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The Effect of Using Graphing Calculators in Complex Function Graphs

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This study investigates the role of graphing calculators in multiple representations for knowledge transfer and the omission of oversimplification in complex function graphs. The main aim is to examine whether graphing calculators were used efficiently to see different cases and multiple perspectives among complex function graphs, or whether graphing calculators were used only as a mechanical tool to push buttons and execute memorized steps. Twenty individuals chosen from seven college calculus I classes (148 students) participated in this study. A survey was administered to students in order to find their attitudes and prior use of using graphing calculators. Data was gathered from the video-taped interviews with students to determine how the graphing calculator was used in the tasks and to get a deeper understanding of college students' engagement process with graphing calculators. The results indicated that experience with the graphing calculator was important factor in solving the tasks with the graphing calculator, while attitude seemed to have no effect on task solving steps. Results clearly show that in order to use the graphing calculator in complex function graphs to implement the multiple representations of knowledge, the students need to know characteristics of features on the graphing calculator. They have to have some use of skills and good experience on the machine, not just skills of thinking and skills of knowing the concept.

Keywords: Complex, Function, Graph, Attitude, Experience, Calculator.

THE EFFECT OF USING GRAPHING CALCULATORS IN COMPLEX FUNCTION GRAPHS

The use of graphing calculators is becoming common in mathematics classes. However, little is known about why and how graphing calculators make a difference in mathematical understanding. There are two reasons for that. First, much of the initial research on graphing calculators only compared the achievement and attitudes of different student groups using graphing calculators and traditional instruction (non-calculator groups). Secondly, research generally looked at students' basic mathematical ability with very minimal graphing calculator utilization. What is missing from research on the use of graphing calculators is important information about the role the graphing calculators play in the class environment. The students' flexibility on understanding of graphical concepts was mostly ignored.

Hennessy et al. (2001) showed both that graphing calculators can be used mechanically, and manual/paper-pencil work to show the steps of drawing the graphs on the paper is essential for students to develop concepts and skills in a difficult curriculum area. In their survey results, it seemed clear that despite positive immediate feedback, rapid and easy plotting, and visualization with graphing calculators, most students struggled with understanding mathematical concepts. This indicated to authors that some manual (paper-pencil) work and tutor help was needed. Graphing calculators saved both time and space, but Hennessy et al. (2001) concluded that both graphing activities and examinations need some kind of

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conceptual understanding rather than the sole use of graphing calculators.

Graphing calculators give students automatically produced graphs. By using real data, students can get immediate feedback from the graphing calculators, compare and contrast different graphical representations simultaneously. In this way, with immediate input and output, students can improve their own explorations by using the features of graphing calculators. However, it is also possible that graphing calculators might cause automatized procedures (key pressing steps or memorizing to push buttons) rather than enhance students' understanding of complex graphical concepts. Automatized procedures might affect students' understanding of graphical concepts. Using graphing calculators might cause students to memorize key stroking steps (and only produce answers) for graphical tasks, without understanding the drawing steps of graphs.

Some researchers expressed their concern about the use of graphing calculators as a mechanical process (e.g., memorization of key pressing steps). They stated that this process might lead students to avoid the process of drawing a graph and turn their attention to only the graph itself (Yerushalmy & Schwarts, 1993; Hennessy et al., 2001). Even with the easy plotting with graphing calculators, students may not get beneficial ideas from the multiple representations of functions. Doerr & Zangor (2000) found that students saw graphing calculators as a "black box" and they only used it as a private tool. In their study, as a private tool, students showed frequent failure to join group discussions while using graphing calculators. Moreover, students did not show a meaningful strategy for the use of the graphing calculators. They failed to produce meaningful interpretations of the task situation with the graphing calculator.

Kwon (2002) focused on students' graphing ability in terms of interpreting, modeling, and transforming, and indicated that calculator-based range activities enhanced students' ability to understand graphs. Actually, these three components (interpreting, modeling, transforming) were based on Leinhardt et al.'s (1990) action-task classification of graphing and functions. In this classification, there are two components (interpretation and construction of the graph) in action and four components (prediction, classification, translation, and scaling of graph) in tasks. When graphing calculators are used, scaling and construction processes (and partially translation) are totally lost. However, interpretation, classification, and prediction have the potential to be improved efficiently when using the features of graphing calculators for complex graphs. Literature does not say explicitly whether using graphing calculators cause students to lose some of these components. Especially, when the graphs become complex, we do not know how much graphing calculators can make positive contribution to students' graphical understanding.

Spiro et al. (1991), in their cognitive flexibility theory, gives considerable attention to complex domains in the learning process. They suggest that learners integrate different aspects of the knowledge to increase transferability to different learning contexts in order to create new representations. However, one would appreciate how it is difficult to make these transformations in the reasonable class time. It is not very clear whether using different cases and examples (with graphing calculators) in their full complexity facilitate learning in complex function graphs. Someone would expect that students will be able to classify, translate and interpret the complex function graphs flexibly. Since some studies argue that the graphing calculator can be used mechanically, it is important to see how this mechanic procedure occurs in students' use of the graphing calculator. Seemingly, using the graphing calculator might help student to see and master categorization of graphing tasks and affect their understanding in the process of constructing graphs, when the graphical tasks become complex.

There is some research on how students use graphing calculators and what kind of patterns/modes emerge (e.g., as a tool for exploratory or confirmation tool, and /or for graphical representation or numerical representation) on complex functions that have not been always examined. Hennessy et al. (2001) identified three roles of graphing calculators: a catalytic role, a facilitating role, and a checking role. However, Doerr & Zangor's (2000) description was more detailed. Through their analysis of the data, they identified five categories of patterns and modes of calculator use: computational tool, transformational tool, data collection and analysis tool, visualizing tool and checking tool. Similarly, Kwon (2002) highlighted three patterns: interpreting, modeling, transforming. From a different perspective, Choi-Koh (1999), in his case study with one student, used Bloom's taxonomy for cognitive domain in the use of graphing calculators. He identified six patterns while the student was working on graphing calculators: evaluation, synthesis, analysis, application, comprehension, and knowledge of terminology.

It can be hypothesized that graphing calculators, by using supporting material, are suitable to understanding and solving complex function graphs and helping students use different representations. Graphing calculators can provide opportunities to solve complex function graphs and by helping students explore functions and their graphs in more than one way. Heid (1988) pointed out that in calculus courses, students are mostly assigned very traditional and straightforward functions to graph such as $y= 2x^2+5x+2$ or $y=2x^2-5x-3$. Without technology, these equations also require considerable time to draw the graphs. Demana, Schoen, &Waits (1993) argue that limiting ourselves to traditional function graphs seriously restricts students from functions that they can manipulate. Students mostly solve linear and quadratic functions in both traditional high school curricula and college calculus classes. However, as Tall (1997) pointed out, traditional calculus curriculum includes mastery of symbolic methods for differentiation and integration and applying these to work with a range of functions. This position makes it necessary to approach calculus in different ways, with a consequent variety of curricula. Moreover, calculus includes a broad range of functional forms that college calculus classes do not cover as much.

Moschkovih, Schoenfeld & Arcavi (1993) argue that there are at least two ways to approach solving calculus tasks (analytic solutions and graphical solutions). The situation becomes more challenging when considering more advanced functions. Additionally, Romberg, Carpenter & Fennema (1993) argued that the creation of most graphs, especially in complex functions, like polynomial or logarithmic functions, is very difficult. The difficulty comes from the fact that many function graphs require many points to be plotted and sketched. Also, the creation of the pair values in a table is a more difficult and advanced situation. Using the graphing calculator allows students to see changes and transformations on more advanced functions and their graphs at the same time. In other words, graphing calculators allow students to see where complex graphs shift, reverse, and stretch; and allow students to see different case examples(function and their graphs) to show multiple perspectives of the content with its complexity and ill-structuredness.

Thinking about the complexity of calculus topics, one would appreciate how it is important to use multiple representations of information. An assumption underlying this study was that graphing calculators are suitable to do these multiple representations. It is vital to see which representation (graphical, numerical, or algebraic) or combination of representations students choose to use and how they use them when they solve complex function graphs with graphing calculators. Therefore, there is a need for a study to document the ways graphing calculators were used by individual students. It is necessary to look at the students while using graphing calculators to examine how they used them, when they were using them, for what kind of purposes, and whether calculator use enhanced their understanding/learning in complex function graphs.

Tall (1997) argued that a student's development of cognitive flexibility in calculus requires significant constructions and re-constructions of knowledge. He mentioned that the way in which numeric and symbolic representations develop involves an interesting form of cognitive flexibility. In his study, calculus was summarized as the study of "doing" and "undoing" the process. In this process, the flexibly in switching from one representation to another seemed very difficult for the average student. Students managed to move from one representation to another, but failed to move flexibly back and forth. Graphing calculators can be capable of providing multiple representations of many calculus tasks to help students learn to think about calculus concepts flexibly. Additionally, Boers & Jones (1993) studied students' use of graphing calculators to find the graph of

$$f(x) = \frac{x^2+2x^2-3}{2x^2+3x-5}$$

Results indicated that 80%of the students had difficulty reconciling the graph with the algebraic information. Moreover, Quesada (1994) introduced graphing calculators into a calculus class. However, 60% of the students received a grade of D or F, or withdrew from the course, which the author interpreted as students' lack of a clear understanding of the basic function graphs that they could not read basic graphs, after calculators were introduced.

Mostly, in traditional math classes, students are supposed to stick with certain types of functions. However, graphing calculators can give students a chance to see different and more complex function graphs by using different representations. Actually, the teacher, in a class environment, needs some kind of environment in which multiple representations are used efficiently to transfer knowledge, and that oversimplification must be avoided. Since most existing studies only compared the use of graphing calculators with the use of paper and pencil methods on the same kind of tasks in a very short class time, the question of how the use of graphing calculators can be used to enhance cognitive flexibility is unanswered.

This study looks at the role of graphing calculators in multiple representations for knowledge transfer and the omission of oversimplification. For example, thinking about the transfer of knowledge through the lens of literature, it is interesting to see the reactions of students to basic types of transformations (e.g., horizontal shift, vertical shift, reflection about the x-axis/y-axis/ the origin) done using graphing calculators. Thinking about the notation y=f (x), even in simple function graphs, the importance of using graphing calculators is still unknown. The idea of using multiple representations of knowledge fits well with using the graphing calculator. That is, graphing calculators are capable of providing multiple representations of mathematical concepts. Students can easily switch among tabular, algebraic, and graphical representations, allowing them to observe patterns and relations. By building tables, tracing along curves, and zooming in on critical points, students may be able to process information in a varied and meaningful way.

Purpose of the Study

By connecting the ideas from current research literature on graphing calculator, this study investigates how college students use graphing calculators to construct and understand complex function graphs in calculus. The main aim is to examine whether graphing calculators were used efficiently to see different cases and multiple perspectives among complex function graphs, or whether graphing calculators were used only as a mechanical tool to push buttons and execute memorized steps. It can be expected that allowing students to use graphing calculators will enable to look at the introduction of complex function graphs without oversimplification. Or, as some researchers argue, college students might become too dependent on the graphing calculator and lose rich and flexible understanding in calculus topics when the graphing calculator is used.

Spiro et al. (1991) and Resnick (1988) argued that most teachers heavily rely on the simplification of the topic. It is not saying that learning should begin with mass complexity, because that can lead to confusion. Theoretically, using graphing calculators can accelerate the understanding of experiences with different function graphs so students are better prepared to apply their knowledge to new or similar cases.

The literature suggests that students must build a broad knowledge base and flexibility of thought that facilitates learning in complex, non-linear functions. Therefore, it was hypothesized that graphing calculators would help students enhance their understanding of function graphs in calculus classes. Moreover, this study will examine whether students' understanding is affected by their prior knowledge of and attitudes toward graphing calculators. Following questions were addressed by this study:

1. How are the patterns students follow in constructing complex function graphs related to complexity and difficulty level of tasks when the students work with graphing calculators?
2. To what extent flexible thinking and/or rote memorization of knowledge occur when students are working with complex functions on graphing calculators?

METHOD

Participants and Setting

Twenty individuals chosen from seven college calculus I classes (148 students) in Upper New York State participated in this study. This study was conducted in at three institutions: private college, community college, state university. In order to diversify the range of students, I chose classes with differing ability of using graphing calculators. Seven classes were from a community college (one class=22 students), a private college (three classes=total 53 students), and a state university (three classes=total 73 students).

A survey was administered to students in order to find their attitudes and prior use of using graphing calculators. Attitude survey questions were based on the work of Meriwather & Tharp (1999) and Milou (1999) who used the Attitude towards Graphing Calculators (UATGC) survey for attitudes/beliefs about graphing calculator. This survey was used before and established content validity. The prior knowledge survey was a set of 8 statements that were intended to find students' prior/initial knowledge and expertise in use of graphing calculators. The survey consisted of four yes/no items, two Likert Scale items, one application question, and one qualitative question. For this survey, based on Hennessy et al. (2001) and Hubbard (1998), a Prior Knowledge with Graphing Calculators (PKGC) rating scale was developed. Two experts in mathematics education independently reviewed the instrument and indicated that, in their opinion, it had content and construct validity. The surveys were administered during the first two weeks of classes. I used mean (average) scores for the cutoff between positive attitude-negative attitude and high experience- low experience. For attitude scores (for 148 students), the average score was 55.5(minimum=37, maximum= 74). For prior experience scores, the average score was 10.9 (with a 0 minimum score and 16.5 maximum score). However, 0(zero) represented the group of the students who were unlikely to use the graphing calculator at all. Thus, the minimum non-zero prior knowledge score was 2.5, and students with a scale score of 0 were excluded.

Twenty students (from the students who agreed to be interviewed) were chosen based on four groups: positive attitude+low experience, positive attitude+high experience, negative attitude+low experience, and negative attitude+high experience. There were several reasons for the choice of these four groups. These groups were selected in accord with practical considerations. That is, based on students' prior knowledge and attitudes, these groups were crucial to goal of determining whether students' interaction with the graphing calculator was taking place and, if so, identifying the patterns of the interaction (graphing calculator use) and the practices facilitating it. Additionally, it was vital to use these groups to ensure a valid sample and how each group responded to using graphing calculators based on complexity and transferring knowledge. A total 88 students (out of 148) agreed to come to interviews. After that, I began to email students to choose 20 targeted participants (out of 88 students).

Students Interviews

Videotapes were used to record the interviews. In this way, the tape was replayed several times for analysis. Also, videotapes were used for comparisons. The video-tape only showed students' work and captured their calculator strokes. Video camera was set up and operated by researchers. Videotapes were needed to show exactly students' work and key strokes while students were using graphing calculators. In order to describe students' interactions with the graphing calculator, these interviews were vital in terms of how each student responded to certain situations on the tasks.

Targeted students were interviewed after their mid-term exams about their intentions for the use of graphing calculator, the tasks they received during the interview, and their general reactions while working on the tasks. During the first tasks, the questions were easier and shorter (low level tasks) compared to the questions in the latter tasks. There were two reasons for that. First, the goal was to minimize the pressure on students and get them more engaged with the tasks. Moreover, the goal was to try to understand student's fluency with graphing calculators and to gather information about students' familiarity with the graphing calculator. Secondly, by giving students more complex tasks in latter tasks (high level tasks), I was be able to see their proficiency in calculator use more closely. The time period ranged from 26 to 52 minutes for the interviews. The number of tasks in the interviews ranged between 4-6(no less than 4, and no more than 6) tasks depending on students' enthusiasm and on what they were capable of. As soon as the interview begins, I provided only one task to the student. When the student is done, I provided another task (Questions were given separately). I arranged tasks based on their difficulty level. All students did not receive the same sequence of tasks. Tasks were given based on the students' ability so that they were working on tasks that matched their level of expertise. Practically, it was not very possible to give the students same tasks since they had different mathematical knowledge and different tendency to use the graphing calculator. Mostly, I decided give next task while the students was working on previous one. I asked two mathematics professors(not from the study classes) independently to review and categorize each task as *low level*, *medium level*, and *high level* task based on its difficulty and complexity. Complexity and difficulty are based on required conceptual and procedural understanding in the tasks. Categorization was collected from two professors, compared with each other and each task was given a difficulty level. There are 5 low level, 3 medium level, and 6 high level questions, out of 14 tasks.

Data Analysis

Each interview was coded on following steps:

- Quantitative data was obtained by measuring the variables (e.g., calculator use, calculator fluency, graph of the function, mathematical understanding, solution etc.) being studied along a scale that indicated how much of the variable was present. Researcher coded each interview according to rubric developed. Higher score indicated that more of the variable (such as 2 for mathematical understanding) was present than do lower score (0 for mathematical understanding).
- Categorical data was obtained for: whether they graph on the graphing calculator, the features they used on the graphing calculator, how much they did calculations on the paper, how much they did calculations on the graphing calculator, representations they used in the process of solving task. Categorical data simply indicated that the total number of events (e.g., the features used) researcher found in solving the problem. In order to measure the features students used and the representations they used, researcher used a frequency table and nominal scale to get the percentages.
- Qualitative data: Field notes were taken for each video- taped interview to reflect each research question. Researcher mostly wrote a paragraph or passage, sometimes a label, describing what was seen in each task that is more important. Moreover, researcher, by using the notes from the interviews, compared pair of students who did things/scored on the problems differently. Second, I transcribed all interviews for qualitative data and tried to find some patterns among the groups. I looked at the interview transcripts to examine patterns in students' task solving activities with the graphing calculator.

In order to find out how much using graphing calculator played an important role in high-level tasks, researcher also looked at 3 different item difficulty scores for each group. Accordingly, same trend was found for negative *attitude-high experience*, *negative attitude-low experience* and *positive attitude-low experience* groups (Table 1). These groups' overall scores on medium level tasks were higher than low level tasks. However, scores on high level tasks were lower than medium level tasks. While moving from low level tasks to medium level tasks, these groups were more successful to solve the problems with the help of a graphing calculator; however, there was a decrease from moving medium level tasks to high-level tasks.

On the other hand, positive attitude-high experience group showed different trend (Figure 1). This group scored low on medium level tasks, and almost equal on low and high level tasks. There is no clear evidence to suggest that one group overall score on high level tasks was quite distinctive then medium and low level tasks. Actually, low experience groups (negative attitude-low experience, positive attitude-low experience) scored even lower on high level tasks than low level tasks. Results suggest that the harder the question is, the lower the students' ability to handle the question by using the graphing calculator.

In order to verify quantitative data, I also looked at categorical data for percentage on item difficulties. It means in what percentage low, medium, and high level tasks were graphed on the graphing calculator. When looking at the percentage on item difficulty, there is slightly difference between negative attitude-high experience and positive attitude-high experience groups on high level tasks (Table 2). Other than that, both high experience groups preferred to use the graphing calculator in almost every question. Negative attitude-low experience group has the lowest percentage on three levels. There is clear evidence that low experience students (negative attitude-low experience, positive attitude-low experience) followed same trend; because these groups' calculator use on medium level tasks were higher than low level tasks but there was again a decrease on using the calculator for high level tasks.

When looking at the percentage in terms of students' preference to use the graphing calculator in high level tasks, there is little evidence to say that low experience students showed more flexibility in high-level tasks (Figure 2).

The features(on the graphing calculator) students used in the questions and representations they used were coded to see how much they graphed the function on the graphing calculator and on the paper as well as how much they made calculations on the paper and graphing calculator. For high experience groups (negative attitude-high experience, positive attitude-high experience), there seems to be some dependency on the graphing calculator; because both groups' score for calculations on the graphing calculator were higher than other two groups (Table 3). Positive attitude-high experience group scored highest for calculations on the graphing calculator (%14). However, this group also scored second highest for calculations on the paper. This tendency shows that positive attitude-high experience group followed more flexible ways by switching from paper- pencil method to calculator use or vice-versa.

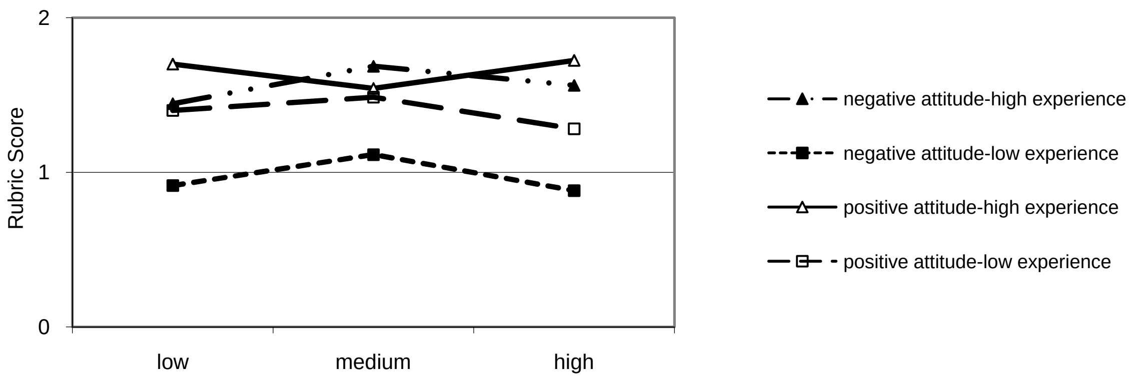

Figure 1. Mean scores on item difficulty

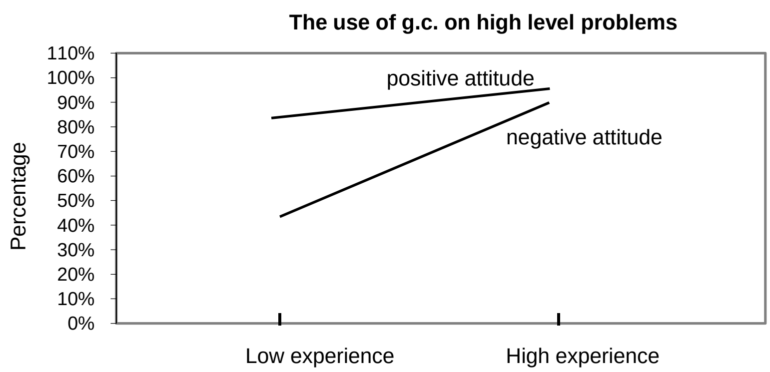

Figure 2. Percentage for the use of the g.c. on high level problems

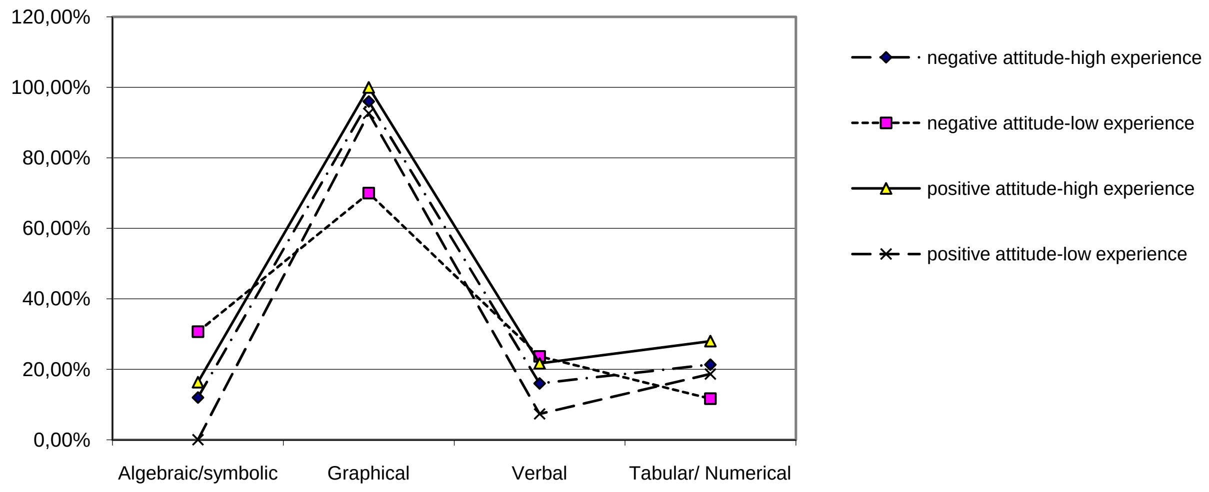

Figure 3. The use of the representations

Table 1. Groups' mean on item difficulty (level of the tasks)

	Item difficulty		
	Low level	Medium Level	High Level
Negative attitude- Low experience	0.91	1.11	0.88
Positive attitude-Low experience	1.4	1.48	1.28
Negative attitude-High experience	1.44	1.68	1.56
Positive attitude-High experience	1.7	1.54	1.72

Table 2. Groups' percentage on item difficulty (level of the question)

	Item difficulty		
	Low level	Medium Level	High Level
Negative attitude-Low experience	40.00%	70.00%	43.33%
Positive attitude-Low experience	70.00%	100.00%	100.00%
Negative attitude-High experience	100.00%	100.00%	93.33%
Positive attitude-High experience	100.00%	100.00%	100.00%

Table 3. Groups' percentage for calculations on the paper and on the graphing calculator

	Did they do calculations on the paper?	Did they do on the g.c. (scientific)?
Negative attitude- Low experience	25.00%	0.00%
Positive attitude-Low experience	4.00%	3.33%
Negative attitude-High experience	4.00%	8.00%
Positive attitude-High experience	13.33%	14.00%

Table 3 shows that negative attitude-low experience group scored highest for calculations on the paper. This group never made calculations on the calculator (scientific mode). This group mostly preferred to use paper pencil work for the solution of the problem. Percentage for calculations on the paper was same for positive attitude-low experience and negative attitude-high experience groups.

Total, 12 graphing calculator features were identified while the students were solving the problems with the graphing calculator. Table 4 shows that students, generally, used GRAPH (80.17%), WINDOW (32.92%), TABLE (31.75%), TRACE (19.83%) functions on the calculator. Especially, high experience groups (negative attitude-high experience, positive attitude-high experience) used TRACE, TABLE, CALC, MATH features extensively. However, these features were not extensive for the other two groups (negative attitude-low experience, positive attitude-low experience).

TABLE feature seemed somewhat important in the interviews that this feature helped a lot to find the y axis and x axis coordinates for the function on the table. Moreover, some students were able to discover any discontinuity on the function (hole) by using this feature. Relatively, many students preferred to get y values from this feature rather than making table on the paper, and giving some values for x to get y values. This pattern was quite extensive among students. I mostly saw that, even in low level questions like one degree function, many students tried to get intersections by looking at TABLE feature or using TRACE feature to spot the intersections on the function graph. Most students solved the problem in this mode by using TABLE to get the x and y values or WINDOW, ZOOM, TRACE features to get a better picture of the graph and look at graph to identify the critical points (maxima, minima, and hole).

Students' preference to use graphical representation (%89.67) was reasonably higher than algebraic, verbal and tabular representations (Figure 3). Second highest representation use was on tabular representations (%19.92). However, there is a clear pattern on using graphical representation that students mostly tried to get the function graph on the graphing calculator and explained to solve the problem verbally by looking at the calculator. Positive attitude-high experience group scored highest on using tabular representations, while negative attitude-low experience group scored highest on algebraic/symbolic representations.

DISCUSSION AND CONCLUSION

This exploratory study investigated students' interaction with complex function graphs in using graphing calculators. In this sense, this study looked at college students' use of graphing calculators and tried to see whether graphing calculators were used efficiently to see different cases and multiple perspectives among complex function graphs, or whether graphing calculators were used only as a mechanical tool to push buttons and get answers (graphs) while working on the tasks.

Research suggests that an instructional method must be as complicated as is necessary to give the students necessary information and learning goals. It was expected that using the graphing calculators will enable to look at the *introduction of complex function graphs* without oversimplification in calculus topics. Tasks were chosen around the first five chapters of Calculus I and administered to students. Students had the option to use the graphing calculator, which allowed to discover their preference for the representation (graphical, analytic, etc.), and their dependency on the graphing calculator. I investigated what kind of patterns/modes of graphing calculator use emerged in students' use of graphing calculators with calculus tasks of varied difficulty.

The findings from the interviews clearly showed that students who had more experience and knowledge on graphing calculators were more flexible in solution strategies than students who had limited experience on the graphing calculator. In other words, high experience groups showed their flexibility in *multiple case examples* by moving from one representation to another (e.g., their flexibility to move from paper- pencil work to graphing calculator use or vice versa).

Students mostly were confused when the tasks were getting complicated; and their translation *skills* did not improve while moving from one representation to another by using the graphing calculator. Moreover, it was clear in interviews that the graphing calculator and mathematical understanding must work together for the solution of the task. Without understanding the task's underlying principles, using the calculator is not enough for students to reach an acceptable outcome. Understanding of the mathematical concept and using the graphing calculator are related to each other, and there is a positive correlation between these two variables. In other words, it is hard to master the task without having initial concept knowledge or a general principle of the concept. Although the order of tasks is arranged in accord with the its complexity and difficulty, giving low level and then high level tasks to the students (especially for low experience students) did not work very well since each task needed some kind of "situation-based" or "case-based" knowledge to be solved.

Interviews clearly showed that students' class experiences regarding graphing calculator use effected students' use of the graphing calculator in the tasks. Students' explanations of the task solving procedures revealed that students seemed to follow the methods they learned in the classes or they followed the methods that are shown by teachers in the classes.

In this study, as indicated by previous research, students used the graphing calculator as a visualization tool to get a clear picture of the function graphs; as a checking tool to see whether the graph they produced on the paper is correct or not; and as a comparing tool to compare different function graphs and see the changes at the same time. However, regarding using the graphing calculator in high level tasks, there is little evidence to say that using the graphing calculator promoted students' understanding of the high level tasks. Rather, using the graphing calculator mostly caused students (especially for low experience students) to produce prepared graphs and copy those graphs on the paper, without finding critical points of those high level function graphs.

High experience groups were better prepared to use the graphing calculator in the tasks and successfully to *go beyond low level knowledge* (when high level tasks were given). The *positive attitude-high experience group* was more flexible in the use of the graphing calculator. Previous experiences with the graphing calculator appeared to allow students to find the answer quickly, without hesitation and error. This group also showed more work on the paper; and explanation was more clear and understandable than in the other groups. This group was the most successful in connecting algebraic work with the result on the graphing calculator. High experience students' use of the graphing calculator seems to fit well in complex tasks and seemed to allow students to create *multiple representations of knowledge*. The results indicated that experience with the graphing calculator was important factor in solving the tasks with the graphing calculator, while attitude seemed to have no effect on task solving steps. Low experience students mostly missed the critical analysis and complexity of high level tasks and only focused on getting the image from the graphing calculator. Although the students had sufficient mathematical knowledge on the tasks, the adequate and necessary skills on using the graphing calculator were needed to understand the tasks' underlying principles and to get the correct solution. For example, *negative attitude-low experience* group was the lowest group for mathematical understanding, solution process of the tasks, and graphing calculator use in high level tasks. However, this group followed more algebraic ways for the solution of the tasks. In other words, low experience and negative attitude on the graphing calculator enforced this group to work on the paper. Low experience groups mostly used the graphing calculator as a visual help to get the graphs; however because of the unfamiliarity with the features on the calculator, low experience students made errors finding critical points of the graphs and made calculation errors on the paper. *Negative attitude-high experience* group members scored higher than *positive attitude-low experience* group members. Results clearly show that in order to use the graphing calculator in complex function graphs to implement the *multiple representations knowledge*, the students need to know characteristics of features on the graphing calculator. They have to have some use of skills and good experience on the machine, not just skills of thinking and skills of knowing the concept.

There is considerable evidence in this study that students who had low experience on the graphing calculator did not give adequate attention to critical analysis of the tasks. That is, because of the limited knowledge on features of the graphing calculator, primary use of the calculator for students with low experience was to graph the functions (by only using Y= and GRAPH); without finding the critical points of the graph, or exploring other points that made the graphs complex and complicated. Thus, it is quite critical for teachers to allow students to use the graphing calculator in class environment. Teachers also must be ready to help students learn how to use the graphing calculator with its full complexity and potential. Moreover, teachers should consider students with different abilities and experience with the graphing calculator and try to minimize these gaps among students.

Some low experience students did not prefer to use the calculator in the tasks since they were not sure what the tasks meant for them. Moreover, some students who did not use the graphing calculator indicated that they already knew the task. Therefore, teachers must give proper attention to mathematical methods they use when graphing calculators are used in the class. It is crucial for teachers to recheck how the subject is taught when the graphing calculator is used. There is a clear indication in the interviews that mere availability of the graphing calculator in the task solving process does not affect or change students' task solving strategies. Rather, the kind of understanding and knowledge students have of the task (students' experience with the tasks) shapes students' approach to tasks. There was a common belief among students that the graphing calculator does not help teach a new concept; but everything must be done on the paper to show that they understood the problem. Thus, teachers should clearly indicate how much graphing calculator use is required and how much written work is needed for the task. Students must get clear direction on how to integrate the use the graphing calculator in the classroom and with the written work required. Students need instruction in how one representation relates to and inform the other.

From this study, it is not possible to say that using the graphing calculator enhanced students' understanding of graphing ability in given high level tasks. Some students were able to get the correct answers (graphs) although they did not understand the task entirely. It did not mean that the use of the graphing calculator gave a flexible understanding of the task; it just gave a quick and prepared answer for the student. Regarding introduction of complex function graphs early, only students with high experience and positive attitude seemed successful. Other than that, there is no clear indication that the use of the graphing calculator improved students' understanding as students move from well simple knowledge to complex knowledge. This study suggests that to introduce domain complexity early can be problematic for the students, combining with the lack of experience on the problem with the lack of experience on the graphing calculator. Some interviews clearly showed that students, sometimes, struggled with the technical details of the graphing calculator (etc. using the parenthesis incorrectly, setting up WINDOW feature incorrectly when different question was given, wrong use of ZOOM and TRACE features while trying to get better picture of the graph, little knowledge about CALC and MATH feature).

Some research assumes that using the graphing calculator will automatically improve students' understanding of the mathematics. It is the major problem in the literature. Rather, research should focus on the ways to better understand how effective use of the graphing calculator can be established in high level of mathematics. Research must focus on broad generalization of the cases by looking across schools and content areas as well as school districts and different grades. There is a need to look and identify cases in broad surveys and interviews, which can help to interpret specific cases.

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