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The role of affective learner characteristics for learning about abstract algebra: A multiple linear regression analysis

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Abstract

Recent research has boosted the inclusion of introductory group theory into secondary and undergraduate mathematics education due to manifold potentials, e.g., with regards to the promotion of students' abstract thinking. However, in addition to research on cognitive processes, learners' affective characteristics have largely remained unexplored in the context of teaching and learning group theory to date. In this paper, we contribute to closing this gap: We report on an empirical study investigating n=143 students' affective characteristics within a two-weeks course program–the Hildesheim teaching concept. In our study, this concept was used to introduce pre-service primary teachers into group theory. A multiple linear regression analysis reveals that neither mathematics-specific ability self-concept nor subject interest are significant predictors of the achieved conceptual understanding of group theory after the intervention indicating that group theory is not reserved for only the mathematically interested students or students with a high mathematics-specific self-concept.

Keywords: mathematics-specific self-concept, subject interest, situational interest, group theory

INTRODUCTION

In recent years a body of research has emerged, entirely dedicated towards exploring educational aspects of abstract algebra (Wasserman, 2014, 2016, 2017, 2018) and group theory in particular (Melhuish, 2015, 2019; Melhuish & Fagan, 2018; Pramasdyahsari, 2020; Veith & Bitzenbauer, 2022; Veith et al., 2022a, 2022b, 2022c). Even though group theory is mostly taught on university level mathematics, numerous connections to primary and secondary school mathematics have been uncovered (cf. Even, 2011; Wasserman, 2016) and with it the great potential it offers for mathematics educators in all fields alike. Consequentially, many studies focused on mathematics teachers and how they responded to abstract algebra courses deepening their content knowledge (cf. Veith et al., 2022b). The importance of group theory in mathematics teacher education is underpinned by a study conducted by Wasserman (2014): With a mathematics for teachers' course, Wasserman showed that dealing with the concepts of algebraic structures had a significant impact on the participants' beliefs and their practices of teaching. In this article we want to enrich these findings by shedding light onto how a conceptual understanding of group theory might be connected to and influenced by affective learner characteristics.

RESEARCH BACKGROUND

Learning and Teaching Group Theory

So far, two primary research interests of abstract algebra education can be observed: On the one hand, researchers investigated potential learning hurdles the concepts of abstract algebra pose. On the other hand, it was examined how knowledge of algebraic structures can be of use for mathematics educators in all fields.

The first aspect is comprised of learning difficulties primarily located in the fundamental basics of group theory. For example, learners have trouble with the newly presented vocabulary of group theory (cf. Veith et al., 2022a), especially *composition* and *operation* are potentially confusing terms that were shown to lead to

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Contribution to the literature

- Exploration of affective learner characteristics in the context of group theory. The results are situated in the body of prior research and linked to results regarding cognitive charcteristics, allowing for a holistic perspective on group theory education.
- Results indicate that neither ability self-concept nor subject interest are significant predictors of conceptual understanding of group theory contents.
- Uncovering the central role of situational interest and its influence on developing group theory concepts.

fundamental misunderstandings. Additionally, generalizing the notion of *inverses* from inverse functions to *inverse elements* in algebraic structures revealed to be a non-trivial step for students dealing with abstract algebra for the first time (cf. Wasserman, 2014). Most pronounced, however, were learning difficulties that are tied to the binary operations of groups, namely associativity and commutativity. Here, learners tend to conflate and overgeneralize these properties as shown by Melhuish and Fagan (2018) as well as Larsen (2019). This is substantiated by an earlier study where in the context of an in-service professional development course the participating teachers showed to have trouble distinguishing between both properties and to some extend are even convinced, they are logically dependent on one another (cf. Zaslavsky and Peled, 1996).

As for the second aspect, a comprehensive and detailed summary is provided by Wasserman (2018). This summary can be viewed as an extension of his 2016 study, where he explored the potential of abstract algebra for the teaching of school algebra: By outlining a progression line across elementary, middle, and secondary mathematics he demonstrated how knowledge of “algebraic structures may transform teachers’ elementary conceptions of number and operation as related to early algebra concepts” (Wasserman, 2016, p. 31). Regarding elementary education, for example, he explored how the teaching of arithmetic properties is influenced by knowledge about abstract algebra. The importance of this exploration is twofold: Firstly, as mentioned beforehand, these arithmetic properties have shown to be the most problematic aspect when entering abstract algebra. Secondly, Chick and Harris (2007) found that primary school teachers displayed unsatisfying knowledge of how the mathematics they teach build the foundation for later algebra. They further argued that this deficit might be caused by the educational background primary teachers are presented with and thus demanded to bridge this gap by additional learning opportunities. This conclusion was also derived by Wasserman (2016) who saw the need to foster teachers’ understanding of algebraic structures to enhance their ability to reflect on elementary mathematics content.

In conclusion, the literature suggests that pre-service primary teachers’ abstract algebra education is to be improved, especially regarding introductory aspects of group theory such as inverses, binary operations, associativity, and commutativity. In order to better facilitate abstract algebra education, however, we need to know how it is connected to affective learner characteristics. As mentioned earlier, Wasserman (2014) already provided evidence for the educational impact of group theory by exploring the transformation of teachers’ beliefs and teaching practices by arguing that teachers should engage with abstract algebra to “help them more fully understand the vertical development from arithmetic properties to algebraic structures and gain an understanding of the mathematical horizon, which is an important knowledge component for teaching” (Wasserman, 2014, p. 210). And in another survey by Even (2011) mathematics teachers participating in an advanced mathematics course explained how deepening their algebraic knowledge resulted in a deeper knowledge of what mathematics actually is. These observations result in a multitude of pressing questions: Do primary pre- service teachers experience group theory as a relevant part of mathematics education? Do they perceive it as a difficult branch of mathematics and is their success in understanding these abstract concepts dependent on their subject interest?

While the psychological constructs occurring in these questions are already well operationalized in the literature, namely as *relevancy of content*, *ability self-concept*, *subject interest*, and *situational interest*, the clarification of these questions require

(a) a teaching concept for primary pre-service teachers dedicated to group theory and

(b) a test instrument to assess conceptual understanding of group theory. Both requirements are met by the literature, and we will present them in the following.

The Hildesheim Teaching Concept

A teaching concept specifically designed for the aforementioned target group is presented by the Hildesheim teaching concept. The Hildesheim teaching concept aims at introducing secondary school students and first semester students to introductory group theory. It is derived from the literature (cf. Veith & Bitzenbauer, 2020), merging viewpoints from the new math era (i.e., Griesel, 1965; Kirsch, 1965; Steiner, 1966) as well as contemporary works on abstract algebra education (i.e.,

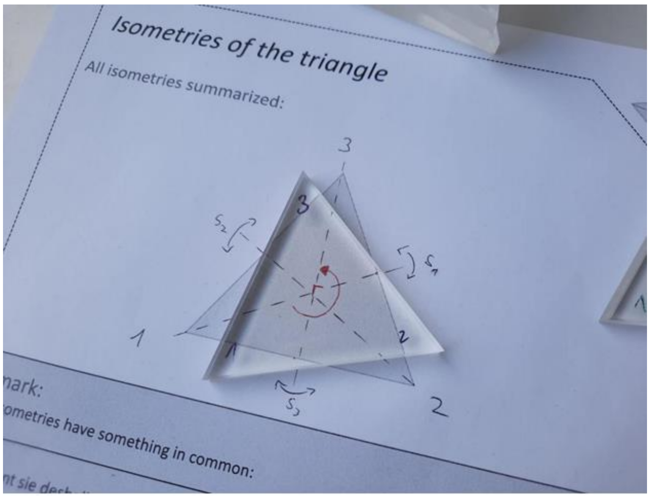

Figure 1. Example image of the hands-on learning material in the Hildesheim teaching concept

Lee & Heid, 2018; Wasserman, 2014; Weber & Larsen, 2008), for example Larsen's (2013a, 2013b) TAAFU materials. It has been subject to both formative (Veith et al., 2022a) and summative evaluation (Veith et al., 2022c). The core idea lies in introducing learners to groups from a geometric perspective by studying the dihedral groups D_3 and D_4 as well as cyclical groups $\mathbb{Z}_n$. This introduction is further motivated through a hands-on approach, e.g., by working with triangles and squares made from acrylic glass to reenact the transformations of D_3 and D_4 in a haptic way (cf. **Figure 1**). It has been shown that this approach is conducive to learning and guides learners to an adequate conceptual understanding of introductory group theory. The formative evaluation further showed that the instructional elements within the concept are well accepted by Thus, our study used this teaching concept as a guideline.

Measuring Conceptual Understanding of Group Theory

In order to facilitate quantitative research into educational aspects of group theory and enrich findings from qualitative research (cf. Baldinger, 2018; Cook, 2018; Even, 2011; Suominen, 2018), two concept inventories have been developed in recent time: the GTCA (Group theory concept assessment) by Melhuish (2015) and the CI²GT (Concept inventory for introductory group theory) by Veith et al. (2022b), respectively. Here, we conceive that conceptual understanding "reflects knowledge of concepts and linking relationships that are directly connected to (or logically necessitated by) the definition of a concept or meaning of a statement" (Melhuish, 2019, p. 2). While the GTCA is focused primarily on undergraduate mathematics students, the CI²GT is focused primarily on secondary school students and pre-service mathematics teachers. The latter assesses conceptual understanding of introductory group theory namely by addressing the definitional fundamentals (neutral element, inverses, binary operations, associativity, and commutativity) as well as Cayley tables and isomorphisms in the context of the groups D_3, D_4, and $\mathbb{Z}_n$. In other words, the CI²GT addresses precisely the contents covered by the Hildesheim teaching concept.

Interim Conclusion

In total, we conclude that the need to foster pre-service primary teachers' algebraic education derived from the literature can be tackled by combining the Hildesheim teaching concept with the CI²GT. To further investigate the aforementioned affective learner characteristics, the CI²GT is to be complemented with additional scales from the literature. In this regard, we understand

1. *relevancy of content* as a "student perception of whether the course instruction/content satisfies personal needs, personal goals, and/or career goals" (Frymier & Shulman, 1995)
2. (Ability) *self-concept* as mental representations of persons about themselves (Baumeister, 1999),
3. *subject interest* as "characterized by intrinsic desire to understand a particular topic that persists over time" (Schraw et al., 2001, p. 24), and
4. *situational interest* as a "spontaneous interest that appears to fade as rapidly as it emerges and is almost always place-specific" (Schraw et al., 2001, p. 24).

As part of a two-week program into introductory group theory, we investigated the above-mentioned constructs with a sample comprising n=143 pre-service primary school teachers. The intervention of this program was based on the Hildesheim teaching concept and the CI²GT. While mathematics-specific self-concept and subject interest are fairly stable variables, the construct of situational interest is "changeable and partially under the control of teachers" (Schraw et al., 2001, p. 212) and therefore of great importance of educators. Thus, we decided to feature it in both research questions.

RESEARCH QUESTIONS

With this contribution, we approach a clarification of the following research questions:

1. **RQ1:** How is group theory introduced via the Hildesheim teaching concept perceived by learners regarding
   a. situational interest,
   b. relevancy of content, or
   c. perceived difficulty?
2. **RQ2:** How may affective learner characteristics such as
   a. mathematics-specific self-concept,

Table 1. Example item of the CI²GT

Item 4: Let $G=(M, \circ)$ be a non-abelian group and $a, b \in M$. The inverse of $a \circ b$ is…				
□ … $b^{-1} \circ a^{-1}$				
□ … $a^{-1} \circ b$				
□ … $a^{-1} \circ b^{-1}$				
□ Very confident	□ Confident	□ Undecided	□ Unsure	□ Guessed

Table 2. Overview of adapted scales for this research as well as their internal consistencies expressed by Cronbach's alpha.

Construct	Number of items	α	Adapted from
Mathematics-specific self-concept	7	0.72	Hoffman et al. (1998)
Subject interest	6	0.81	Hoffmann et al. (1998)
Situational interest	5	0.77	Pawek (2009)
Relevancy of content	4	0.71	Winkelmann (2015)
Perceived difficulty	1 per sub-domain of CI²GT	-	-

Note. The items of the scales can be found in **Appendix A**.

b. subject interest, or

c. situational interest

or prior knowledge be used as predictors of students' conceptual understanding of group theory acquired by participating in the Hildesheim teaching concept?

METHODS

Study Design and Sample

To clarify the research questions, a field study in a pretest-posttest design was conducted. The sample comprised n=143 pre-service primary school teachers in their first academic year who were introduced to group theory via the Hildesheim teaching concept. None of the participants had participated in any course on abstract algebra prior to the intervention.

Instruments

The participants' conceptual understanding of introductory group theory was assessed using the CI²GT–a concept inventory consisting of 20 dichotomous items. Its internal consistency expressed by Cronbach's alpha is α=0.76 and the development and analysis of the CI²GT is documented in Veith et al. (2022b). The 20 items are designed in a two-tier way: In the first tier the respondent has to select exactly one of three answer options. And in the second tier, the respondent has to rate the confidence in their given answer on a 5-point rating scale (1=guessed, 2=unsure, 3=undecided, 4=confident, and 5=very confident). A point is awarded if and only if the correct answer option was selected and the respondent was confident or very confident. An example item is provided in **Table 1**.

In addition, we assessed five affective variables using 5-point rating scales (1=lowest trait level and 5=highest trait level), which were adapted from the literature (cf. **Appendix A**). The questionnaire containing these scales was administered alongside the CI²GT such that the participants in a first step responded to said scales before moving on to the group theory items provided by the CI²GT. The internal consistencies of the scales used to assess the affective learner characteristics are presented in **Table 2**. It is noteworthy that the *perceived difficulty* was obtained by the pre-service teachers' ratings of the difficulty of each sub-domain of group theory included in the Hildesheim teaching concept. As such, this does not represent a psychometric scale.

Data Analysis

Analysis carried out to answer RQ1

To explore the interaction between affective learner characteristics and conceptual understanding of group theory, we applied a correlation analysis to the data. As the data is ordinally scaled we used Spearman's correlation coefficient ρ. According to Hemphill (2003), correlations ρ with

1. $|\rho|<0.20$ are considered as weak.
2. $0.20<|\rho|<0.30$ are considered as medium.
3. $|\rho|>0.30$ are considered as strong.

Analysis carried out to answer RQ2

To explore possible predictors for the assessed conceptual understanding of introductory group theory, we investigated multiple linear regression models. Therefore, the CI²GT score in the post-test serves as the dependent variable. The model under investigation includes the following variables: mathematics-specific self-concept, subject interest, situational interest, and prior knowledge expressed by the CI²GT score in the pre-test. To check the underlying assumptions of the resulting models, we followed Bitzenbauer (2020) and

1. examined linear dependence of the included variables via scatterplots.
2. ruled out multicollinearity of the variables by ensuring that tolerance ≥0.2 and variance inflation factor *VIF*<5 (Kutner et al., 2004).
3. verified normal distribution of residuals via a P-P-diagram (Michael, 1983).

4. checked for homoscedasticity of the residuals by plotting standardized residuals against the (unstandardized) predicted values.
5. conducted a Durbin-Watson test to check autocorrelation. With a value of DW=1.76 for the Durbin-Watson statistic, autocorrelation can be ruled out according to Stoetzer (2017).

RESULTS

Descriptives and Correlation Analysis

The descriptives on the assessed affective learner characteristics are provided in **Table 3** and are summarized in **Figure 2**.

The correlations among the affective learner characteristics themselves are provided in **Table 4**.

Remarkably, all observed correlations are significant ($p<0.05$) with the exception of the one between the mathematics-specific self-concept and the relevancy of content and, according to Hemphill (2003), they can mainly be classified as strong.

Alongside **Table 3**, the correlation analysis allows for a first insight into how these constructs interact with respect to learning about group theory.

Multiple Linear Regression Analysis Results

An overview of the multiple linear regression model is presented in **Table 5**. An F-test verifies statistical significance of the model [$F(4, 132)=5.81, p<0.001, \omega^2=0.02$]. The effect size $\omega^2=0.02$ indicates a small effect according to Cohen (1988). With $R^2=0.15$ the model explains 15% of variance in the dependent variable.

It can be observed that mathematics-specific self-concept and subject interest are not significant predictors, while pre-test score (i.e., students' prior knowledge), and situational interest are statistically highly significant predictors.

Table 3. Mean values (μ) of the assessed constructs alongside the standard deviations (σ) as well as each correlation coefficient (ρ) to the CI²GT score in the post-test.

Construct	μ	σ	ρ
Mathematics-specific self-concept	3.38	0.44	0.18
Subject interest	3.65	0.57	0.05
Situational interest	3.35	0.63	0.23
Relevancy of content	3.72	0.68	0.12
Perceived difficulty	2.88	0.50	-0.24

Note. 5-point rating scales used to measure the constructs were adapted so that 1 represents the lowest trait level and 5 the highest

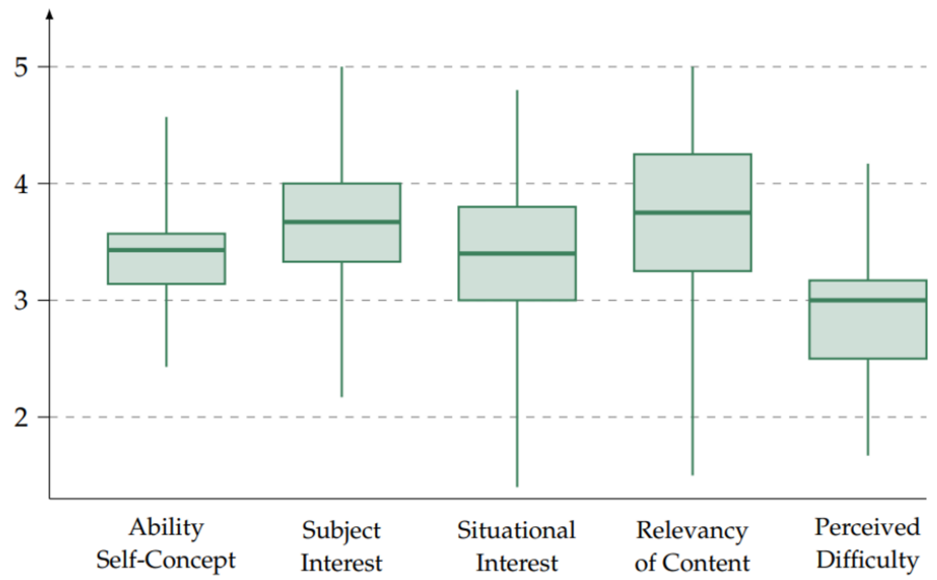

Figure 2. Box plots of the data for each affective learner characteristic

Table 4. Correlation coefficients among the affective learner characteristics

	MSSC	SuI	SiI	RC	PD
MSSC	1	0.48	0.32	0.16	-0.33
SuI		1	0.36	0.35	-0.25
SiI			1	0.59	-0.46
RC				1	-0.29
PD					1

Note. MSSC: Mathematics-specific self-concept; SuI: Subject interest; SiI: Situational interest; RC: Relevance of content; & PD: perceived difficulty

Table 5. Multiple linear regression model with estimates *B* and standardized estimates *β*

Predictor	*B*	SE	*β*	Lower	Upper	t	p
Intercept	0.55	2.62	--	--	--	0.21	0.833
Mathematics-specific self-concept	0.63	0.80	0.08	-0.11	0.26	0.78	0.446
Subject interest	-0.35	0.61	-0.05	-0.24	0.13	-0.57	0.570
Situational interest	1.58	0.51	0.27	0.10	0.44	3.09	0.002
Pre-test score	0.30	0.11	0.23	0.06	0.40	2.70	0.008

DISCUSSION

Discussion of RQ1

All affective learner characteristics are above the center of the scale (cf. **Figure 2**). The students possessed proper mathematics-specific celf-concepts (μ=3.38) and subject interest (μ=3.65). Moreover, the values for situational interest (μ=3.35) and relevancy of content (μ=3.72) lay above the center of the scales, and indicate that the intervention based on the Hildesheim teaching concept was able to

(a) invoke situational interest in the concepts of group theory and

(b) provide plausible evidence for learners that applications of group theory are manifold both inside and outside of mathematics.

In addition, the perceived difficulty was well below the center of the scale (μ=2.88), insinuating that the abstract concepts of algebra (cf. Veith & Bitzenbauer, 2022) were didactically conveyed in a comprehensible manner and did not evoke a sense of overload.

The performance in the CI²GT is connected with the affective learner characteristics, expressed by the correlations in **Table 3**. For example, the perceived difficulty correlates negatively at ρ=-0.24. This is not surprising–the more challenging the concepts are perceived the more cognitive load (cf. Sweller et al., 2011) is required to comprehend the mathematical problems and thus the test performance declines. A similar but positive correlation is observed regarding the situational interest (ρ=0.23) which is, as mentioned earlier, "changeable and partially under the control of teachers" (Schraw et al., 2001, p. 212). This suggests that a higher test performance in the CI²GT is connected to engaging teaching concepts and learning materials. This connection has already been found in other studies (cf. Hidi, 1990; Vainikainen et al., 2015) and it has been demonstrated that situational interest is a significant predictor of learning outcome. Thus, regarding group theory the question arises as to which affective factors are connected to situational interest. A first insight into this question is provided by **Table 4**. Here, with ρ=0.59 a strong correlation can be observed between situational interest and relevancy of content, hinting at the importance of emphasizing the use of group theory and its applications. As a mathematical model of symmetry, the applications are multifaceted even outside of mathematics, i.e., in chemistry, physics, computer science and even musical set theory. We argue that expounding those connections might enrich this abstract theory with meaningfulness which in turn increases the situational interest as empirical results suggest. Additionally, the strongest negative correlation can be observed between situational interest and perceived difficulty (ρ=-0.46). Thus, teaching group theory should focus on adequate didactic reduction and low threshold learning opportunities to further facilitate the development of situational interest.

Lastly, the data suggest that subject interest is nearly uncorrelated to the test score in the CI²GT. On a positive note, this could be interpreted as group theory not being a domain solely accessible for mathematically interested students.

This result is especially important for motivating the demands to foster pre-service teachers' abstract algebra education by Chick and Harris (2007) and Wasserman (2016) as it demonstrates that large intrinsic mathematical interest does not constitute a requirement for making sense of the notions of group theory. In other words, even though group theory vastly remains a topic taught to mathematicians only, it is accessible for other audiences as well. And, as outlined by Wasserman's (2016) study, group theory may well serve to deepen the understanding of arithmetic properties which is precisely what primary mathematics teachers teach.

Discussion of RQ2

The results (cf. **Table 5**) show that neither mathematics-specific self-concept nor subject interest are significant predictors of achieved conceptual understanding of group theory after the intervention. This indicates that these constructs do not play a crucial role in learning environments regarding group theory. This finding is of particular interest as it suggests that group theory is not reserved for only the mathematically interested students or students with a high mathematics. However, as the effect of these two control variables is not statistically significant more research needs to be done to empirically substantiate this finding.

On the other hand, prior knowledge expressed by the pre-test score and situational interest have been revealed to be highly statistically significant predictors of achieved conceptual understanding of group theory after the intervention. This is in line with the findings regarding **RQ1**, where this construct showed first signs of being very influential for student learning about group theory. Situational interest has the highest correlation to the post-test score and with the highest weighted value B=1.58 in the linear regression model also contributes the most. The pre-test score is significant but its contribution to the post-test score is relatively low at *B*=0.30 This underpins our earlier findings where the learning gain in two groups (one with prior knowledge, one without) was comparable (cf. Veith et al., 2022c).

In conclusion, the results of the multiple linear regression analysis align well with the correlation analysis: Instructors of group theory should focus on invoking a high situational interest in the learners as this revealed to be the most contributing factor to an adequate conceptual understanding of this mathematical theory.

Limitations of This Study

The results in this article allow for a first exploration of the interaction of various affective learner characteristics in the context of group theory and how they are connected to a conceptual understanding. In this respect, our results may not be regarded independent from the specific intervention we used, which is a common obstacle in educational research. Additionally, the results are further limited in three aspects: Firstly, the presented study is a field study and as such the data gathered is strongly dependent on the sample. Secondly, our results are not meant to yield set causal relationships. Instead, the results obtained from our exploratory study conducted in the field setting allows for the formulation of hypotheses. These may be the starting point for future investigations in the laboratory setting where the effects can be investigated and distinguished from dark noise. Lastly, no similar research has been conducted yet (to the best of our knowledge) so the results cannot be compared with findings from the literature. Thus, this contribution should be seen as a pure exploration study to set a course for future research in this field.

CONCLUSION AND OUTLOOK

In summary, a coherent picture emerges from the data: Instructions in group theory should focus on fostering learners' situational interest of the mathematical objects as it strongly predicts to which degree conceptual knowledge of group theory can be developed. Due to its high correlation to relevancy of content our findings suggest that fostering such interest can be attained by laying out the various applications groups offer both inside and outside of mathematics. On a positive note, as mentioned before this is precisely within the scope of action of mathematics instructors. And variables outside of the instructors' reach such as mathematics-specific self-concept do not seem to be relevant in group theory learning environments. However, as pointed out before, these findings are limited by statistical significance. Thus, in future research the results are to be substantiated and complemented by increasing sample size and expanding the linear regression model, i.e., by including the learners' self-efficacy and beliefs. Lastly, we want to emphasize that introductory group theory was perceived as a highly relevant part of mathematics by the participating pre-service primary teachers and also evoked situational interest regarding the notions of dihedral and cyclical groups, independently of general subject interest. We therefore argue in favor of enhancing group theory education for this audience to

(a) better address their deficits outlined in prior research and

(b) leverage the opportunities and benefits uncovered in the presented literature.

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APPENDIX A

Table A1. Adapted scale for the Construct Mathematics-specific Self-Concept (1 ≙ I do not agree, …, 5 ≙ I agree)

1.1	I understand mathematical contents well.	□ 1	□ 2	□ 3	□ 4	□ 5
1.2	I can remember mathematical contents well.	□ 1	□ 2	□ 3	□ 4	□ 5
1.3	In school I participated in mathematics classrooms very frequently.	□ 1	□ 2	□ 3	□ 4	□ 5
1.4	My performances in mathematics are good in my opinion.	□ 1	□ 2	□ 3	□ 4	□ 5
1.5	I believe my peers think I am very good in mathematics.	□ 1	□ 2	□ 3	□ 4	□ 5
1.6	I expect my future scores in mathematics to be very good.	□ 1	□ 2	□ 3	□ 4	□ 5
1.7	I think I am gifted in mathematics.	□ 1	□ 2	□ 3	□ 4	□ 5

Table A2. Adapted Scale for the Construct Subject Interest (1 ≙ I do not agree, …, 5 ≙ I agree)

2.1	I find mathematics interesting.	□ 1	□ 2	□ 3	□ 4	□ 5
2.2	I think doing mathematics is fun.	□ 1	□ 2	□ 3	□ 4	□ 5
2.3	I am interested in mathematical connections.	□ 1	□ 2	□ 3	□ 4	□ 5
2.4	I like mathematics puzzles and riddles.	□ 1	□ 2	□ 3	□ 4	□ 5
2.5	It is important for me to learn about mathematics.	□ 1	□ 2	□ 3	□ 4	□ 5
2.6	I like to engage with mathematics in my spare time	□ 1	□ 2	□ 3	□ 4	□ 5

Table A3. Adapted Scale for the Construct Situational Interest (1 ≙ I do not agree, …, 5 ≙ I agree)

3.1	I would like to learn more about Group theory.	□ 1	□ 2	□ 3	□ 4	□ 5
3.2	I would like to learn more about applications of Group theory.	□ 1	□ 2	□ 3	□ 4	□ 5
3.3	I would like to learn more about other algebraic structures.	□ 1	□ 2	□ 3	□ 4	□ 5
3.4	I feel like I understood the contents of the past two weeks	□ 1	□ 2	□ 3	□ 4	□ 5
3.5	The contents of the past two weeks have been very interesting.	□ 1	□ 2	□ 3	□ 4	□ 5

Table A4. Adapted Scale for the Construct Relevancy of Content (1 ≙ I do not agree, …, 5 ≙ I agree)

4.1	I feel like Group theory is a very important part of mathematics.	□ 1	□ 2	□ 3	□ 4	□ 5
4.2	I feel like Group theory is very important for science in general.	□ 1	□ 2	□ 3	□ 4	□ 5
4.3	Engaging with Group theory enabled a deeper look into mathematics.	□ 1	□ 2	□ 3	□ 4	□ 5
4.4	Group theory made me see mathematical connections that were not oblivious to me.	□ 1	□ 2	□ 3	□ 4	□ 5