Towards a Dialogical Pedagogy: Some Characteristics of a Community of Mathematical Inquiry

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This paper discusses a teaching model called community of mathematical inquiry (CMI), characterized by dialogical and inquiry-driven communication and a dynamic structure of intertwined cognitive processes including distributed thinking, mathematical argumentation, integrated reasoning, conceptual transformation, internalization of critical thinking "moves," and collectively constructed concepts. As a form of pedagogy, community of inquiry is non-hierarchical, democratic, pluralistic, ethically and culturally sensitive, and inherently egalitarian. In addition, the structure of the inquiry process in CMI is understood as one in which every individual has an effect on the system as a whole, which is therefore emergent, self-correcting, self-directed, and self-organizing. This paper draws some implications of this form of pedagogy for mathematics education.

DIALOGICAL PEDAGOGIES: THE VYGOTSKIAN TRADITION

It is commonly accepted that the Russian psychologist Lev Vygotsky was the one of the first to articulate an understanding of learning and development as dynamic processes in dialectical relationship, and to emphasize how the relationship between the individual and the social mediates these processes. Over the last few decades, Vygotskyan scholars have introduced alternatives to cognitive and developmental individualism based on a model that features participation in a shared activity. Since at least the early 90's , participative, dialogical pedagogies such as apprenticeship (Rogoff, 1990; Lave & Wenger, 1991), guided participation (Rogoff, 1990), distributed

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thinking, community of inquiry (Lipman, 1991) and many more have been experimented with in one form or another.

There has also been a change of focus in the area of mathematics education from individualistic learning to learning in the social context of the classroom. The theory and practice of community of mathematical inquiry are coming to be recognized as offering possibilities for rich pedagogical activities and creative approaches in mathematics teaching and learning. In keeping with the goal of constructing a pedagogical system which both allows for and encourages the fundamental notion of learning as cognitive reconstruction in a social context, several prominent instructional theories, both inside and outside of mathematics teaching, have emerged in the last two decades. Brousseau's theory of "didactique des mathematiques" (1986), understands learning as adaptation to new situations, and seeks to define the systemic conditions necessary for it to take place. Artigue (1994) uses the term "didactic engineering" to refer to the teacher's work in developing the conceptual and methodological means for controlling interacting

phenomena in the classroom, and their relation to the construction and functioning of students' mathematical knowledge. Lave's and Wenger's theory of "situated learning" (1991) emerged from the idea of cognitive apprenticeship—a notion widely popular in 1980s. Based on Leontiev's activity theory, situated learning theory focuses on the relationship between learning and the social situations in which it occurs. Lave and Wenger (1991) situate learning in certain forms of co-participation, emphasizing the kinds of social engagements which provide the proper context for learning, rather than just the cognitive processes and conceptual structures involved. Situated learning assumes the relational character of learning, the negotiated character of meaning, the engaged nature of learning activities for the participants involved, and a highly interactive role for participants in the learning process.

Magdalene Lampert's understanding of teaching and learning through what she calls the "problem approach" has elements in common both with Brousseau's "didactique" and Lave and Wenger's theory of situational learning. The problem approach to teaching involves creating learning situations for students, evaluating them in terms of students' various levels of sophistication, and modifying the situation to keep it "challenging, but attainable" (Lampert, 1990). Lampert also introduces the idea of mathematics teaching as operating in a "community of discourse," which she characterizes as a little-explored territory (Lampert, 1990). In addition, Deborah Ball (1999) takes up Brousseau's idea that part of the teacher's role is to take noncontextualized mathematical ideas and to embed them in a context for student inquiry, a process that she calls "community of reasoning" (Ball, 1999).

Paul Cobb, Terry Wood and Erna Yackel are probably among the first mathematics researchers in the USA to have drawn analogies from the philosophy and sociology of science for understanding classroom life as a community of inquiry (Cobb, Wood, & Yackel, 1991). Their sociological analysis focuses on the creation of "taken-as-shared knowledge" in the community, and the establishing of classroom social norms (Cobb, Wood, & Yackel, 1991; Cobb, Wood, & Yackel, 1993; Cobb et al, 1997; Wood, 1985). According to these authors, "taken-as-shared" implies that participants achieve a sense that some aspects of knowledge are shared within a collective interpretative framework that constitutes the basis for communication among the participants in the community. The notion of "social norms" implies common agreement as to the expectations that the participants--including the teacher--have of themselves, and a shared conception of what it means to practice mathematics in a community, which includes an understanding of the ways that mathematical validity can be established.

In a practice analogous to that of establishing procedural norms in a community of philosophical inquiry, Cobb et al. describe negotiating what constitutes effective and appropriate mathematical practice in the classroom through engaging the learning community in conversations about how to practice mathematics collaboratively. This kind of discussion corresponds to Brousseau's (1994) *contract didactique*, or negotiated agreement between students and teacher. The initial student-teacher contract is a prerequisite to creating a specific mathematics classroom culture. As Schoenfeld (1994) suggests, one of the major goals of teaching is to create, together with the group, a "classroom culture" with a shared linguistic medium, and to help students "acculturate" to this particular context. Other interpretations of and approaches to community of mathematical inquiry have been under development for at least twenty years (e.g. Goos, 2004; Siegrist, 2005). None of them are incompatible with the approach adopted and the form of communal inquiry described here.

Community of Mathematical Inquiry

Community of inquiry may be broadly described as the collective execution of a dialogical, language-based activity whose goal is to reach communal agreement through argumentation. The model of community of philosophical inquiry developed by Mathew Lipman and Ann Sharp in the 1970's at the IAPC (Institute for the Advancement of Philosophy for Children) at Montclair State University (Lipman, 1991; Sharp, 1992), a question-based (as opposed to a propositional or apodictic) approach to teaching and learning, is eminently adaptable to disciplines other than philosophy—or rather, it offers a straightforward and comprehensive way to approach other disciplines from a philosophical perspective. This formulation of Community of mathematical inquiry (CMI) described here embodies most of the essential characteristics of Lipman's model of a community of philosophical inquiry, which has its roots in a combination of John Dewey's ideas of communal inquiry and C.S. Pierce's notion of scientific community of inquiry. In keeping with Lipman's formulation, it is a communal discursive event which is *dialogical* and *inquiry-driven* (Dewey, 1910, 1939; Pierce, 1958, 1966). Its main objective is the construction of meaning and the formation of concepts, not through teacher transmission, individual reflection or debate, but through what is referred to as "building on each other's ideas"— that is through *distributed thinking* in a dialogical context.

Lipman and Sharp's approach to communal inquiry clearly resonates with the Vygotskyan approach applied to education, which views learning and development as dynamic, dialectically driven. In such a context, the

individual subject and his cognitive processes must be understood in terms of their incorporation into different systems of collective practical and cognitive activities (Bauersfeld, 1994). A pedagogical approach attuned to Vygotskyan psychology pays as much attention to mediated group process as to the individual, capitalizes on the notion of distributed thinking, and recognizes the collective subject and its dialectical, systemic processes as vital to learning and development (Toulmin, 1999; Lushyn & Kennedy, 2000).

Such a modality of social cognition suggests the form of social organization that we know as education in the sense of the German word *bildung* (i.e. culture itself understood as an educational process), and considers this dimension of education to be fundamental to conceptual development and all that implies for human development as a whole. Vygotsky viewed learning as a process of acquiring a cultural sign system, which he characterized as a "tool" for filling in the "cognitive gaps" within one's own developmental zone. Since language is the most powerful cultural sign system, a complex and dynamic relationship connects language and thinking; and given that language is a social phenomenon, it follows that thinking is deeply embedded in social activities and cultural practices (Vygotsky, 1962). Language and thought are understood as overlapping activities—that is, the verbalization of one's thought is not only making the implicit explicit, but also generates thought.

Discussion in community of mathematical inquiry advances through identifiable critical thinking interventions or "moves," including questioning, offering examples and counter-examples, asking for justification, giving reasons, offering clarifications, making propositional statements, exploring alternative positions and hypotheses, drawing conclusions, reasoning syllogistically, making inferences, and many others (Kennedy, 2005) . As it enters the conversational system, the verbalized material undergoes a continual process of *translation* that involves listening and responding, clarification and reformulation, taking turns, remaining sensitive to context and open to new interpretations, translating between various expressive, cognitive, and discursive styles, entertaining different perspectives, and self-correcting (Kennedy, 2004).

Along with the "technical" moves that comprise the dialogue and are listed above, several key assumptions are immanent to the inquiry, which resonate with the notion of "social norms" discussed above. There might be more than one perspective or interpretation, and in dialogue those perspectives interact as equal interlocutors. Each perspective enters dialogue with the possibility of being modified or changed by the others. Moreover, dialogue presents a possibility for reconstruction, not only of perspectives and ideas, but also of values, modes of practice, beliefs, attitudes,

dispositions, and relationships. Finally, the individuality of each interlocutor is recognized and valued as unique in communal dialogue, but only through its relation to each other individuality. In dialogue "one thinks for oneself and with others" (Kennedy, 1999).

The ideal mathematical inquiry proceeds through a form of argumentation which, because it is inherently dialogical, is thus by implication a dialectical process, which is to say a process which moves forward through encountering and attempting to resolve inadequacies or inconsistencies. *Argumentation* is understood here as a new form of collective classroom discourse, not as a debate but as a cooperative competition in constructing a collective argument whose purpose is to arrive at commonly agreed-upon conclusions by way of open and free deliberation, which is characterized by distributed thinking and communal scaffolding. Any given argument is built on a previous argument or entertained as a counter-argument to a previous one. As such, argumentation in community of inquiry is inherently both chaotic and teleological. It can be influenced by any single element of the system—for example by any single participant—as well as by any element in the cognitive medium, for example the initial problem under consideration, by specific examples and counterexamples, or by the presence of conscious or unconscious assumptions.

Community of Mathematical Inquiry and Mathematical Classroom Culture

Mathematical argumentation relies on processes such as reasoning and explicit justification of claims and inferences. Getting students accustomed to justifying their claims and the mathematical operations they use is also a form of acculturation, and constitutes an aspect of the specific "mathematical classroom culture" that recent theorists in mathematics education emphasize (Schoenfeld, 1992). Such a culture cannot be implicitly assumed. Fischbein (1982), for example, comments that most high school students have not been enculturated into the practice of giving reasons. Coe and Ruthven (1994) found that when a proof context is data-driven and students are expected to form conjectures through generalization or counterexample, their justificatory strategies are primarily based on examples or counterexamples. Similarly, Finlow-Bates, Lerman and Morgan (1993) found that even many first year undergraduates had difficulties following chains of reasoning. There are studies of elementary and middle school students that suggest that if students are systematically and consciously initiated, in a suitable environment, into the practice of making mathematical arguments and justifying their ideas and procedural moves, their ability to make inductive and deductive

judgments shows progressive development (e.g. Maher & Martino, 1996; Zack, 1997; Lampert, 1990).

In addition, Cannon and Weinstein (1993) understand the process of reasoning as manifesting primarily through four of its dimensions—formal, informal, interpersonal, and philosophical–some of which seem to be completely absent in current school practices. In the context of CMI, I have argued (Kennedy, 2006) that in fact communal mathematical inquiry is conducive to a form of multi-dimensional reasoning that includes formal, informal, interpersonal, and philosophical/metacognitive dimensions, and have suggested ways of introducing argumentative discourse through the practice of what I call integrated reasoning.

Indeed, the connection between mathematical thinking and reasoning in the teaching and learning of mathematics tends to be obscured when the process of doing mathematics is not only removed from the need to develop any habits of inference, but it is stripped of the opportunity for or encouragement of conscious guessing, the tracing of conjectures, exploration of hypotheses, argumentation, or of any attempt to assume a mode of inductive or deductive reasoning (Schoenfeld, 1994; Lampert, 1990). In addition, it seems that students generally believe that practicing mathematics is a quick and predetermined process in which one either knows or doesn't know "the answer," when in fact finding the answer demands continuous cognitive reconstruction and cognitive efforts. In this respect, I would argue that CMI is a form of mathematical practice that carries the potential for individual and collective reconstruction of habits of reasoning, not only of beliefs about mathematical practices, but of attitudes and dispositions towards mathematics in general.

Dialogue and Dialectic in Community of Mathematical Inquiry

Patterns of argumentation in CMI are understood and practiced as dialogue rather debate, for dialogue provides the conditions for the emergence of new perspectives within and between interlocutors (Forman et al., 1998). Tolerance and even encouragements of a diversity of perspectives prompts the awareness of oppositions between the views or beliefs of participants, and triggers reflection on the information they are provided with. Numerous studies suggest that the experience of being exposed to conflicting views in a context of argumentation leads to significant restructuring of participants' understanding of a topic (Forman et al, 1998; Leitao, 2000; Van Eemeren and Grootendorst, 1994; Krummheuer, 1995; Resnik et al., 1993; Pontecorvo, 1993). Other researchers note that examining opposite sides of an argument does not always lead the participants to cognitive change and to a change of views, but rather to further polarization (Stein

& Miller, 1993; Perkins, Allen, & Hafner, 1983; Kuhn, 1991). Toulmin (1969) offers something of an explanation of this discrepancy by emphasizing the importance of developing "proper" inferring-habits and "rational" canons of inference, which can serve as stepping-stones for knowledge-building mechanisms. But he emphasizes that such habits and cannons must be preceded by the development of proper attitudes towards mathematical practices, and by the presence of dispositions toward reflective thinking.

The chief pedagogical significance of the constructive process of community of inquiry is that it operates in the collective zone of proximal development, which acts to "scaffold" concepts, skills and dispositions for each individual. The concept of the zone of proximal development, which represents the distance between actual and possible development that can be bridged when learning is facilitated by someone with greater expertise than the learner—neatly operationalizes the educational implications of Vygotsky's theory. The scaffolding process functions through subprocesses such as clarification, reformulation, summarization, and explanation, as well as through challenge and disagreement. The emergence of different perspectives inevitably gives rise to oppositions, inadequacies, or contradictions, and thus forces discrimination and the production and resolution of differences.

In this context, *collective concept transformation* is understood to operate through the emergence of cognitive conflict and the ongoing resolution of that conflict in a dialectical manner—which is to say through the recognition and articulation of contradictions and inconsistencies, and their mediation through the processes already discussed—communal dialogue, integrated reasoning, distributed thinking, collective argumentation, and their dynamic interplay within the CMI. Consistent with Vygotsky, the process of concept transformation or conceptual building proceeds from participants' "spontaneous" or "everyday" concepts towards more scientific concepts, i.e. in a "bottom-up" fashion (Vygotsky, 1962).

It has also been argued (Lipman, 1991) that community of inquiry represents the ideal situation for Vygotsky's notion of the intrapersonal appropriation of the interpersonal—or "*internalization*"—not only on the conceptual but on the behavioral level, i.e. in the development of habits of both cognitive and behavioral self-control and self-regulation. Furthermore, community of inquiry as an open, emergent, inquiring system is continually mediating further cognitive advancement, through the re-externalization of the internal in the ongoing discourse of the community, followed by further internalization, and so on in an ascending spiral of development. Given that we view the community of inquiry as a complex and dynamic

system of interrelated subjects, mutually intertwined individual and collective processes, distributed thinking, argumentation, and concept transformation, we reflect on and analyze the external conceptual and argumentation processes as they are manifested in the *collective subject*, which reveals itself through practical activities and collective cognitive processes.

The Role of the Teacher in Community of Mathematical Inquiry

From a systemic perspective, community of inquiry is an open, interactive system, and all of its elements exercise what Lushyn and Kennedy (2000) call "ambiguous control" over each other. The role of a facilitator in such a system is also ambiguous, since she has, if necessary, to encourage the scaffolding process without providing direct answers or authoritative perspectives, but more through a form of the Socratic elenchus—that is, through provocative questioning, reformulation, and the offering of counterexamples and counter-perspectives. Vygotsky's notion of appropriate intervention in the process of concept formation and advancement is obviously more subtle and indirect than in traditional pedagogy, which typically satisfies itself with a behaviorist model and leaves it at that.

The ultimate achievement of a community of inquiry as a pedagogical system is to move the group as a whole and each member in it in the direction both of enhanced cognitive/conceptual and behavioral self-organization and self-regulation, a movement which has implications, not only for students' mathematical learning, but also for student empowerment through the development of democratic skills and dispositions and the skills of communal deliberation.

One primary goal of the facilitator in a community of mathematical inquiry practice has been to create a context for mathematical inquiry ("contextualizing") to be used as springboard for discussions of mathematical ideas that are meaningful to students and which correspond to their mathematical knowledge—that is, which are challenging and yet still accessible to students' inquiry. This is what we might refer to as problematization, and it is at least analogous to what Brousseau calls "devolution," and Balacheff refers to as "toward a problematique" (Balacheff, 1990). Its basic goal is to embed the mathematical idea in a context which "perplexes" students and evokes the student's felt responsibility for the pursuit of meaning through offering a stimulus as a starting point for inquiry (Dewey, 1910). A stimulus presents a "rich" mathematical problem—a problem which might be is set in or evokes a narrative context, and which not only requires calculation, but offers possibilities for interpretation. It could be referred to as a "thinking story"—whether presented as a short narrative, a video clip, or an image (a painting by Escher, for example).

One of the operative assumptions of a pedagogy that is more appropriate to human beings and their learning processes than the traditional model must, I would argue, be that the acquisition of new concepts is most meaningful to students when they have the opportunity to construct those concepts and their relationships for themselves, through interactive participation in activities which provide motives and goals for them. In the Vygotskyan model, the role of the facilitator is to construct *with* the students opportunities for interacting with meaningful ideas, and for collaborating with others in activities that define meaningful goals. One of the challenges for such a facilitator is to identify activities which scaffold students' learning to a more advanced level of their potential development.

Such a view would implicitly suggest that any rigid or formulaic kind of instructional planning in a CMI faces an inherent tension. Most of the researchers reviewed above suggest that students must have the freedom to respond to learning situations on the basis of their past knowledge and of their current understanding of the problematized situation, rather than being expected to give either uniform answers or answers which are merely expected by the instructor (e.g. Resnick, 1980). If this tension is taken seriously, it implies the necessity for teacher adaptation to the paradigm shift from "teaching as telling" to a dialogical model, which is the prime characteristic of community of inquiry theory.

The community of inquiry teacher is not just a planner but also an organizer—the initiator of a process of negotiation aimed at establishing social norms for the communal practice of mathematics. She is the one who initiates students into mathematical discourse--or the "language game" which provides the fundamental meaning-context for mathematical symbols and ideas (Wittgenstein, 1966). Furthermore, she does not introduce it as a static form, but is continually modeling and shaping the classroom discourse through offering restatements, clarifications, examples, and summarizations, and asking students to do so as well, even as she is all the while actively listening.

CONCLUSION

The perennial problem of pedagogical sterility in mathematics education can be traced to a set of much larger epistemological and ontological beliefs, which have come increasingly to be challenged over the course of the last half century. One of the greatest challenges to these beliefs is presented by Vygotsky (1978) and his concept of "developmental teaching," the fundamentals of which have been sketched here, and which presents a great challenge to mathematics teachers and teacher education in general—the challenge of coming to

understand themselves as agents of such an emergent pedagogy.

Understanding mathematical knowledge construction as an emergent process suggests the idea of a dynamic, non-linear pedagogy, from which it follows that the learning process produced by such a pedagogy would be dynamic and non-linear as well. The teaching/learning process can be altered at any moment when a confrontation of multiple contradictory perspectives presents itself. The resolution of this confrontation represents, not the mutual acceptance of one imposed unilateral perspective, but a "sublation" (that is, the overcoming of contradiction through dialectical negation), which emerges as a result of the recognition of all presented perspectives, and transforms the whole system to a new level of development. This presents a sharp contrast to traditional mathematics instruction, which is compartmentalized into segments representing units of instruction, made uniform by mathematics textbooks, focused on one idea at a time, and aimed at forming certain skills through practicing planned exercises. In contrast to the traditional teaching model, the goal of the teacher who facilitates mathematical learning in a community of inquiry is to support the development of students' constructive abilities, their self-concept as learners, and their capacities for internally driven, self-organized cognitive transformation through the practice of argumentation.

As a discursive form, community of inquiry pedagogy is distinguished from traditional practice by its multilogical as opposed to monological style and character. Since everyone in the system can exercise control to some degree, and every characteristic of the system--whether social, psychological, logical, conceptual, linguistic or some other—can change it, the system undergoes a continual dialectical process of deconstruction and reconstruction. This identifies it as an open, emergent system, which in turn describes it as a system in continual transition, over which no one can exercise anything but "ambiguous control." Thus construed, the process of teaching/learning in a community of inquiry is implicitly understood as a developmental and a dialectical process often marked by uncertainty and lack of clarity, which itself implies the capacity to trigger system change and self-organization, and is often associated with the emergence of new forms of knowledge.

The inquiring system described and analyzed here offers the possibility of fulfilling—as much as is possible for a normative ideal—the prerequisites for what Habermas (1990) has called the "ideal speech situation," which requires that all its members have equal opportunity to participate in and contribute to the dialogue, free from internal constraints or external coercion. This implies the need for a pedagogy which

not only develops communicative competence, but which models a form of argumentation that understands itself as a *cooperative* competition in constructing a better collective argument—with the major goal of an agreement arrived at collectively through open, free communication. In short, the model of collective inquiry whose developmental and transformative potential has been described here offers the institution of education an egalitarian and democratic model that stresses the equality and freedom of each participant, that can function as a matrix for collective knowledge construction and, through its promotion of integrated reasoning, represents a more sophisticated approach to learning than is currently in place in the vast majority of schools. Finally, it offers an outline of a methodology and a pedagogy which understand mathematical development as a dialectical, emergent phenomenon, and thus represents a new direction for mathematics education.

Community of inquiry theory and practice offer new ways of understanding and rethinking the teaching and learning of mathematics, and new insights into how school mathematics might be reconstructed as collaborative dialogical inquiry. Its emphasis on communal dialogue makes of it an ideal medium for the interplay between individual and collective cognitive and psychodynamic processes in the development of mathematical concepts, and in the development of the skills and dispositions of argumentation. In addition, it offers a promise for the transformation of mathematics teaching and learning from a rigid, transmissional model to one which is student-centered, self-regulatory, and inquiry-driven.

That the CMI model points to the advantages of sensitivity to social setting, to collaboration, and even to some form of dialogue, is nothing new. It is the radical epistemological difference—which in turn is determinative of differences in learning theory—which distinguishes it from the transmission or even the individual problem-solving model. Community of inquiry takes the notion of distributed learning and thinking with the utmost seriousness, which amounts to the epistemological claim that knowledge constructed in an inquiring system—a group whose chosen activity is collaborative, dialogical deliberation—has qualitative differences from knowledge attained individually, or even as a result of a dyadic interaction. Such knowledge construction demands skills, dispositions, and even fundamental beliefs on the part of teachers that require a radical reconstruction of the logical terms of teaching and learning itself.

On a practical level alone, the role of the facilitator in a community of mathematical community of inquiry is far more complex than the traditional teacher's, requiring as it does sensitivity, flexibility and creativity in the organization and planning of content and activities,

the courage to take risks and to endure suspense in the facilitation and scaffolding of the inquiry, and trust in the inherent self-organizing capacity of groups in the management of communal dynamics. As such, the application of the community of mathematical inquiry model to mathematics education poses a profound challenge, given both the nature of the discipline and the pedagogical traditions that still dominate it. It also offers the possibility of the development of a form of classroom practice capable of transforming the field of mathematics education.

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