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The Role of Cognitive Ability and Preferred Mode of Processing in Students' Calculus Performance

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The present study sought to design calculus tasks to determine students' preference for visual or analytic processing as well as examine the role of preferred mode of processing in calculus performance and its relationship to spatial ability and verbal-logical reasoning ability. Data were collected from 150 high school students who were enrolled in Advanced Placement calculus courses. The measures of preferred mode of processing did not correlate with the measures of spatial ability and verbal-logical reasoning ability, suggesting that cognitive abilities did not predict the students' preference for visual or analytic processing. Multiple regression analysis revealed that spatial visualization ability, verbal-logical reasoning ability, preference for visual processing contributed significantly to the variance in calculus performance. Correlations between calculus performance and the measures of preferred mode processing suggest that the nature and complexity of mathematical tasks might have influenced the students' degree of preference for using visual processing.

Keywords: cognitive ability, preferred mode of processing, visual processing, analytic processing, calculus performance, high school

INTRODUCTION

Studies have examined the relations between measures of cognitive abilities and processes and mathematical performance for several decades (Dean & Morris, 2003; Galindo, 1994; Hegarty & Waller, 2005; Krutetskii, 1976; Massa & Mayer, 2006; Presmeg, 2006; Stylianou, 2002). Research into students' conceptual understandings of fundamental concepts of calculus has provided comprehensive analyses of students' difficulties (e.g., Bremigan, 2005, Ferrini-Mundy, 1987, Haciomeroglu & Chicken, 2012; Haciomeroglu, Chicken, & Dixon, 2013). For instance, Aspinwall, Shaw, and Presmeg (1997), Haciomeroglu, Aspinwall, and Presmeg (2010), and Samuels (2010) and reported the cases of calculus students, who experienced different difficulties associated with their preferred modes for visual or analytic processing. However, within this large body of research, few studies explored the quantitative relationships between calculus performance and

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preference for visual or analytic processing. Moreover, measures of processing preference consist of items related to real-life situations (Dean & Morris, 2003) and do not take account of visual or analytic processing involved in solving calculus tasks. Therefore, the goal of the present study was to design calculus tasks to determine students' preference for visual or analytic processing as well as examine the role of preferred mode of processing in calculus performance and its relationship to cognitive abilities.

State of the literature

- Few studies have explored quantitative relationships between cognitive abilities, calculus performance, and preference for visual or analytic processing.
- Most previous studies have failed to take into account preferences for visual or analytic processing, which may account for inconclusive research findings between cognitive abilities and mathematical performance.
- Algebra tasks and questionnaires were designed to measure students' preference for visual or analytic processing in cognition but no adequate measures for calculus were available.

Contribution of this paper to the literature

- This study contributes to the existing research on factors affecting mathematics performance by generating new information about the relationships between measures of spatial ability, verbal-logical reasoning ability, calculus performance, and preferred mode of processing mathematical information.
- This study contributes to the mathematics education research field by designing calculus tasks and corresponding questionnaires, which have the potential to be used as a measure of students' visual and analytic tendencies in calculus.
- The measures of preference did not correlate with the measures of cognitive abilities, suggesting that cognitive abilities did not predict students' preference for visual or analytic processing.

BACKGROUND AND PURPOSE

Preference for visual or analytic processing

Research concerning the use of visualization has received attention in different content areas of mathematics (e.g., Aspinwall et al., 1997; Battista, 1990; Bremigan, 2005; Lean & Clements, 1981; Lowrie & Kay, 2001; Moses, 1977; Presmeg, 1985; Suwarsono, 1982; Zazkis, Dubinsky, & Dautermann, 1996;). Krutetskii (1976) identified and described types of thinkers based on students' preferences for two cognitive processes: visual-pictorial or verbal-logical. Following the work of Krutetskii, Lean and Clements (1981), Moses (1977), Presmeg (1985), and Suwarsono (1982), have recognized that individuals could be placed on a continuum (i.e., degree of mathematical visuality) according to their preference for visual processing and defined mathematical visuality as the extent to which a learner prefers to use visual processes to solve mathematics problems. The position is taken that visualizers are considered as learners who prefer to use visual solutions, and analyzers as learners who prefer not to use visual solutions when there is a choice on a specific task. As we designed calculus tasks to measure the students' preference for visual or analytic processing, a solution was classified as a visual method of solution if it involved graphic representations. A solution was classified as an analytic method of solution if it involved analytic (or algebraic) representations.

Mathematical performance, cognitive ability, and preferred mode of processing

There are various studies that have examined the relationships between cognitive abilities, preferred mode of processing, and mathematical performance in different content areas. Battista (1990), with high school students, found that spatial visualization and verbal-logical reasoning abilities were significant factors of geometry achievement and geometric problem solving. Bremigan (2005) investigated the frequency and nature of diagrams in 600 students' written solutions to three free response problems on the Advanced Placement (AP) Calculus Examination. The results indicated that the more frequent drawing of diagrams was associated with low AP scores, suggesting that high-scoring students might have strong visualization skills and might have used visualization without drawing or modifying a diagram. A similar finding was reported by Ferrini-Mundy (1987), who found a correlation between spatial ability and certain aspects of calculus. Kozhevnikov, Hegarty, and Mayer (2002) and Kozhevnikov and Thornton (2006) pointed to spatial visualization ability as a significant predictor of performance on physic problems involving graph interpretation.

However, other research has shown inconsistent results. In the studies by Moses (1977) and Suwarsono (1982), mathematical performance significantly correlated with spatial ability, but not with preference for visual processing. Suwarsono also found that verbal reasoning ability and spatial ability were not related to preference for visual processing. Lean and Clements (1981) reported a similar finding: spatial ability and knowledge of spatial conventions were not factors significantly affecting mathematical performance of engineering students. Galindo (1994) compared preferred mode of processing and performance of students enrolled in sections of first semester calculus using different instructional approaches (i.e., graphing calculator, Mathematica, and no technology) and concluded that there was no significant relationship between preference for visual processing and calculus performance for all sections. Hegarty and Kozhevnikov (1999) examined sixth grade students' problem solving performance and preference for visual processing. Their results revealed that preference for visual processing did not correlate with problem solving performance and was negatively associated with verbal ability, nonverbal reasoning, and spatial ability.

Our contention is that calculus requires adequate understanding of visual representations and the ability to visualize objects in two or three dimensions, and that aspects of visual processing, which play an important role in calculus performance, may not be measured accurately by existing questionnaires consisting of tasks that do not involve calculus. Although various tasks and questionnaires have been designed to measure individual differences in the tendency to use visual or analytic processing in cognition (e.g., Krutetskii, 1976; Mayer & Massa, 2003; McAvinue & Robertson, 2006-2007; Richardson, 1977; Suwarsono, 1982), no adequate measures for calculus were available. Thus, calculus tasks with questionnaires were first constructed to determine students' preferred mode of processing (i.e., visual or analytic processing). Then, a battery of spatial ability, verbal-logical reasoning ability, and mathematical tests along with questionnaires were administered to high school students enrolled in AP calculus courses. More specifically, the purpose of this study was to design calculus tasks that could reliably measure the students' preference for visual or analytic processing, and then examine the role of preferred mode of processing in calculus performance and its relationship to spatial ability and verbal-logical reasoning ability.

It is important to note that preferred mode of processing refers to how individuals prefer to process information, not skills or abilities. This distinction is important because preference and ability may not correspond (Presmeg, 2006; Sternberg & Grigorenko, 2001). Specific cognitive abilities (i.e., spatial and verbal ability) are assumed to be related to preference for visual or analytic processing. Thus, spatial ability and verbal ability have been included in correlational studies investigating the role of visual and analytic processing in cognition (Hegarty & Kozhevnikov, 1999; Kozhevnikov et al., 2002; Kozhevnikov, Kosslyn, & Shepard, 2005; Lean & Clements, 1981; Moses, 1977, Suwarsono, 1982). However, we agree with Battista (1990) that compared to verbal ability, as measured by a vocabulary test, verbal-logical reasoning ability—the ability to reason from premise to conclusion—is more related to mathematical performance. Thus, this study omitted verbal ability tests and instead used verbal-logical reasoning ability as the counterpart to spatial ability.

METHOD

Participants

The participants were 150 were twelfth grade students (17-18 years of age) who were enrolled in Advanced Placement (AP) calculus courses at four high schools. AP Calculus courses are designed for students who prepare for college-level mathematics and plan to take the AP Calculus Exam to earn college credit and advanced placement. Thus, overall the participants in this study were high achieving and motivated students. There were 82 male and 68 female students. The students' ethnicity was as follows: 89 were White, 8 were African-American, 28 were Hispanic, and 21 were Asian. The remaining 4 students indicated "Other" as their ethnic group.

Materials

The six tests, measuring spatial orientation ability (Cube Comparisons and Card Rotations), spatial visualization ability (Form Board and Paper Folding), and verbal-logical reasoning ability (Nonsense Syllogisms and Diagramming Relationships), are part of the Kit of Reference Tests for Cognitive Factors (Ekstrom, French, & Harman, 1976). Preference measures consisted of questionnaires for calculus (i.e., derivative and antiderivative) tasks and algebra word problems taken from the Mathematical Processing Instrument (MPI) (Suwarsono, 1982). The calculus tasks were also used to assess the students' mathematical performance. The students' scores on the AP Calculus AB Exam were collected from their teachers at the end of the study. The AP Calculus Exam is an important standardized test. High school students who perform well can earn college credit and advanced placement. It covers differential and integral calculus topics, and scores are reported on a 5-point scale (5 is the highest, and 1 is the lowest).

Spatial ability measures

The Cube Comparisons Test consists of 21 items and requires the participant to view two drawings of a cube and determine whether or not the two drawings can be of the same cube. The internal reliability of the Cube Comparisons Test is 0.84 (Ekstrom at al., 1976). An example is given in Figure 1.

The Card Rotations Test consists of 10 items, each of which presents a two-dimensional figure and eight other drawings of the same card. The participant indicates whether each of the eight cards, without reflecting, is the same or different from the original figure. The internal reliability of the Card Rotations Test is 0.80 (Ekstrom at al., 1976). An example is given in Figure 2.

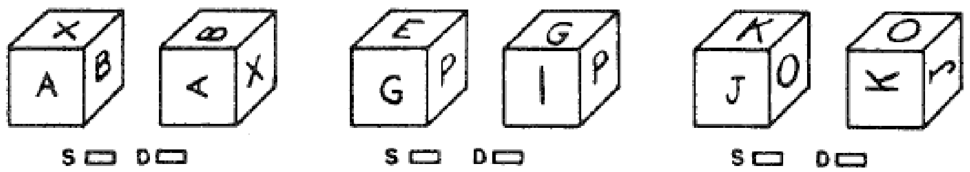


Figure 1. Items from cube comparisons test

Figure 2. Items from cube comparisons test

The Form Board Test consists of 24 items. Each item presents five shaded drawings of pieces and requires the participant to decide which of the shaded figures, from two to five, can be used to make the given geometric figure. The internal reliability of the Form Board Test is 0.81 (Ekstrom at al., 1976). An example is given in Figure 3.

The Paper Folding Test consists of 10 items each of which illustrate folds made in a square sheet of paper and a hole punched in it. The participant selects one of the five drawings that shows the position of the holes when the paper is completely unfolded. The internal reliability of the Paper Folding Test is 0.84 (Ekstrom at al., 1976). An example is given in Figure 4.

Verbal-logical reasoning ability measures

The Nonsense Syllogisms Test consists of 15 items. Each item is a formal syllogism, in which statements are nonsense and cannot be solved by reference to past learning. The participant determines whether or not conclusions drawn from the statements show good reasoning. The internal reliability of the Nonsense Syllogisms Test is 0.64 (Ekstrom at al., 1976). An example is given in Figure 5.

The Diagramming Relationships Test consists of 15 items. In each item, three groups of things (e.g., objects, animals) are given, and the participant selects one of five diagrams, which shows the correct relationships among the three groups. The internal reliability of the Diagramming Relationships Test is 0.79 (Ekstrom at al., 1976). An example is given in Figure 6.

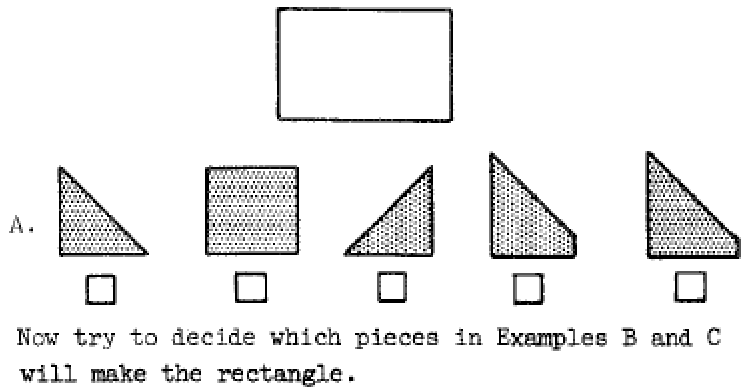


Figure 3. Items from form board test

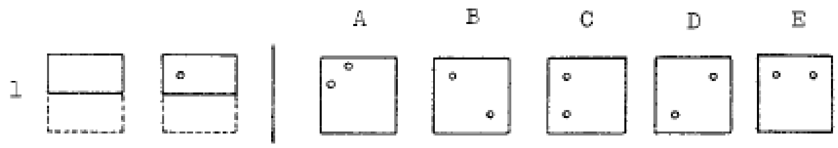


Figure 4. Items from paper folding test

Mark the G if the conclusion shows good reasoning.
Mark the P if it is poor reasoning.

1) All trees are fish. All fish are horses
Therefore all trees are horses. G P

Figure 5. Items from nonsense syllogisms test

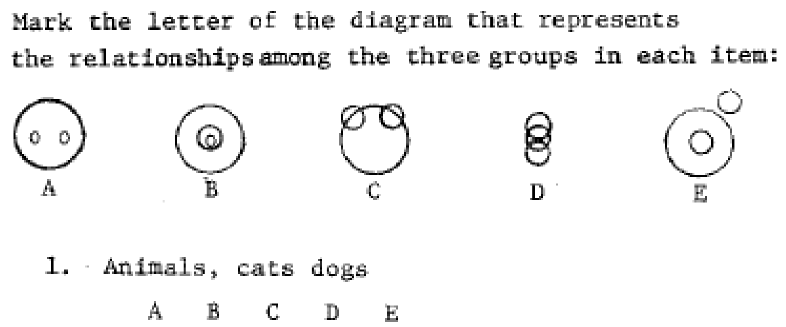


Figure 6. Items from diagramming relationships test

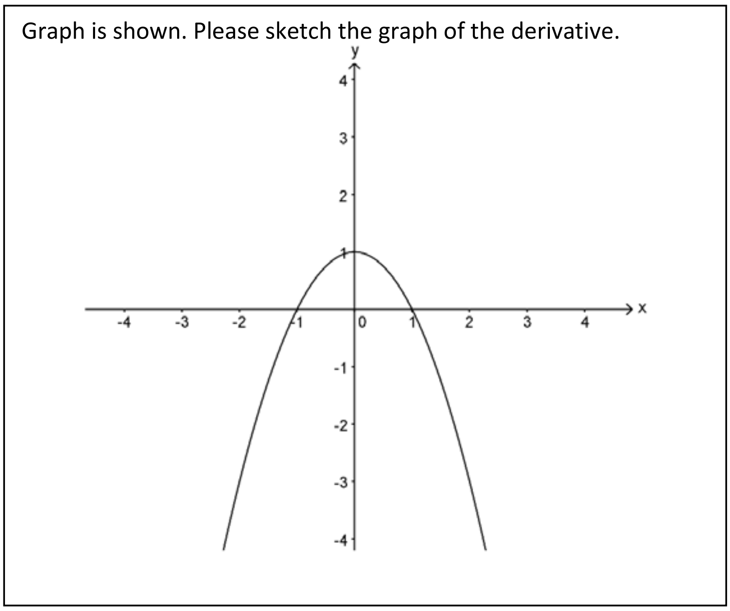


Figure 7. Derivative task

Preference measures

Questionnaires corresponding to calculus and algebra tasks were used to determine the degree to which students preferred visual or analytic processing. In this study, these calculus and algebra tasks were administered in a packet. The calculus packet consists of two parts. The first part is a test consisting of ten derivative and ten antiderivative tasks, and there are seven graphic and three algebraic tasks in each test. The second part is a questionnaire consisting of a visual and an analytic solution for each task. Upon completion of each test, the students were given the questionnaire and were asked to choose for each task a method of solution that most closely describes how they solved the tasks. The same procedure was followed to administer the algebra packet, a modified version of the Mathematical Processing Instrument (MPI) (Suwarsono, 1982). To illustrate the use of the calculus packet, an example of one of the derivative tasks presented graphically (see Figure 7) and the corresponding item in the questionnaire are provided in Figure 7. In this study, the students' thinking was considered as visual when they prefer to use visual methods and as analytic when they prefer not to use visual methods when there is a choice on a specific task.

Analytic solutions are generally equations-based. An analytic solution to a task presented graphically typically may involve translation to an equation, computing the derivative of the equation, and then using this new equation to draw the derivative graph. It was observed that instead of estimating equations precisely, analytic students referred to basic groups of functions such as linear, quadratic, or cubic functions and their derivative graphs associated with odd or even powers of x respectively. The following is the analytic solution given on the questionnaire for the derivative task in Figure 7:

> Analytic Solution: I estimated the equation of the graph (or recognized the equation of the graph). For example: This could be the graph of $f(x) = -x^2 + 1$ so I computed the derivative as $f'(x) = -2x$ and drew the derivative graph using this equation.

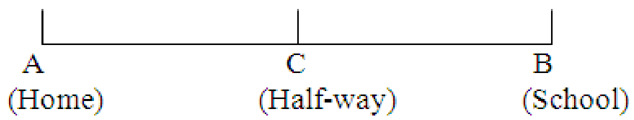
Visual solutions are image-based. They are able to visualize the changing slopes of tangent lines to the function and accordingly are able to construct an entire derivative graph with no need to consider individual parts of equations at critical points or intervals. These individuals are able to determine the shape of derivative graphs based on their visual estimates of slopes. The following is the visual solution given on the questionnaire for the derivative task in Figure 7:

> Visual Solution: I estimated the slopes (or the slopes of tangent lines) at various points on the graph of the function and used this to draw the graph of the derivative. For example: The slopes of tangent lines are positive and decreasing as x approaches 0 from the left. The slope is zero at $x = 0$ because the graph of the function has a horizontal tangent line at (0, 1). The slopes of tangent lines are negative and decreasing as x approaches positive infinity.

For the tasks presented algebraically, the students' thinking were considered as analytic when they preferred to calculate the derivative or integral, and used this equation to draw a possible graph of the derivative or antiderivative. On the other hand, their thinking was considered as visual when they preferred to draw the graph of the given function on paper (or in mind) and estimate the slopes of tangent lines at various points on this graph to draw a possible graph of the derivative or antiderivative. For instance, one of the algebraic tasks requires sketching a possible graph of the antiderivative, given $f'(x) = 3x^2 + 1$. An analytic solution involves computing the integral as $f(x) = x^3 + x^2 + c$ and drawing the graph of f(x) using this equation, whereas a visual solution involves drawing the graph of $f'(x) = 3x^2 + 1$ on paper (or in mind) and using the y values (or slopes of tangent lines) to draw the graph of the antiderivative.

The algebra packet, or the MPI, consists of two parts. There are eight algebra word problems in the first part that can be solved by visual or analytic methods. The second part is a questionnaire consisting of three to five visual or analytic solutions for each problem. An example of one of the problems from the test and its possible solutions from the questionnaire are provided below. Solutions 1 and 2 are considered visual, and Solution 3 is considered analytic.

> Problem: One morning a boy walked from home to school. When he got half way, he realized that he had forgotten to bring one of his books. He then walked back to get it. When he finally arrived at school, he had walked 4 km altogether. What was the distance between his home and school?
>
> Solution 1: To solve this problem, I imagined the route travelled by the boy that morning. When he finally arrived at school, he had walked twice the distance between home and school. This was equal to 4 km, so the distance between home and school was 2 km.
>
> Solution 2: I drew a diagram representing the route between his home and school.



The distance covered by the boy was AC, then CA, then AB. This means that when he finally arrived at B (school) he had walked twice the distance between his home and school. This was 4 km, so the distance between his home and school was 2 km.

Solution 3: I solved this problem by using symbols and equations.
Suppose the distance between home and school = x
Then half the distance = 1/2 · x
The total distance travelled that morning
= 1/2 · x + 1/2 · x + x
= 2 · x
This was equal to 4 km. Thus, x = 2 km, which was the distance between his home and school.

Calculus performance measures

Three calculus performance scores were included in the analyses. The students' scores on the AP Calculus AB Exam were collected from their teachers at the end of study. The students' calculus performance was also assessed by the derivative and antiderivative tests presented graphically and algebraically, yielding two scores labeled PGraphic (Performance on 14 Graphic tasks) and PAlgebraic (Performance on 6 Algebraic tasks).

Procedure

All students received standardized instructions and were tested in groups of 12 to 30 in their intact classrooms. All participating students gave their informed consent and were debriefed at the end of the study. The paper-and-pencil tests were administered to measure spatial ability, verbal-logical reasoning ability, preferred mode of processing, and performance on calculus derivative and antiderivative tasks. After the students took the Advancement Placement Calculus AB Exam, the interviews with seventy-nine students, who were willing to participate, were conducted. We did not know the students' scores at the time of the interviews. Two tasks—one derivative and one antiderivative—were presented to determine their preference for visual or analytic processing. The students' scores on the AP Calculus Exam were collected from teachers at the end of the study. The students were given 8 minutes for Form Board, 4 minutes for Nonsense Syllogisms (NS) and Diagramming Relationships (DR), and 3 minutes for Cube Comparisons (CC), Card Rotations (CR), and Paper Folding (PF) tests. Completion of the derivative, antiderivative, and algebra tests and their questionnaires was not timed. The total scores for CC, CR, FB, and NS tests were determined by subtracting the number of incorrect answers from the number of correct answers. Since there were 5 response options for each item on PF and DR, the total scores were determined by subtracting one-fourth the number of incorrect answers from the number of correct answers.

Scoring of preference measures

The calculus packet consisted of 14 graphic (7 derivative and 7 antiderivative) and 6 algebraic (3 derivative and 3 antiderivative) tasks. In assessing the students' performance on the calculus tasks, they were given a score of 0 for each incorrect answer and 1 point for each correct answer. In determining visual preference scores regarding these calculus tasks, the students were given a score of 0 for each analytic solution and 2 points for each visual solution, regardless of whether the answer was correct or incorrect. If a solution does not give any indication of method or both methods were used, a score of 1 was given. Thus, four scores for each student were obtained from these graphic and algebraic calculus tasks: PGraphic (performance on graphic calculus tasks), PAlgebraic (performance on algebraic calculus tasks), VPG (visual preference for graphic calculus tasks), and VPA (visual preference for algebraic calculus tasks).

In determining visual preference scores for the algebra word problems on the MPI, the students were given a score of 0 for each analytic solution and 1 point for each visual solution, regardless of whether the answer was correct or incorrect. Thus, for the MPI questionnaire, the total possible score was 8 points. The MPI was not used to measure mathematical performance because it consisted of algebra word problems, which were easy for the participants to solve, and may not reflect the differences in their mathematical performance. The internal reliability of the visual-analytic preference measures VPG, VPA, and the MPI were 0.92, 0.71, and 0.22, respectively.

RESULTS

Means and standard deviations for each of the twelve measures appear in Table 1. In order to determine the extent of the relationships between the measures of preference for visual or analytic processing and the other variables, Pearson product-moment correlations were computed. The correlations between all variables are presented in Table 2.

Correlational analysis

There were significant correlations between the three measures of calculus performance. The MPI did not correlate with the other two measures of visual preference on algebraic and graphic tasks (i.e., VPG and VPA). Of the three measures of visual preference, VPG had positive correlations with all other measures and significantly correlated with AP and Performance on Graphic tasks (PGraphic). There was a significant but small correlation between VPA and PGraphic. The MPI had non-significant negative correlations with the three calculus performance measures. The correlations between the three measures of visual preference and the measures of spatial ability and verbal-logical reasoning ability were either negative or non-significantly low.

Table 1. Means and standard deviations of measures (N = 150)

Measure	Label	M	SD
1. AP Calculus Exam Score	AP	2.69	1.55
2. Performance on Graphic Calculus Tasks	PGraphic	0.46	0.26
3. Performance on Algebraic Calculus Tasks	PAlgebraic	0.24	0.18
4. Cube Comparisons Test	CC	9.99	4.70
5. Card Rotations Test	CR	59.01	15.35
6. Form Board Test	FB	7.74	5.68
7. Paper Folding Test	PF	6.52	2.23
8. Nonsense Syllogisms Test	NS	2.55	4.50
9. Diagramming Relationships Test	DR	8.69	3.74
10. Visual Preference for Graphic Calculus Tasks	VPG	1.09	0.67
11. Visual Preference for Algebraic Calculus Tasks	VPA	0.60	0.56
12. Visual Preference for the MPI Tasks	MPI	0.62	0.18

Table 2. Correlation matrix for twelve measures

Measure	1	2	3	4	5	6	7	8	9	10	11	12
1. AP	—											
2. PGraphic	.62*	—										
3. PAlgebraic	.42*	.54*	—									
4. CC	.23	.28*	.20	—								
5. CR	.16	.24	.04	.50*	—							
6. FB	.38*	.40*	.28*	.45*	.23	—						
7. PF	.33*	.33*	.15	.36*	.35*	.47*	—					
8. NS	.30*	.40*	.17	.20	.14	.27	.10	—				
9. DR	.36*	.40*	.23	.34*	.18	.37*	.30*	.41*	—			
10. VPG	.31*	.51*	.18	.09	.08	.19	.17	.18	.16	—		
11. VPA	.11	.28*	.11	.02	.01	.15	.08	.12	.11	.40*	—	
12. MPI	−.08	−.08	−.05	.07	.04	.12	.09	−.08	.11	−.09	.06	—

Note: AP = Advanced Placement Calculus exam score; PGraphic = mathematical performance on graphic calculus tasks; PAlgebraic = mathematical performance on algebraic calculus tasks; CC = Cube Comparisons Test; CR = Card Rotations Test; FB = Form Board Test; PF = Paper Folding Test; NS = Nonsense Syllogisms Test; DR = Diagramming Relationships Test; VPG = visual preference for graphic calculus tasks; VPA = visual preference for algebraic calculus tasks; MPI = visual preference for algebra tasks on the Mathematical Processing Instrument (MPI).
**p < .05 (adjusted).*

Among the spatial ability measures, only Form Board (FB) had a significant correlation with Performance on Algebraic tasks (PAlgebraic). Cube Comparison (CC) and Card Rotation (CR) had the lowest correlations with the calculus performance measures. Form Board (FB), Paper Folding (PF), Nonsense Syllogism (NS), and Diagramming Relationships (DR) significantly correlated with AP and Performance on Graphic tasks (PGraphic). The correlation between CC and PGraphic was significant, but CR was correlated neither with AP nor with PGraphic. Except the correlation between CR and FB, the other correlations between the four measures of spatial ability were significant. The two measures of verbal-logical reasoning ability significantly correlated with each other. DR correlated three of the four measures of spatial ability, CC, FB, and PF, whereas NS only correlated with FB, suggesting that FB was the only spatial ability measure correlating with both measures of verbal-logical reasoning ability.

Factor analysis

An exploratory factor analysis was conducted to examine relationships between the variables used in the present study. The factor analysis produced four factors with eigenvalues greater than 1, explaining 14%, 12%, 9%, and 9% of the variance respectively. Factor loadings for each of the twelve measures are presented in Table 3. The four factors were labeled as spatial ability, calculus performance, verbal-logical reasoning ability, and preferred mode of processing. Loadings with magnitude 0.30 or more are indicated in bold. Of the three measures of visual preference, the MPI did not load heavily on any of the four factors, whereas visual preference on graphic and algebraic tasks (i.e., VPG and VPA) loaded on the fourth factor, labeled as preferred mode of processing. The four measures of spatial abilities CC, CR, PF, and FB loaded on the first factor (spatial ability) while the two measures of verbal-logical reasoning ability loaded on the third factor (verbal-logical reasoning ability). The performance measures, PGraphic, PAalgebraic, and AP

Table 3. Factor loadings for twelve measures

Measure	Factor			
	1: Spatial ability	2: Calculus performance	3: Verbal-logical reasoning ability	4: Preferred mode of processing
4. CC	*.717*	.075	.190	−.016
5. CR	*.639*	.033	.017	.040
7. PF	*.523*	.142	.184	.110
6. FB	*.495*	.224	.297	.117
2. PGraphic	.261	*.721*	.255	*.426*
3. PAlgebraic	.121	*.612*	.142	.048
1. AP	.235	*.582*	.261	.174
9. DR	.244	.150	*.760*	.085
8. NS	.128	.250	*.416*	.148
10. VPG	.093	.232	.045	*.729*
11. VPA	.017	.049	.078	*.522*
12. MPI	.115	−.175	.133	−.041

Note: CC = Cube Comparisons Test; CR = Card Rotations Test; PF = Paper Folding Test; FB = Form Board Test; PGraphic = mathematical performance on graphic calculus tasks; PAlgebraic = mathematical performance on algebraic calculus tasks; AP = Advanced Placement Calculus exam score; DR = Diagramming Relationships Test; NS = Nonsense Syllogisms Test; VPG = visual preference for graphic calculus tasks; VPA = visual preference for algebraic calculus tasks; MPI = visual preference for algebra tasks on the Mathematical Processing Instrument (MPI).

Table 4. Standard multiple regression analysis of AP exam scores

Variables	B	SE	β	t	p-value
1. VLR	0.33	0.10	0.25	3.17	.00
2. SO	−0.01	0.14	0.00	−0.04	.97
3. SV	0.46	0.13	0.29	3.49	.00
4. VPG	0.20	0.07	0.22	2.78	.01
5. VPA	−0.06	0.09	0.05	−0.69	.49

Note: VLR = composite verbal-logical reasoning ability score; SO = composite spatial orientation ability score; SV = composite spatial visualization ability score; VPG = visual preference for graphic calculus tasks; VPA = visual preference for algebraic calculus tasks.

loaded strongly on the second factor (calculus performance). PGraphic also loaded heavily on the fourth factor.

Multiple regression

The scores on the tests of spatial orientation ability, spatial visualization ability, and verbal-logical reasoning ability were scaled and averaged to create three composite scores for each student: composite spatial orientation ability score (SO) made up of CC and CR; composite spatial visualization ability score (SV) made up of FB and PF; and composite verbal-logical reasoning ability score (VLR) made up of NS and DR. A standard multiple regression was performed between AP exam scores as the dependent variable and spatial orientation ability (SO), spatial visualization ability (SV), verbal-logical reasoning ability (VLR), visual preference for graphical calculus tasks (VPG), and visual preference for algebraic calculus tasks (VPA) as independent variables (see Table 4). The five predictor variables contributed to

25.3% of the variance in AP, $F(5, 144) = 11.1$, $p < 0.01$. Three variables—VLR, SV, and VPG—contributed significantly to the prediction of AP exam scores. In a stepwise multiple regression analysis with a .05 level of significance required for a variable to be entered into the equation, when AP was regressed on the same variables, VLR (beta = 0.33, $p < 0.01$), SV (beta = 0.46, $p < 0.01$), and VPG (beta = 0.19, $p < 0.01$) would enter the equation again. The predictor variables explained more than a fourth of the variance in AP exam scores (adjusted R-squared = 0.26).

DISCUSSION AND CONCLUSION

This study contributes to the existing research on factors affecting mathematics performance by examining the relationships between measures of spatial ability, verbal-logical reasoning ability, calculus performance, and preferred mode of processing mathematical information. The correlational matrix revealed that spatial orientation ability, measured by Card Rotation and Cube Comparisons tests, did not correlate with calculus performance. Unlike the measures of spatial visualization ability and verbal-logical reasoning ability, spatial orientation ability seems to be unrelated to calculus performance although visualizing mathematical objects from different perspectives is crucial to understanding calculus. Multiple regression analysis also revealed that spatial visualization ability, verbal-logical reasoning ability, preference for visual processing contributed significantly to the variance in calculus performance. The results support the idea that spatial visualization ability and verbal-logical reasoning ability are related to students' ability to solve problems in physics and mathematics (Battista, 1990; Bremigan, 2005; Ferrini-Mundy, 1987; Kozhevnikov et al., 2002, 2006; Moses, 1977). The three measures of preference did not correlate with the measures of spatial ability and verbal-logical reasoning ability, suggesting that cognitive abilities did not predict the students' preference for visual or analytic processing, and vice versa. This is consistent with previous research (Hegarty & Kozhevnikov, 1999; Kozhevnikov et al., 2002; Lean & Clements, 1981; Suwarsono, 1982). Presmeg (2006) also observed that students with strong spatial abilities might prefer to use analytic methods if the use of visual methods or imagery is not required.

Factor analysis on the twelve variables provided interesting results. Using the varimax rotation, eleven of these variables load onto four easily interpretable factors: a calculus performance factor; a spatial ability factor; a verbal-logical reasoning factor, and a preferred mode of processing factor. The measures of visual preference, spatial ability, and verbal-logical reasoning ability loaded strongly on different factors and did not load on the factor of which calculus performance measures loaded strongly. A modified version of the Mathematical Processing Instrument (MPI) (Suwarsono, 1982) did not load on any of the four factors and did not correlate significantly with any measure. On the other hand, preference for visual processing regarding the calculus tasks presented graphically loaded substantially on the preferred mode of processing factor and correlated significantly with calculus performance measures, suggesting that this questionnaire is measuring an important component of cognition. This might be because the graphic calculus tasks were used to measure both calculus performance and visual preference of the students in the sample. However, the important role of visual preference is also evident in consideration of the results of multiple regression, which suggested that preference for visual processing was one of the three variables contributing significantly to the prediction of AP exam scores. It should be noted that the MPI consists of thirty algebra problems, but only eight problems were used due to time constraints, and this might be the reason for low reliability and the lack of correlations in this study.

Galindo (1994) observed similar findings from a study of the relationship between preferred mode of processing and calculus performance, in which 18 algebra problems out of 24 from the version of the MPI revised by Presmeg (2006) were used to measure preferred mode of processing. Galindo did not report the reliability of the MPI for his study but concluded that preference for visual processing was not related to calculus performance. Although preferred mode of processing as assessed by the MPI was not related to calculus performance in both studies, when the students' preference for visual or analytic processing regarding the calculus tasks were assessed, significant correlations between preference for visual processing and calculus performance were found in this study. Thus, another possible explanation for the lack of correlation is that the nature and complexity of algebra and calculus tasks might have influenced the students' degree of preference for using visual processing. That is, algebra tasks involve solving verbal problems presented in algebraic context, whereas calculus tasks are presented graphically and require sketching the graph of the derivative or antiderivative. Moreover, visual processes used in solving algebra word problems involve constructing or physically modifying figures, whereas visual processes used in solving calculus tasks involve mentally modifying or transforming figures (e.g., visualizing tangent lines to the graph of a function and then transforming them into the derivative graph). Thus, visual processes that are relevant to calculus may not be fully captured by algebra tasks, and thus individual differences in visual preferences may not be reflected accurately. As Dean and Morris (2003, p.268-269) noted "Investigations into the role of imagery in cognition using self-reports are possible but must pay careful attention to item content on the different measures."

This study with high school calculus students has generated new information about the relationships between spatial ability, verbal-logical reasoning ability, preferred mode of processing, and calculus performance. Moreover, analyses of data obtained with cognitive ability tests and questionnaires have produced results worthy of continued study, and thus the calculus tasks and the corresponding questionnaire have the potential to be used as a measure of students' visual and analytic tendencies in calculus. It is suggested that these tasks be used to provide learning opportunities for students with different preferences. By discussing functions and their derivatives presented graphically and algebraically, it is possible that students will synthesize visual and analytic mode of thinking that will enhance their conceptual understanding of calculus.

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