipezeze, the goalkeeper, who is *6 meters* away from him. Chipezeze kicked the ball to Selemela, the left-hand striker who is *16 meters* away from him. Position of three players formed a *36 m*$^2$ area triangle. Show that sin of the angle formed by three players at Chipezeze equals to $\frac{3}{4}$. | 1. Routine/procedures without connection<br>2. Complex/procedures with connections |

**Table 1 (Continued).** Activities given to learners

| Week | Activity | Questions | Cognitive level (DBE, 2012) |
|---|---|---|---|
| 3 & 4 Lessons were about activities that involve sin & cosine rules | 5 | 1. Two wires support a flagpole side by side from one point. The first wire is *8 m* long and makes a 65º angle with the ground. The second wire is *9 m* long. Find the angle that the second wire makes with the ground.<br>2. Bonolo standing at point S, sights an electric pylon in front of her. She observes that the angle of elevation from where she is standing to the top of the pylon is 35º. She then walks *20 meters* towards the pylon and realised that the angle of elevation from this new point to the top of the pylon is 45°. Find the height of the pylon. | 1. Routine/procedures without connection<br>2. Complex/procedures with connections |
| 3 & 4 Lessons were about activities that involve sin & cosine rules | 6 | 1. Nando and Galito are standing *1,069 meters* apart on a straight horizontal road. They observe an aeroplane between them directly above the road. The angle of elevation from Nando to the aeroplane is 60° whereas from Galito is 75°. Find the height of the aeroplane at that point to the nearest meter. | 1. Problem solving/doing mathematics |
| | 7 | 1. A soccer goal posts are 8 *m* wide.<br>a. A player is directly in front of the goalposts such that he is 12 *m* from each post. Within what angle must he kick ball to score a goal?<br>b. A second player takes an angled shot at 25°. This player is 12 *m* from the nearest post. How far is he from the furthest post? | a. Routine/procedures without connection<br>b. Complex/procedures with connections |
| | 8 | 1. Lion and Mercy walk separate paths that diverge from one another at an angle of 48°. After three hours Lion has walked 7.9 *km* and the distance between them is 6.8 *km*. Find the distance Mercy walked at that time, correct to the nearest metre. | 1. Problem solving/ doing mathematics |

**Table 2.** Total percentages of all the cognitive demands

| Cognitive level | Number of questions | Percentage % |
|---|---|---|
| Knowledge | 0 | 0.00 |
| Routine procedures | 7 | 50.00 |
| Complex procedures | 5 | 35.71 |
| Problem solving | 2 | 14.29 |

## Data Analysis

### *Quantitative data analysis*

The quantitative data were analyzed using descriptive statistics: the mean ($\bar{x}$) and the standard deviation (SD), and inferential statistics: t-test, ANCOVA, and the Mann Whitney U-test. T-test was to identify differences between pre- and post-test means for EG and the CG, while Mann-Whitney U-test was to identify differences between gender (boys and girls) performance in EG. ANCOVA was used to show that the differences observed between the two groups in post-test were not by chance but due to the intervention. All analyses were done using SPSS version 27 as a tool. Finally, the effect size was calculated and interpreted using Cohen's d to measure the differences between the means of EG and CG (Sullivan & Feinn, 2012).

### *Qualitative data analysis*

Classroom interactions were analyzed using deductive analysis (Miles & Huberman, 1994). A start list was created from interview questions related to the research questions and literature. Through the coding process, core concepts formed themes (Bradley et al., 2007).

FGDI were analyzed using Groenewald's (2004) explicitation process. The explicitation process with five steps (Groenewald, 2004) was used:

1. Bracketing (epoche) (Moustakas, 1994), which suspended the researchers' views, experiences, biases, beliefs, and preconceptions about the phenomenon under investigation.
2. Horizontalizing, where the researcher carefully listened to the interview recordings to extract important and relevant statements from each interview (Moustakas, 1994).
3. Clustering similar meanings together to form themes (Creswell, 2013).
4. Summarizing all themes and did member checking by returning the summaries to the participants to confirm that what was captured was a true reflection of the participants' views (Groenewald, 2004).
5. General and unique themes were presented as the main findings (McMillan & Schumacher, 2010). Where learners answered in their mother tongue (Sepedi), those answers in Sepedi were translated into English and later "back-translated" into Sepedi to ensure efficacy.

**Table 3.** t-test results of the pre- and post-test of EG and CG (*significant p=.05)

| Test | Group | n | M | SD | t | p | d |
|---|---|---|---|---|---|---|---|
| Pre-test | Experimental | 45 | 8.36 | 9.87 | 0.55 | 0.58 | 0.11 |
| | Control | 57 | 7.33 | 8.91 | | | |
| Post- test | Experimental | 45 | 22.29 | 18.21 | 3.95 | <.01 | 0.79 |
| | Control | 57 | 10.51 | 11.79 | | | |

**Table 4.** Analysis of covariance summary of the EG and CG post-test

| Tests of between-subjects effects | | | | | | |
|---|---|---|---|---|---|---|
| Dependent variable: Post-test | | | | | | |
| Source | Type III sum of squares | df | Mean square | F | Sig. | Partial Eta squared |
| Corrected model | 17,112.44[a] | 2 | 8,556.22 | 96.78 | <.01 | .66 |
| Intercept | 2,563.22 | 1 | 2,563.22 | 28,.99 | <.01 | .23 |
| PRE-TEST | 13,622.76 | 1 | 13,622.76 | 154.08 | <.01 | .61 |
| EG AND CG | 2,765.76 | 1 | 2,765.78 | 31.28 | <.01 | .24 |
| Error | 8,752.74 | 99 | 88.41 | | | |
| Total | 51,026.00 | 102 | | | | |
| Corrected total | 25,865.18 | 101 | | | | |

Note. [a]R squared=.66 (adjusted R squared=.66)

**Table 5.** A Mann-Whitney U-test EG for boys and girls in pre-test

| Test statistics[a] | | | |
|---|---|---|---|
| | Boys & girls | Boys | Girls |
| Mann-Whitney U | 195.00 | | |
| Wilcoxon W | 601.00 | | |
| Z | -1.03 | | |
| Asymp. Sig. (2-tailed) | .30 | | |
| Exact Sig. [2*(1-tailed Sig.)] | .30[b] | | |
| [a]Grouping variable: EG male & females | | | |
| [b]Not corrected for ties. | | | |
| Median | | 8.00 | 4.00 |

## RESULTS

The post-test results show that using ABA improved learners' performance better than using TTA t(102)=3.95; p=0.05) with a Cohen d=0.79 (**Table 3**). The results indicated no gender differences in the EG learners' post-test performance. The main findings from the FGDI were divided into positive experiences and negative experiences. The positive experiences included collaborative group work, solving trigonometry problems, developing trigonometry vocabulary, different ways of solving the problem and improved performances. The negative experiences included group work and time consumption. Below are the results of the quantitative part showing t-test results of EG and CG pre- and post-test performances.

**Table 3** indicates no statistical differences between the pre-test of the EG (M=8.36, SD=9.87) and CG (M=7.33, SD=8.91), [t(100)=.55; p=.58] with Cohen d=.11. However, there were significant differences after intervention between CG (M=10.51, SD=11.79), and EG (M=22.29, SD=18.21) and [t(100)=.3. 95; p=.05] with Cohen d=.79.

**Table 3** further shows that the t-statistics of the post-test is 3.95 with the p<.01 at $a$=.05. These results indicate a significant difference between post-test scores of EG (M=22.29, SD=18.21) and CG (M=10.51, SD=11.79). These indicate that EG scores were higher than the CG. The EG learners' post-test mean is 11.78 points higher than CG post-test mean [95% CI (5.86-17.7)]. We, therefore, reject the $H_{02}$ and conclude that there are differences in the post-test scores of EG and CG, with EG performing better than CG.

### ANCOVA of the EG and the CG Post-Test

The dependent variable is the post-test scores, the independent variable is the groups (denoted by EG and CG), and the pre-test scores covariate. ANCOVA was used to show whether the differences observed between two groups in the post-test were not by chance but due to the intervention.

In **Table 4**, the p<.01 indicates a significant difference in post-test scores between the groups, when adjusted for the pre-test score. The adjusted mean indicates that the EG performed significantly better than the CG. It indicates that indeed, the improvement in the EG group is the result of the ABA intervention (p<.01).

Comparing the EG girls' and boys' pre-test and post-test performance using a Mann-Whitney U-test.

In **Table 5**, a Mann-Whitney U-test shows boys' performance in the pre-test was higher (Mdn=8.00) than girls (Mdn=4.00), U=195.00, p=.30. However, the performance of boys and girls is not significantly different (Mann Whitney U-test, p=.30).

A Mann Whitney U-test EG results for boys and girls in the post-test are presented in **Table 6**.

In **Table 6**, a Mann-Whitney U-test shows males' performance in the post-test was somewhat higher (Mdn=23.00) than females (Mdn=14.00), U=180.00, p=.17. However, the performances of boys and girls are not significantly different (Mann Whitney U-test, p=.17).

**Table 6.** A Mann-Whitney U-test EG for boys and girls in post-test

| Test statistics[a] | | | |
|---|---|---|---|
| | Boys & girls | Boys | Girls |
| Mann-Whitney U | 180.00 | | |
| Wilcoxon W | 586.00 | | |
| Z | -1.36 | | |
| Asymp. Sig. (2-tailed) | .17 | | |
| Exact Sig. [2*(1-tailed Sig.)] | .17[b] | | |
| [a]Grouping variable: EG male & females | | | |
| [b]Not corrected for ties. | | | |
| Median | | 23.00 | 14.00 |

We, therefore, do not reject the $H_{03}$ and conclude that there are no gender differences in test scores for EG learners' post-test performance.

The lessons followed the SEEDS phases. During the stimulation phase, the teacher began by reading the activity to the learners and then encouraged them to reread the activity in their groups to make sense of what was needed. In activity 1, for example, learners were given the following activity:

> A ladder leaning against the side of a house forms an angle of 65$^{o}$ with the ground. The foot of the ladder is 8 meters from the building. Find the length of the ladder to the nearest meter.

During the elucidation phase, learners gave an interpretation of the given problem. For example, L2G1, a learner in group 1, explained the group's understanding of item 1 in the following manner:

> A ladder is always on the wall (L2G1 demonstrating to the class, showing by her hand on the chalkboard wall). A wall is vertical, right. The ground is always horizontal, okay? Therefore, the ladder together, with the ground and the wall, forms a triangle.

When the teacher interacted with the learner's explanation during activity 1, she asked the following questions to the whole class:

> L2G1 indicated that a triangle is formed; which type of triangle is formed?
>
> On which side of the triangle are you going to put eight meters?
>
> At which angle are you going to put 65$^{o}$?

During the execution phase, learners were given a platform to engage with the activity's presentations. Immediately after learners were presented with the activity, different groups reacted differently to make sense of what the activity was all about. For example, in group 7, a learner demonstrated using hands, where one hand was a wall, and the other was a ladder. A learner in group 4 leaned against a wall, trying to model a scenario. In all other groups, learners developed autonomy and started talking to each other, and the class was noisy.

During the discussion phase, the groups were selected to present their solutions to the whole class after the teacher realized that most groups had finished discussing the activity. The selection was based on the errors or misconceptions portrayed in the group's solution. Learners were provided with a platform to engage with each group's presentation. The groups with the most errors or misconceptions in their solutions were allocated to present first, followed by those whose answers were partially correct. For this article, we only presented G3 presentation. Their solution was partially correct, they however did not realize that. The solution drew attention of the whole class and provided more interactions.

The proceedings of activity 1 are, as follows:

> L3G3: From our group, we thought that we should use tangent to find the length of the ladder, so we realized that we calculated the length of AB instead of AC. That's where our mistake lies.
>
> Teacher: Is group 3 wrong?
>
> All groups started to discuss. Ultimately, L3G5 said "they are wrong mam, they calculated for the height of the wall not the length of the ladder".
>
> L4G1: If we were asked to calculate the height of the wall, they were correct.
>
> Teacher: Let us look at their solution in connection with the way to answer the given question. How does their solution help in answering the question? Discuss in your groups.
>
> The teacher moved from group to group listening to their discussions.
>
> G4, G5 and G7 maintained that they were wrong because they calculated the wall's height not the ladder's length. G2 seemed confused by the question, and G6 discovered that G3 solution was partly answering the question. Below is their representative answer to the teacher's question.
>
> L6G6: We think G3 is not wrong, but their solution to the problem is incomplete. We think if we can calculate sine 65°since we now have two sides, the opposite and the adjacent we can get the answer correct.
>
> Teacher: In your groups, use sine 65° as they indicated and share your findings. This time we want G5 to answer.

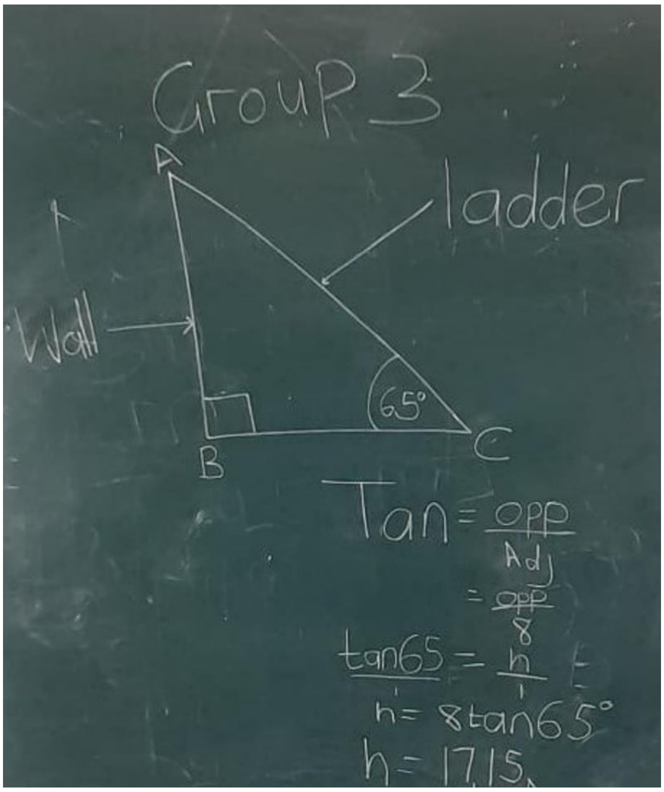


**Figure 2.** Group 3 presentation of activity 1

> L5G5 After a few minutes: The answer we got is 18.92 and to the nearest is 19 meters; this is the same as group 2's answer.
>
> L3G1: Mam, we also got 19 but using a different method.
>
> Teacher: What is your method? Is it the Pythagoras' theorem?
>
> Group 1: Yes, mam.
>
> (Learners laughed)
>
> Teacher: Please share with us.
>
> Teacher: Can all of you please use Pythagoras' theorem to answer the question?
>
> L4G3: Yes, they are correct; the answer is 18.9 rounded off to 19 meters.
>
> Teacher: Group 3, what have you learnt about your solution?
>
> L3G3: We have learnt that we have partially answered the question and that we were still on the right track by starting with the tangent ratio.

Although group 3 used a tangent ratio, they did not realize they were on the right track (**Figure 2**). They thought they were wrong to calculate the height of the wall. After the intervention by the teacher and other groups interacting with their presentation, they realized that they should have applied Pythagoras' theorem after they got the height of the wall. In the summary phase, a learner was asked to summarize what they had learned in the lesson, specifically reflecting on the main concepts of the lesson. L3G3 summarized the lesson: When solving the 2D trigonometric problems, you should always look for the missing side or angle in the triangle and relate that to the trigonometric ratios. Again, we should always strive to use different ways of solving the problem.

## Focus Group Discussion Interviews

Themes were identified from the FGDI during Groenewald's (2004) explicitation process. The themes were classified according to the TDS. Detailed ABA obstacles and affordances to learning and performance were identified according to each situation of the TDS and informed the development of the subthemes which are reported below:

### *Situation of action*

**Reading, analyzing, and drawing information from the activity:** Some groups analyzed and devised strategies for sharing ideas within their groups and getting helped by their peers in the group. For example, members in group B said:

> We were divided into groups to discuss and answer the questions. After the teacher had read the activity to us, we also read and devised strategies for solving the problems. For example, we said, let us all solve the problem in our scribblers, and then, we compare and discuss the answers. In that way, we found that everyone was involved, and we were sure that we understood what was happening before class presentations.

Group C members indicated that ABA assisted them developing an understanding of the problem given in the activity. For example, a member in this group said:

> Before you could answer the questions, you should read the statement, make sure that you understand it and then draw the diagram this will make you to know as to whether you should use trigonometric ratios or rules, because without a diagram you would not understand what is needed.

### *Situation of formulation where learners form concepts*

**Solving trigonometry problems**: Focus group members reported that learning in an ABA environment helped them understand how to solve trigonometry problems. For example, for group D, a member said:

> In the first place, we did not understand how to answer the questions, but after we were taught using ABA, we could now answer that type of questions. This helped us in the post-test as we could use the expertise we gained when we were taught using ABA. This approach gave us the skill of representing trigonometric problems using diagrams before solving them.

Though some groups reported being able to help each other during groupwork, others reported their frustrations with working in groups. For example, a member in group C talked about the fear of asking questions as he thought other classmates would think he was dumb. He said:

> During group work, I normally feel ashamed to tell my classmates that I do not understand because they will say I am stupid, so I kept quiet, knowing that during presentation time, when others are presenting, and others are asking questions, that is where I will get something. However, even now, there are some things I do not understand.

All members in group A agreed with a member who indicated, "during the lesson, some learners in their groups were making noise arguing about the solutions that others are wrong, and others are right, and we could not concentrate in our group".

**Developing trigonometry vocabulary:** Gaining trigonometry vocabulary also emerged as one of the experiences that most participants mentioned they had in an ABA learning environment. Participants cited trigonometric terms they felt they did not know before the intervention and the terms included angles of elevation and depression. A member in group E indicated that:

> Last year, we were just given diagrams to interpret without telling us the names of the angles. We only knew the straight, acute, obtuse and the reflex angles, but not the special angles like elevation and depression in trigonometry.

Group C members indicated that ABA assisted them in developing representation skills, which correctly impacted how they communicated the problem during the post-test. A member in this group said, "we have learnt how to draw the graphs and after that which trigonometric ratio to apply". Another member plainly explained:

> Before answering the questions, read the statement, make sure you understand it and then draw the diagram. This will make you to know whether you should use trigonometric ratios or rules because, without a diagram, you would not understand what is needed.

### *Situation of validation which involves learners verifying concepts*

**Verifying concepts during discussion:** A member of group C explained:

> During the pre-test, we did not know what elevation is and what is depression. However, during the lessons and discussions, when you asked us about what inclination is, I realized that the angle of elevation is the same as the angle of inclination and the angle of depression is the angle of declination. At first, I thought the inclination angle was for analytical geometry only, but now I understand why we are using tan [tangent] to calculate it in analytical geometry.

### *Situation of institutionalization where concepts, symbols and knowledge are used*

**Solving trigonometric problems:** A member of group C indicated: "Now we are able to solve trigonometric problems, especially those involving trigonometric ratios". Another member from group C indicated that:

> I enjoyed working with these real-life problems, mam. Even after school, when we were going home, we shared this with our schoolmates, showing them that we can use trigonometry to calculate the height of the pole and the mountain. We enjoyed it more when we were telling the grade 12 learners. To me, to draw a diagram and interpret it is no longer a problem.

For group D, a member said:

> In the first place, we did not understand how to answer the questions, but after we were taught using ABA, we could now answer that type of questions. This helped us in the post-test as we could use the expertise we gained when we were taught using ABA. This approach gave us the skill of representing trigonometric problems using diagrams before solving them.

**Different ways of solving trigonometric problems:** Most focus groups emphasized that learning in an ABA environment allowed them to use different methods when solving a problem. For example, a member of group D summarized their discussion by saying:

> Group work helped us in understanding because we were given a lot of different methods of how to solve one problem because each group was given a chance to present their workings on the board if it is different to those already written on the board.

A member of group E indicated,

> We learnt a lot when sharing ideas, many ways of solving the question, there are many ways to kill a cat, there are many ways to go to a place you want to go. We could see the easy methods of solving the questions [problems].

**Improved performances:** Most focus group learners had a feeling that ABA improved their performances. Below are how some members of the groups sharing their experiences.

One member of group C made a statement that represented most comments of many members in this group by saying:

> In the pre-test, we did not perform well because we were not used to teaching how we were taught trigonometry in grade 10; we were allowed to work in groups. Our teacher used to give us an example; after that we would solve it together with him or sometimes solve it, asking us questions here and there. In the beginning, we did not understand what we were supposed to do; however, at least in the post-test, we improved our performances.

Members of group D shared the same feeling felt by a member of this group, who commented that:

> In the first place, we did not understand how to answer the questions, but after we were taught using ABA, we could now answer that type of questions. This helped us in the post-test as we could use the expertise we gained when we were taught using ABA. This approach gave us the skill of representing trigonometric problems using diagrams before solving them.

Other group members highlighted that the approach helped them understand the trigonometry, though it is time-consuming. For example, all members in group D agreed with the member who said:

> The challenge was that we had to understand under compressed time because the time of the period was short. We took too much time discussing each group's presentation, even if it differed slightly from the others. We think this method helps us understand, but it needs much time.

Members of group F shared the same experience; however, they put it in another way. Although they praised ABA and were excited about how learning took place in an ABA environment, their manner of praise indicated that ABA is time-consuming. A member of group F highlighted,

> The way we were taught was excellent. The lesson is so slow that everybody in the group will understand because we do few things in class.

## DISCUSSION

The study explored the effect of the ABA on grade 11 learners' performance in solving two-dimensional trigonometric problems. The t-test results indicate EG and CG learners performed at the same level in the pre-test. The study shows no significant differences between the pre-test CG and EG scores (**Table 3**). It, therefore, suggests that we fail to reject $H_{01}$ and conclude that there are no differences in the EG and CG pre-test scores. This implies that at the beginning of the intervention, the EG and the CG were homogeneous (Edmonds & Kennedy, 2013) and of similar ability concerning their knowledge of 2D trigonometry.

This study further revealed that learners whose learning was facilitated using ABA performed better than the CG who were taught using TTA (**Table 3**). Therefore, we reject $H_{02}$, which states that there are no differences in the post-test scores of EG and CG. These results support those obtained by other researchers in their studies, where learners exposed to ABA performed significantly higher than those taught using other methods (Emaikwu, 2012). The learners in the EG worked together in their groups, developed, and applied problem-solving strategies, and discussed and arrived at conclusions about the solution to the problems. The gains achieved agreed with the TDS that learners interact with each other by acting, formulating, and validating their ideas (Brousseau, 1997). It indicates that ABA is effective in improving learners' performance. The .79-point estimate of Cohen's d indicates that large effect, according to Sullivan and Feinn (2012). After adjusting the pre-test scores, ANCOVA results show that the improvement in performance was because of ABA intervention and agreed with Celik (2018).

The results show that the EG boys and girls performed at the same level in the post-test (**Table 6**). This indicates that ABA does not segregate between boys and girls. This, therefore, suggests that we fail to reject $H_{03}$, which states that there are no gender differences in EG learners' post-test performance. These findings concur with Barai (2018) and Emaikwu (2012), who found that an ABA does not segregate between males and females. These results contradict the DBE (2020)'s diagnostic report, which indicated that from 2016 to 2020, South African male candidates performed better than female candidates in mathematics. The results, therefore, suggest that ABA can ease the problem of gender imbalance in science, technology, engineering and mathematics (STEM) in South Africa.

During the implementation of the SEEDS phases, learners were stimulated by the activity to interact with each other and act out the given problem in the execution phase which happened in the adidactical phase of the TDS. The classroom interactions provided an understanding of how the ABA learning environment offered learners an opportunity to work collaboratively with each other during all phases of the lesson (Noreen & Rana, 2019). Learners could talk to each other within their groups during the discussion phase. This was in line with the TDS, which indicates that learners become active participants when working with their peers (Brousseau, 1997). During this discussion phase of the lesson, learners acquired mathematical process skills: connection, communication, reasoning, and representation as they engaged in groups presentations. This supports previous research in this field, concluding that ABA fosters critical thinking and mathematical process skills (Pokhrel, 2018). Learners' frustrations were cleared and their strategies for solving problems were enhanced during classroom interactions in an ABA environment. For example, during the intervention, learners thought there was only one way of solving a problem. They thought group three was wrong to use the tangent ratio to calculate the height of the wall instead of the length. The whole class was given an opportunity to relook at the solution to establish other strategies to lead to the correct answer. This was didactical-adidactical as during this phase the teacher asked questions which gave learners opportunity to use other methods of solving the problem.

The following paragraphs summarize all the actions the learners and the teacher performed while applying the SEEDS procedure and their connections to mathematical process skills.

1. During the stimulation phase, learners were presented with real-life activities which activated their thinking.
2. During the elucidation phase, learners gained connection skills through explanations of the problem by connecting the real-world problem with triangles. They also communicated the scenario in their own words, modelling the situation and converting it into algebraic statements.
3. In the execution phase, learners develop communication skills through various representations and problem-solving strategies such as drawing a diagram, applying a correct ratio or a rule, reasoning and questioning the decisions taken. Learners also gained representation skills. They represented the ladder by leaning against the wall in a slanting way, demonstrating the scenario using a ruler and a notebook, and using hands to model a wall and a ladder.
4. In the discussion phase, learners gained reasoning skills as they debated: Diagrams drawn, procedures used, errors and misconceptions discovered, conclusions drawn, and the relevance of the answers provided.
5. In the summary phase, learners gained problem-solving skills when they reflected on the answer. In addition, they reflected on the process that led to the solution and sought to perfect it.

From the FGDI interviews, learners indicated they worked collaboratively to solve problems when learning in an ABA environment. This concurs with Kaur and Sankhian (2017), who stated that using an activities-based method provides better mathematical learning opportunities and improves learners' collaboration and self-expression. Other participants indicated they devised strategies to solve the problem when working in groups during the execution phase. This shows that learners were self-reliant during the adidactical situation of formulation. The EG learners also mentioned that they gained in-depth knowledge and increased their vocabulary of trigonometry, hence improvement in their performance. This indicates that learners were able to formulate and institutionalize concepts when learning in ABA environment. These findings support Kaur and Sankhian (2017), who revealed that ABA improves the learners' content knowledge in mathematics and performance.

During the situation of action, focus groups indicated that they read, analyzed, and drew information from the given activities. The findings also indicated that some learners claimed to have experienced difficulties with participating in groups during the teaching and learning process. This indicates that the learners thought that during lessons, the teacher should feed them with information. TDS indicates that these learners' didactical contracts have not been broken (Warfield, 2006). They did not trust themselves and their classmates during the execution and discussion phases. They thought that there should be an authority to tell them what to do and what not to do when learning.

The interviews indicated that some learners felt ABA was time-consuming, as they did not have sufficient time to deal with and explain the questions. This observation agrees with Kaur and Sankhian (2017), who, in their study, concluded that though Activity Based method simplifies learning and assists learners to gain permanent knowledge, it is a time-consuming and lengthy teaching method.

## CONCLUSION

The findings established that learners in the EG whose lessons were facilitated using ABA achieved higher scores than those in the CG, where TTA was used. Therefore, ABA has positive effect on learners' academic performance in solving two-dimensional trigonometric problems than TTA. Although ABA was time-consuming, it was worth the effort because it improved learners' autonomy and performance. The study concludes that with determinations to work hard, those challenges could be overcome to implement ABA to improve learners' performance.

### Recommendations

Teachers should adopt ABA in teaching trigonometry as there is a possibility that learners would participate actively and collaboratively with little guidance from their teachers, which could improve learners' academic performance in trigonometry. Teachers should also caution not to prepare many trigonometric real-life activities to be carried out in one lesson, as learners reported ABA to be time-consuming.

### Limitations

The authors acknowledge that control group was not interviewed, it can be better for further studies to interview the control group after the intervention.

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**Declaration of interest:** No conflict of interest is declared by authors.

**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

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