MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2024, 20(3), em2409
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/14281

Transforming sociomathematical norms in a South African grade 11 classroom to enhance learners' mathematical proficiency

Lekwa Mokwana [1*], Satsope Maoto [1], Kabelo Chuene [1]

[1] Department of Mathematics, Science and Technology Education, School of Education, University of Limpopo, Limpopo, SOUTH AFRICA

Received 04 August 2023 ▪ Accepted 29 January 2024

Abstract
In this paper we use parts of qualitative data from the first author's doctoral study to explore how transforming existing sociomathematical norms enhance learners' mathematical proficiency. The study was conducted in a grade 11 mathematics classroom comprising of 23 learners, facilitated by the first author as learners engaged in a mathematical discourse on analytical geometry. Data were gathered through video recording, documents and researcher journal. We adopted Yackel and Cobb's (1996) interpretive framework and Kilpatrick et al.'s (2001) notion of mathematical proficiency as lenses, which guided the data analysis. We analyzed the data following Polkinghorne's (1995) narrative analysis method. We found that transforming existing sociomathematical norms to enhance learners' mathematical proficiency involved a three-stages process: negotiating entry into learners' existing sociomathematical norms, disrupting learners' existing sociomathematical norms and constituting 'new' sociomathematical norms. As learners developed new taken-as-shared meanings regarding acceptable mathematics explanations, justifications and mathematically different solutions they enhanced their *conceptual understanding*, *procedural fluency*, *adaptive reasoning*, and *strategic competence*.

Keywords: mathematics education, mathematical proficiency, sociomathematical norms

INTRODUCTION

Department of Basic Education (DBE) (DBE, 2018) in South Africa developed and published the '*Mathematics teaching and learning framework for South Africa: Teaching mathematics for understanding*' (TMU). The framework is aimed at providing guidance to mathematics teachers for all school grades (grade R-12) in teaching mathematics for understanding. TMU in a way provides standards for teaching school mathematics in South Africa, just like there are standards for teaching mathematics in other countries such as the United States, which are *problem solving*, *communication*, *reasoning and proof*, *representation* and *connections* (National Council of Teachers of Mathematics [NCTM], 2000). TMU framework is underpinned by five interwoven strands of mathematical proficiency; *conceptual understanding*, *procedural fluency*, *strategic competence*, *adaptive reasoning* and *productive disposition* (Kilpatrick et al., 2001). In this article, we argue that enhancing learners' mathematical proficiency may require the transformation of existing norms in a classroom setting.

Sociomathematical norms are aspects of mathematics classroom culture, a set of distinct instructional practices, concerned with the shared understandings and behaviors related to mathematical activities that are encouraged and enacted within a classroom (Cobb & Yackel, 1996). However, classroom culture is implicitly learned through observation and participation, and not by deliberate study. As such, to achieve the goals of TMU, amongst other implications, teachers are expected to create learning opportunities to enhance learners' mathematical proficiency. Creating learning opportunities implies orchestrating classroom social interactions and negotiating classroom culture (Cobb & Yackel, 1996). Consequently, Cobb and Yackel (1996) define learning as "a constructive process that occurs while participating in, and contributing to, the practices of the local community" (p. 185).

✉ Lekwa.Mokwana@ul.ac.za **(*Correspondence)** ✉ Satsope.Maoto@ul.ac.za ✉ Kabelo.Chuene@ul.ac.za

Contribution to the literature

- Importance of teachers' awareness of sociomathematical norms enacted in their classrooms.
- Enactment of sociomathematical norms in a classroom setting and its contribution to learners' development of mathematical proficiency.
- An illustration of how sociomathematical norms can be disrupted and renegotiated to enhance learners' mathematics learning.

Participating in classroom social interactions comes with assumed individuals' obligations and expectations, which are regarded as social norms and constitute the classroom culture (Yackel, 2001). These norms are sociomathematical when they relate to work that is specific to mathematics (Wester et al., 2015) and "mathematical aspects of students' working" (Morrison, 2021, p. 1). On one hand, students can express such norms when showing how to do calculations, while teachers' expected norms are more than calculations (Wester et al., 2015). On the other hand, the teachers' sociomathematical norms may include naming and comparing, checking whether students' mathematical arguments are sufficient and adequate, and pushing for students' understanding (Van Zoest et al., 2012).

Sociomathematical norms contribute to conditions that make meaningful learning of mathematics possible (Yackel et al., 2000). Correspondingly, Gülburnu and Gürbüz (2023) argue that mathematics learning opportunities emerge through the negotiation of sociomathematical norms and contribute to collective mathematics learning by shaping the interaction among class members Although teachers do not teach sociomathematical norms, it is important for them to be aware of, and understand the sociomathematical norms of their classrooms (Kang & Kim, 2016; Zembat & Yasa, 2015). Furthermore, teachers should be able to engender productive sociomathematical norms during teaching and learning (McClain & Cobb, 2001; Partanen & Kaasila, 2015; Yackel et al., 2000).

The classroom culture and sociomathematical norms of traditionally oriented and reform-oriented classrooms differs. On one hand, in traditional classrooms, learners are expected to grasp knowledge that a teacher already has. Correspondingly, a teacher's role is to explain and clarify, while learners' role is to figure out what a teacher has in mind. In reformed classrooms, on the other hand, the classroom community's shared role include; explaining and justifying one's solutions, trying to understand other's reasoning, asking questions if one does not understand, and challenging arguments one disagrees with. Even though reform oriented mathematics classrooms are desired, as they support effective learning (Wood, 2016), the shift from traditional to reform oriented classrooms still pose a challenge in South African mathematics classrooms, where language proficiency impedes expressing ones ideas (Robertson & Graven, 2019). It is within the context of shifting from traditional to reform oriented classroom practices and TMU framework that the first author conducted a study from which we harvested this article. Hence, we explored how transforming existing sociomathematical norms enhance learners' mathematical proficiency.

LITERATURE REVIEW

Sociomathematical norms provide criteria for evaluating mathematical activities and discourse unrelated to any particular mathematical idea or concept (Cobb et al., 2010). The norms are also concerned with how the mathematics community makes decisions, talks about, and analyses the mathematical aspects of the activities at hand (Güven & Dede, 2017). Examples of sociomathematical norms include what counts as a mathematically different, sophisticated, efficient, and elegant solution (Cobb & Yackel, 1996). Mathematically, what counts as mathematically different is observable in interactions, where learners present different solutions to the same task (Chuene, 2011). However, their difference should be mathematically sound for solutions to be accepted as mathematically different (Widjaja, 2012). As learners compare and argue their solutions to come up with a mathematically efficient one, they reach a consensus when their discourse lends to a mathematically acceptable solution (Chuene, 2011). Yackel and Cobb (1996) further clarify that:

> Issues concerning what counts as different, sophisticated, efficient, and elegant solutions involve a taken-as-shared sense of when it is appropriate to contribute to a discussion. In contrast, the sociomathematical norm of what counts as an acceptable explanation and justification deals with the actual process by which students contribute (p. 461).

Embodied in this clarification is that the learners' taken-as-shared sense of when to contribute depends on comparing their contributions with those already offered. The comparison focuses on the solutions' differences, sophistication, efficiency, and elegance. For example, the efficiency of solutions has to do with their viability for application. If a learner has offered a long-winded solution, which may be laborious, it would be expected that other learners' contributions are a shorter version of what has been offered already.

The importance of sociomathematical norms in mathematics learning has been recognized, as evidenced by over a decade-long notable upward trajectory of research. Studies were conducted in both elementary (Abdulhamid, 2016; Gülburnu & Gürbüz, 2023; Morrison et al., 2021) and secondary (Partanen & Kaasila, 2015; Yenmez & Erbaş, 2023) classrooms while other studies were conducted at institutions of higher learning (Chuene, 2011; Güven & Dede, 2017; Pi et al., 2023; Sánchez & García, 2014; Yackel et al., 2000). All the studies cited here adopted Yackel and Cobb's (1996) view of sociomathematical norms, but the difference was in their purpose and the research questions. These studies sought to identity, explore, describe, examine and/or investigate the sociomathematical norms in various classrooms. Pi et al. (2023) explored acquisition of sociomathematical norms in an introductory undergraduate proof-writing course. Whereas Güven and Dede (2017) identified social and sociomathematical norms in classrooms at university level. Similarly, Chuene (2011) explored the enactment of social and sociomathematical norms at university level. The difference between the studies is that, in the former study, participants were students being trained to become teachers, while in the latter study the participants were students enrolled in a Bachelor of Science degree.

Through her engagement with the data, Chuene (2011) concluded that students enacted social and sociomathematical norms when they:

(i) contributed to discussions by initiating discussions,

(ii) took responsibility for raising and offering answers, and

(iii) accepted arguments that are efficient to them.

On the other hand, Güven and Dede (2017) did not explain how the sociomathematical norms at play in the classroom they researched advanced students' mathematics learning. However, they argued that student teachers' awareness of norms could benefit the classrooms in which they will teach. Inherent, Güven and Dede's (2017) argument claims that development and enactment of sociomathematical norms has the potential to support students' mathematical learning. The notion that sociomathematical norms support students' mathematics learning is also supported by the results of studies conducted by Van Zoest et al. (2012) and Wadjaja (2012). Furthermore, Yenmez and Erbaş (2023) also discussed how "modeling activities could be a powerful tool for mathematics teachers and students in challenging and transforming traditionally oriented classroom norms to those aligned with reform-oriented mathematical classrooms" (p. 761).

Earlier literature on social and sociomathematical norms focused on, among others, their development (McClain & Cobb, 2001), establishing patterns of interactions (Roy et al., 2014) and how the norms are identified (Güven & Dede, 2017) in mathematics classrooms. Recent studies link sociomathematical norms with mathematical learning and teaching experiences like self-efficacy (Apsari et al., 2020), early numeracy learning (Morrison et al., 2021), abilities in problem-solving (Dini & Maarif, 2022), mathematical process skills (Gülburnu & Gürbüz, 2022), mathematical modelling activities (Yenmez & Erbaş, 2023), and teacher noticing (Baki & Kilicoglu, 2023). This article follows these trends and explores how transforming existing sociomathematical norms enhances learners' proficiency in mathematics. It would then be expected that the new sociomathematical norms need supporting social norms containing teacher's role, students' role and general activity (Wester et al., 2015), which in this article would be mathematical proficiency.

THEORETICAL FRAMEWORK

In this paper, our research explored the question of how transforming existing sociomathematical norms could enhance learners' mathematical proficiency. To guide the analysis, we adopted Cobb and Yackel's (1996) interpretive framework (**Table 1**), which served as a lens to analyze classroom interactions and the constitution of taken-as-shared classroom practices. Additionally, we utilized Kilpatrick et al.'s (2001) notion of mathematical proficiency as another lens to analyze the opportunities created for enhancing learners' mathematical proficiency. The combination of these frameworks provided valuable insights into the dynamics at play in the classroom, shedding light on the transformative potential of sociomathematical norms in fostering learners' mathematics learning.

Cobb and Yackel's (1996) Interpretive Framework

The framework consists of two interrelated perspectives; the social perspective and the psychological perspective. The two perspective allows for a coordinated analysis of collective classroom processes (social perspective) and the individual learners' activity (psychological perspective) as they participated in, and contributed to, the development of these collective processes. This framework views

Table 1. An interpretive framework for analyzing individual & collective activity at classroom level (Cobb & Yackel, 1996, p. 177)

Social perspective	Psychological perspective
Classroom social norms	Beliefs about own role, others' role, & general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs & values
Classroom mathematical practices	Mathematical conceptions & activity

learning as a "process that occurs while participating in, and contributing to, the practices of the local community" (Cobb & Yackel, 1996, p. 185). Furthermore, it views communication as a "process of mutual adaptations that gives rise to shifts of meaning as the teacher and students coordinate their individual activities in the process of constituting the practices of the classroom activity" (Cobb & Yackel, 1996, p. 186). This makes an indirect link between the individual and the social because participation can enable and constrain learning but does not determine learning. But with this approach the unit of analysis is a local community such as a classroom. A teacher's role is therefore not only to proactively support learners' individual constructions but also to proactively support the evolution of mathematical practices. According to Cobb and Yackel (1996), this will enable learners to gradually participate effectively in mathematical practices of wider society.

Table 1 presents this framework that explicitly coordinates two distinct viewpoints on classroom activity. There is an assumed relationship between each construct in the social perspective and the corresponding construct in the psychological perspective. For example, looking at sociomathematical norms raw means that a teacher who initiates and guides the (re-)negotiation of sociomathematical norms is simultaneously supporting the individual learners' (re-)organization of the mathematical beliefs and values (Cobb & Yackel, 1996).

Cobb and Yackel (1996) emphasized that neither the sociomathematical norms nor individual learners' mathematical beliefs and values are given primacy over the other. But should be seen as reflexive and, as a result, cannot exist independently from each other. Therefore, it is neither a case of a change in sociomathematical norms causing a change in individual learners' mathematical beliefs and values nor a case of learners first reorganizing their mathematical beliefs and values and then contributing to the transformation of sociomathematical norms. During analysis we constructed arguments about challenging and transforming existing sociomathematical norms considering the social perspective of the emergent approach as this approach deals with patterns of participation. The psychological perspective, on the other hand, was considered when we accounted for how challenging and transforming existing sociomathematical norms enhance learners' mathematical proficiency.

Kilpatrick et al.'s (2001) Mathematical Proficiency

Kilpatrick et al. (2001) pioneered the notion of mathematical proficiency to denote a disposition demonstrated by people as they do mathematics (Altarawneh & Marei, 2021).

Kilpatrick et al. (2001) describe mathematical proficiency using five interdependent and intertwined strands as *conceptual understanding*, *procedural fluency*, *strategic competence*, *adaptive reasoning*, and *productive disposition.*

The description of each strand captures a range of learners' abilities, which we looked for during analysis because in this paper, a norm is constituted when the response given and agreed on also qualifies as one of the proficiencies.

Conceptual understanding refers to "an integrated and functional grasp of mathematical ideas. … [students organize] their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know" (Kilpatrick et al., 2001, p. 118). Conceptual understanding is achieved when learners are able to

(i) see mathematics as a connected web of concepts,

(ii) explain the relationship between different concepts and make links between concepts and related procedures, and

(iii) apply ideas and justify their thinking (DBE, 2018).

Procedural fluency is defined as "a skill in carrying out procedures flexibly, accurately, efficiently and appropriately" (Kilpatrick et al., 2001, p. 116). Hence, procedural fluency is achieved when learners are able to

(i) carryout mathematical procedures accurately and efficiently and

(ii) know when to use a particular procedure (DBE, 2018).

Strategic competence is defined as the "ability to formulate, represent, and solve mathematical problems" (Kilpatrick et al., 2001, p. 116). Competence in this strand is demonstrated when learners are able to

(i) identify and use appropriate strategies and

(ii) devise their own strategies to solve mathematical problems (DBE, 2018).

Adaptive reasoning is regarded as the "capacity for logical thought, reflection, explanation and justification" (Kilpatrick et al., 2001, p. 116). This strand, which involves both inductive and deductive reasoning, is achieved when learners are able to

(i) explain and justify their mathematical ideas and

(ii) communicate their mathematical ideas through appropriate mathematical language and symbols (DBE, 2018).

Productive disposition refers to "a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick et al., 2001, p. 116).

RESEARCH METHODOLOGY

In this paper we use parts of qualitative data from the first author's doctoral study (Mokwana, 2021) to explore how transforming existing sociomathematical norms could enhance mathematical proficiency. As in the main study, a qualitative case study design was adopted. It is a research design that involves an intensive, holistic, descriptive, and analytical investigation of a specific phenomenon or social unit (Merriam, 1998). The case study approach was deemed relevant, given our focus on sociomathematical norms – a contemporary phenomenon deeply rooted in learners' actions during mathematics learning. This design allowed us to provide rich and thick descriptions, contributing to a better understanding of how sociomathematical norms could enhance mathematics learning.

We employed purposive sampling, a non-random sampling method that allowed us to select information-rich cases for an in-depth exploration (Cohen et al., 2000). It allowed us to discover, understand and gain more insight into crucial issues embedded in classroom interactions. The participants consisted of 23 grade 11 mathematics learners in an urban school located in the Pietersburg Circuit of the Capricorn South District in Limpopo Province, South Africa. They were all in one class allocated to the first author to teach over a period of 12 weeks. We targeted excerpts that focused on learners' mathematical discourse on analytical geometry. The section was taught through two learning activities over two weeks in 12 periods of 45 minutes each. Data presented and analyzed in this paper emanates from one question of learning activity 1, which consisted of five items. The question read as captured in **Figure 1**.

Data collection took place during normal teaching and learning lessons. There were four groups with the number of members ranging from four to five. All groups consisted of members of mixed gender and abilities. Video recording was used, and it enabled a moment-by-moment capturing of unfolding sounds and sights. To ensure comprehensive coverage, two video cameras were deployed–one at a fixed point to record the entire class, and another mobile one to record close-range interactions among groups of learners. Learners' responses documented in their workbooks or on the chalkboard during the lessons were also collected as supplementary data.

Although in this paper the analysis focused on only one question with five sub-items of the learning activity, the excerpts are representative of what transpired throughout the activity. Narrative analysis, as conceptualized by Polkinghorne (1995) and defined by Kim (2015) as "an act of finding narrative meaning" (p. 190), was adopted. The analysis unfolded in two stages. The first stage, narrative analysis, focused on identifying data that revealed the uniqueness, idiosyncrasy, and complexity of the bounded system or individual. Instead of isolating data, we synthesized them into coherent developmental accounts, linking emerging themes during data sorting to provide a comprehensive understanding. The second stage, analysis of the narratives, involved examining common themes within the stories that emerged from the data and organizing them into excerpts.

To ensure ethical integrity, the study adhered to several principles, including informed consent, informed assent, confidentiality, data protection, and consideration of the dual role of the teacher-researcher.

Since the study involved minors, informed consent was sought and granted by their parents and/or guardians. Additionally, the learners were afforded to make an informed assent to confirm their agreement to participate voluntarily. Confidentiality was accorded by maintaining anonymity in reporting and data was stored electronically and accessible to only the researchers. Since the first author was the teacher and the researcher at the same time, measures were put in place to ensure that the research agenda did not interfere with the normal teaching of the learners. Hence, lesson preparation followed the annual teaching plan provided by DBE (2018).

ANALYSIS, INTERPRETATION, & DISCUSSION

We present the analysis in three parts. Firstly, we focus on how the teacher negotiated entry into learners' existing sociomathematical norms. Secondly, we focus on how the learners' existing sociomathematical norms were disrupted. In particular, we trace how the teacher confronted learners' existing sociomathematical norms and how that, in turn, created opportunities for negotiating new norms. Thirdly, we focus on how learners were challenged to change from what was normative to them and jointly constitute new norms, which could be considered as taken-as-shared for the classroom. This could be taken as the shedding off authority by the teacher as learners authored new sociomathematical norms. Throughout the three parts, we scrutinized whether there were opportunities

1. $L(-1;3)$, $M(7;1)$ and $N(x;2)$ are points in a Cartesian plane. Calculate x if
   1.1. the gradient of MN is $\frac{1}{2}$.
   1.2. N is the midpoint of LM.
   1.3. the length of MN is $\sqrt{2}$ units.
   1.4. line MN is perpendicular to the x-axis.
   1.5. L, M, and N lies on the same straight line.

Figure 1. Question 1 of analytical geometry learning activity 1 (Mokwana, 2021, p. 94)

created, used and/or missed for enhancing learners' mathematical proficiency.

Negotiating Entry into Learners' Existing Sociomathematical Norms

After the learners had formed their groups and had settled, there was total silence. Learners started attempting the activity individually, and two different entry points into the solutions for item 1.1. of the learning activity were observed in one of the groups. One learner started off by writing the general equation of a straight line '$y = mx + c$', while another one used the formula for calculating a gradient '$m = \frac{y_2-y_1}{x_2-x_1}$' as a starting point. Excerpt 1 captures the discussion, which unfolded in the group that consisted of four learners. At the beginning only two learners engaged in the discussion while the other two were merely listening but joined in at a later stage.

Excerpt 1

1.1. T: What is this equation that you have written, for?

1.2. L1: It is for the equation of a straight line.

1.3. T: Why do you want to use it?

1.4. L1: Because we are given the gradient and points on the line *MN*.

1.5. T: What do you intend to do with this information?

1.6. L1: I want to find the equation of the straight line.

1.7. L2: Why? What is the question?

[Pause]

1.8. L1: The question is that we must calculate the value of x.

1.9. T: So how do you intend to use the equation of the line that you are calculating in order to calculate the value x?

1.10. L1: Eish sir, I see I made a mistake, I must use the gradient formula because I am given gradient.

1.11. L2: Sir look at how I have done it (the attempt was as below), but I am stuck. My problem is that I do not know how to do this calculation going further but I know that the value of *x* is nine:

$$m_{MN} = \frac{y_2-y_1}{x_2-x_1} = \frac{2-1}{x-7} = \frac{1}{x-7}.$$

1.12. T: How do you know that the answer is nine?

1.13. L2: I think I guessed, so that is why I am asking you to help?

1.14. T: I hear that, just tell me how you guessed the nine?

1.15. L2: Okay sir, the gradient is $\frac{1}{2}$ so already in the numerator I have one (referring to $\frac{1}{x-7}$) and I need two in the denominator. So, I know that *9-7=2*. But sir I know I must show the steps, so I do not know how to do it.

1.16. T: L1, what do you think about her calculation?

1.17. L1: Sir, I was not paying attention because I wanted to do mine first so that I will be able to compare with her.

1.18. T: Take a look at her calculation and listen to her concerns then share your thoughts with us.

[Even though L1 was not done attempting to respond to the question he had already written his new incomplete working out as]:

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{2} = \frac{2 - 1}{x - 7}$$

1.19. L1: [Looking at learner 2's attempt] You did not substitute the gradient of *MN* by ½.

1.20. L2: Why should I do that because I am calculating it?

1.21. L1: You used the information given about *M* and *N*, but you did not use the ½ in your calculation, and the ½, which is the gradient is already given.

1.22. L2: Is he correct sir? [looking at the teacher].

This discussion (1.1-1.6) started with the teacher requesting L1 to explain and justify their choice of strategy or approach. The request was further emphasized by L2 who asked L1 two questions, the first question (why?) called for L1 to justify his approach. L1 did not respond to the why question. Instead, after re-examining the question, he abandoned his initial approach and opted to use the gradient formula (1.10). The latter was because, the second question (1.7) by L2 muted the need to justify, instead, called for L1 to revisit the question they were attempting, suggesting that they missed what was asked. Even though the teacher continued to ask how L1 would use the equation of the straight line to fulfil what was asked (1.9), L1 interpreted question posed as an additional reason for him to justify that his initial approach was not acceptable.

In reflecting on the discussion thus far, it could be argued that L2 asked why, without expecting a response hence she provided direction on what L1 was supposed to revisit. Her dispute to L1's attempt was framed by her own approach to the question, hence L2 indirectly told L1 to use gradient formula or to change his approach because she did not give him time to justify his approach. This argument is supported by how quickly L2 moved to her own attempt (1.11), without asking further questions, after L1 had indicated that he was going to use the gradient formula (1.9). This could be an indication that it was normative for them that their attempts must be the same, something typical for traditionally oriented classrooms.

In the same group, L2 presented her attempt (1.11) and alluded that the answer was nine but also claimed that she was stuck and needed help. When the teacher asked how she knew it was 9 (1.12), the explanation she offered (1.15) made mathematical sense but she was more concerned with the algorithm. Unfortunately for her, she regarded the process she had followed as guessing. However, in her explanation, *conceptual understanding* (Kilpatrick et al., 2001) is reflected, as she demonstrated an understanding of the equal sign as a mathematical operation and the concept of equivalence, in order to get to a correct answer. The conceptual explanation was, however, not sufficient for her. One would have thought that the omission of substitution of $\frac{1}{2}$ could be the reason. However, this was not the case per se as, even after L1 had indicated that she had omitted that (1.16-1.22), she still wanted to have an algorithm representative of her explanation systematically (in a step-by-step fashion). It became apparent that, for L2, a systematic algorithm seems to be what counted as an acceptable mathematical explanation.

Disrupting Learners' Existing Sociomathematical Norms

Observing other groups, we learned that all the groups attempted the question and obtained the correct answer. Generally, the learners substituted m_{MN} by $\frac{1}{2}$ and the coordinates of M and N, and then did a cross multiplication, which produced a linear equation, which they solved and got nine as the answer. At this point, the teacher engaged learners in a whole class discussion based on L2 conceptual explanation. The class discussion, which was a bit rowdy, unfolded as captured in excerpt 2, afforded us an opportunity to establish whether or not L2's explanation was acceptable to the entire class.

Excerpt 2

2.1. T: Ok class, how did you respond to question 1.1. of the learning activity?

[There was noise for some time as learners concluded their discussions and the teacher was also repeating the question.]

[After some time …]

2.2. L3: We used the gradient formula, substituted the given coordinates of *M* and *N*, and the gradient of *MN* by $\frac{1}{2}$.

2.3. T: [Wrote what the learner said on the board and as well as the final answer as below]

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{2} = \frac{2-1}{x-7}$$

$$x = 9$$

2.4. Ls: Where did you get the nine? … You must show all steps. Do a reverse calculation from $x = 9$ and take us back, where we started.

2.5. T: We expect the same fractions on both sides of the equal sign, right?

2.6. Ls: Yes.

2.7. T: Why is that the case?

2.8. L5: Because in step 2 we have two fractions with an equal sign meaning that they are equal.

2.9. T: Okay, good! Can we all see that the numerator one on both sides?

2.10. Ls: Yes sir.

2.11. T: In order to get equal fractions, what should the denominator be?

2.12. Ls: Two.

2.13. T: Good! I have a certain number (pointing at x), which I do not know but know that when I subtract seven from it (pointing at $x - 7$) the answer must be two. What is that number?

2.14. Ls: [Reluctantly] Nine [with others arguing that it is still guessing].

2.15. L4: But sir will they give us all the marks even if we did not show all the steps?

2.16. T: Let us go back a little bit (The teacher wrote $\square$-7=2 on the board)

[Some learners started laughing and mumbling]

2.17. T: What would you write in that box while in primary school, say grade 1?

2.18. Ls: Nine [with others laughing].

2.19. T: Is that guessing?

[Some learners were saying yes with some saying no, while others saying but we are in secondary school now.]

2.20. T: Then how $\square$-7=2 different from $x - 7 = 2$?

2.21. L2: No sir, actually it is the same thing, just expressed differently.

2.22. L4: But sir still we must show all the steps.

[The class shouted yes]

2.23. T: Why? Since it is clear, where our answer comes from?

2.24. Ls: For marks.

2.25. T: Okay, let us prolong the calculation just like most of you did then,

$$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{1}{2} = \frac{2-1}{x-7}$$

$$\frac{1}{2} = \frac{1}{x-7}$$

$$x - 7 = 2$$

$$x = 2 + 7$$

$$x = 9$$

How many marks do you think this calculation is worth?

2.26. Ls: Three marks … two marks.

2.27. T: Let us look at how marks will be allocated if it worth three marks

$m_{MN} = \frac{y_2-y_1}{x_2-x_1}$ ✓ for the formula

$\frac{1}{2} = \frac{2-1}{x-7}$ ✓ for correct substitution

$\frac{1}{2} = \frac{1}{x-7}$

$x - 7 = 2$

$x = 2 + 7$

$x = 9$ ✓ for the answer

2.28. T: Can anyone explain how $x - 7 = 2$ was obtained?

2.29. L4: We did a cross multiplication.

2.30. T: What if I said we equated the denominators?

[Silence]

2.31. L2: That will be correct still, I guess.

2.32. L4: But sir can we at any time equate the denominators or numerators?

2.33. T: I do not know. What do you think?

2.34. L4: Aah! But sir you are the teacher, so you must know.

2.35. T: Tell me what you think first. What do others think?

[Silence]

2.36. T: Can we at any given time equate the numerators or denominators?

[Silence]

2.37. L2: I think in this case we equated them because we are sure that the numerators are equal.

2.38. L3: I agree, if the numerators are equal then we can equate denominators, if the denominators are equal then we can equate the …

2.39. L4: But why?

[Silence]

2.40. L2: Because we are sure that the two fractions are equal, is not we wrote an equal sign between them?

2.41. L4: Sir is that correct?

2.42. T: Do you agree with them or not?

2.43. L4: But sir … [mumbling]

2.44. T: Okay guys let's continue working on the rest of the questions.

The teacher's writing of L3's response (2.2) on the board, and the final answer (2.3) was a deliberate action to disrupt what, at the time, seemed to be the learners' sociomathematical norm of what counts as an acceptable mathematical explanation. The learners' reaction to this disruption (2.4) confirmed that, they felt it was normative to present solutions in a way that steps are linked algorithmically, and not conceptually. This was evident as the learners put forward their demands for the attempt to be acceptable. They wanted all steps to be made explicit, while other learners wanted a reverse calculation to be done. These two demands were reflective of the learners' sociomathematical norms in relation to acceptable mathematical justifications. This was the case as, not only did the learners ask, where the teacher got the nine (2.4), but also provided ways in which they expected the justification.

The first expectation, that of showing all steps, could be aligned to the mathematical proficiency strand of *procedural fluency* (Kilpatrick et al., 2001). Unfortunately for this class, they separated procedures from understanding, an action contrary to Kilpatrick et al.'s (2001) emphasis on the intertwined nature of the strand. Thus, these learners demonstrated *restricted procedural fluency* (Graven & Stott, 2012). In unpacking *procedural fluency*, Kilpatrick et al. (2001) also made mention of procedures carried out efficiently and thus achieving maximum production with minimum wasted effort. The learners' preferred way of working out (2.25) was clearly not an efficient action. Therefore, this norm did not enhance learners' mathematical proficiency.

The second expectation, that of doing a reverse calculation, was concerning and, yet again, revealed the learners' lack of understanding required when doing mathematics. This approach is generally used during the teaching of factorization, where finding the product is the reverse process. Although prone to creating misconceptions, this thinking is prevalent in teaching of mathematics. Other examples include multiplication and division, addition and subtraction, differentiating and integrating, exponents and surds, just to mention a few. However, if over generalized and applied without understanding they mislead learners, and it seemed even in this case, learners over generalized this idea of reverse calculation. This could be regarded as *instrumental understanding* (Skemp, 1976). Justification of answers has to do with adaptive reasoning, and, in this case, the learners preferred approach to justification lacked logical thought (Kilpatrick et al., 2001).

Through a question-and-answer type of discourse (2.5-2.22), the teacher attempted to provide a conceptual justification for the answer. This was an attempt to start a negotiation with learners in order for them to reconsider their normative stance on what counted as acceptable mathematical explanations. However, this was blandly rejected by the learners (2.22) and the majority of them supported that rejection. When probed why they insisted on taking long steps (2.23), they indicated that it was for marks (2.24). That response was indicative of a consensus reached that there was no guessing involved in getting to the answer. Furthermore, the solution, together with its explanation and justification, was acceptable. Learners' concerns about marks, was addressed as the teacher demonstrated that the other steps, which they insisted should be there, were not worth any marks (2.26), and they gave up on the argument. This did not imply they had accepted that long steps were not necessarily important.

As noted in the discussions in except 1 and excerpt 2, there were several opportunities for transforming sociomathematical norms to enhance learners' mathematical proficiency. In excerpt 1, these opportunities were not fully explored as the idea was to negotiate an entry into the learners' existing sociomathematical norms. There were two classroom sociomathematical norms, which were at play during the discussion, namely, what counts as an acceptable mathematical explanation and justification. In excerpt 1, although, the teacher was trying to understand these norms from the learners' perspective, an inconclusive negotiation (transformation) of these norms took place. One of the traits of mathematical proficiency is the ability to translate between various forms of mathematical representation (NCTM, 2000). In particular, in excerpt 2, the discussion yielded the mathematical idea presented as words (2.13), symbols (2.16), and algebraically (2.20). This multidimensional way of looking at the same mathematical idea involves the capacity for logical thought and reflection, which are steeped in *adaptive reasoning* (Kilpatrick et al., 2001).

The learners engaged in reflecting on what was under discussion. Throughout their discussion (2.32-2.38) the learners attempted to make sense and navigate a way to generalize their argument. L4's question (2.32), was actually inquiring whether the argument could be applied to any equation involving fractions. The question (2.32), if explored, could have opened new arguments, which would have assisted learners to develop a better understanding of fractions, equivalence and number sense, in particular. For example, in assisting L4, the teacher could have reverted to the equation; $\frac{1}{2}=\frac{x}{x-7}$ and asked if we could at any time equate the denominators or numerators? If L4 was to follow his argument, then two different values of x would be obtained from the numerator and the denominator. Through mediation, the argument could have been steered to $x=-7$, then the right side of the equation would be $\frac{-7}{-14}$. This would have prompted a new dimension into the discussion, as learners would have been exposed to yet another opportunity to think about the equal sign to mean 'equal' and not necessarily 'same'. These forms of engagement contribute much to attainment of relational understanding (Skemp, 1976).

Constituting 'New' Sociomathematical Norms

The focus is on instances, where learners' were challenged to deviate from their usual sociomathematical norms due to the nature of the mathematical problem at hand. As much as learners have their own acceptable ways of presenting, explaining and justifying their solutions, they could be confronted with problems that create a dilemma for them, of not conforming to their norms. In such cases, although the teacher might offer scaffolds to help learners access the mathematics, he could choose not to put forward the sociomathematical norms as rules to be followed but instead allow them to be interactively constituted as learners engaged in mathematical talk.

Excerpt 3 captures the discussion based on item 1.4., which first began in one of the groups and later drawn to the attention of the whole class because they all struggled with a similar issue. The question required that they calculate x if line MN is perpendicular to the x-axis, given the coordinates as $M(7;1)$ and $N(x;2)$. In each of the four groups there was at least one learner who wrote '$m_1 \times m_2=-1$', however, the learners could not proceed with the calculation. This way of approaching questions was also observed in excerpt 1. For these learners, it appeared that their choice of procedures to follow was not informed by the conception of what was given. Instead, they seemed to have memorized facts about mathematical relations and associated such facts in terms of formulae.

Excerpt 3

3.1. L9: Sir the information given is incomplete for us to answer this question.

3.2. T: Okay, tell us why you are saying so.

3.3. L9: You see sir if we use the formula $m_1 \times m_2 = -1$, we will use coordinates of *M* and *N* for gradient one, so for gradient two, which is for the x-axis we must be given either coordinates of some points there or the actual gradient.

3.4. T: What could be the other way to answer without using that formula?

3.5. L12: No sir, there is no other way, if they say lines are perpendicular, we must use that formula. If they are parallel, we equate the gradients.

3.6. L9: That is why I am saying the information given is not enough.

3.7. T: So you want coordinates of points on the x-axis or the gradient of the x-axis?

3.8. L9: Yes sir.

3.9. T: Draw a Cartesian plane and indicate x-axis as well y-axis, check if you cannot get the coordinates that you want.

[Later]

3.10. L9: We cannot just pick points on the x-axis because we do not know if *MN* is perpendicular to x-axis on the negative or positive side (pointing on her Cartesian plane).

3.11. L11: But we can use any points to calculate the gradient as it is the same throughout the x-axis.

3.12. L9: Okay let's take (2; 0) and (3; 0), ahh the gradient will be zero. If we substitute in the formula then everything is going to be zero.

3.13. L12: Then we can say no solution because we will be getting 0=-1, which is impossible.

3.14. T: No, there is a way to answer that question, forget the formula a little bit.

3.15. L9: Tell us what to do, at least give us a hint then.

3.16. T: No, talk about it and find a way as a group.

[Later]

3.17. L9: Sir, we got it (excited).

3.18. T: That is good let me see your calculation.

3.19. L9: There is no calculation, but we understand it and can explain it, is there any group, which got it?

3.20. T: Not as yet.

3.21. L9: Can I explain it in front to everyone?

3.22. T: Yes, go ahead.

3.23. L9: Guys, guys listen I want to explain 1.4. to you.

[Few seconds later …]

3.24. L9: The question says, we must calculate x if line *MN* is perpendicular to the x-axis. So, we cannot use the formula for gradients of perpendicular lines because we are given information about only one line. Then, in our group we drew a Cartesian plane (drawing it on the board), look at the x-axis and realized it has a gradient of zero, see, it is a reason why we cannot use the formula?

3.25. Ls: No [shouting].

3.26. L9: Check if you substitute by zero on $m_1 \times m_2 = -1$, then you will be left with 0=-1. So, what we did was to try plot the points *M* and *N* on the Cartesian plane. It was easy to plot *M*(7; 1) because we know the exact point. So, if we draw a line passing through *M* such that it is perpendicular to the x-axis, it is like a vertical line (drawing on the board). On this line the value of x at any point is seven, so even, where y is two, the value x is seven.

3.27. L4: So if it was asked in the test what would you write, if they said show all calculations?

3.28. L9: I will just write $x = 7$, because there is no calculation that I can do here.

3.29. L4: What if you just guessed or copied the answer from another person?

3.30. L12: Then you will write an explanation that a line perpendicular to the x-axis is a vertical line and has a constant x value throughout.

When the learners claimed that the information provided was insufficient for the given question (3.1), the teacher asked them why they thought so (3.2). This question required a justification for their claim. Interestingly, the learners provided a procedural justification (3.3). That is, their reason focused on how the formula (or procedure) they chose could not be used on the basis of the information at their disposal. If learners possessed relational understanding (Skemp, 1976), their choice of procedure must have been informed by their understanding of the question and the mathematical concepts involved. Furthermore, according to South African curriculum and assessment policy statement, learners in grade 11 are expected to be able to identify appropriate strategies (DBE, 2011). In this instance, the learners failed to identify an appropriate strategy, which could be an indication that they lacked the *strategic competence* to do so (Kilpatrick et al., 2001).

If an appropriate strategy was not readily available to demonstrate *strategic competence*, learners were expected to devise their own strategies (DBE, 2018). Such abilities required that they had both the understanding of the concepts and the ability to carry out the procedures. Devising own strategies appeared to be something foreign to these learners, as everyone expected the teacher to provide them with a strategy (3.15). Instead, the teacher offered the learners assistance through leading questions (3.4, 3.7, and 3.9). Learners engaged in a discourse that allowed for various mathematical concepts and arguments to be reflected upon (3.10-3.15). Through back-and-forth discussion, learners grappled with devising their own strategy but mostly they fell back to their procedural reasoning (3.12-3.13). Finally, they broke through and arrived at the answer. To corroborate their existing norms, the teacher requested to see their calculation (3.18). Instead, they had no step-by-step calculation but an understandable explanation (3.19). This emerged as the first sociomathematical norm constituted by this group and immediately they wanted to share and compare their attempt with the whole class (3.21).

In presenting the group's attempt, L9 started by justifying why they abandoned an endorsed norm (3.24), a justification, which the class initially rejected (3.25). He then demonstrated how their usual approach could not work and presented how his group approached the question (3.26). Although most learners seemed to be receptive to the explanation, there was an undertone of dissatisfaction, which L4's question (3.27) confirmed. However, L12 justified the explanation and addressed how the attempt could be represented in text. This represented, yet again, another shift from previously endorsed norms to authoring new norms. Initially, learners wanted solutions to be presented in a step-by-step manner for the purposes of marking. Inherent L4's questions (3.27 and 3.29), it became clear that focus was no longer on marks but on presenting solutions textually and in a way that made sense to the reader. This was another sociomathematical norm constituted by the learners, that acceptable mathematical explanations must be understandable when presented textually.

With regard to enhancement of learners' mathematical proficiency, which is the core reason for engaging in mathematical classroom discourse, two opportunities were created. Firstly, learners had to strive towards *strategic competence*, in this case, by not choosing an appropriate procedure, instead, by devising a new one. Secondly, the learners had to engage in *adaptive reasoning*. This was done through navigating multiple representations, and by proving and disproving certain procedures. Implicitly, learners learned the limitations of first choosing a formula based on certain phrases or bits of information given, instead of examining all the given information.

CONCLUSIONS

In this paper we explored how transforming existing sociomathematical norms enhance learners' mathematical proficiency. In most mathematics classrooms, sociomathematical norms involve acceptable mathematical explanations and justification that are limited to algorithm as well as back-and-forth manipulation of equations and expressions. Additionally, learners are expected to present same attempts to solutions and as a result, in most cases, there is an absence of sociomathematical norms associated with mathematically different solutions. These practices limit learners' engagement with meaningful mathematics learning and development of mathematical proficiency. However, we found that transforming existing sociomathematical norms to enhance learners' mathematical proficiency involved a three-stages process: negotiating entry into learners' existing sociomathematical norms, disrupting learners' existing sociomathematical norms and constituting 'new' sociomathematical norms. As learners developed new taken-as-shared meanings regarding acceptable mathematics explanations, justifications and mathematically different solutions they enhanced their *conceptual understanding, procedural fluency, adaptive reasoning*, and *strategic competence*.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that the study obtained ethical clearance certificate TREC/368/2017: PG from Turfloop Research and Ethics Committee at the University of Limpopo. Additionally, permission to conduct this study was sought and granted by the Limpopo Provincial Department of Education. Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Abdulhamid, L. (2016). *Primary mathematics in-service teaching development: Elaborating 'in-the-moment'* [Doctoral dissertation, Witwatersrand University].

Altarawneh, A. F., & Marei, S. I. T. (2021). Mathematical proficiency and preservice classroom teachers' instructional performance. *International Journal of Education and Practice, 9*(2), 354-364. https://doi.org/10.18488/journal.61.2021.92.354.364

Apsari, R. A., Sripatmi, S., Putri, R. I. I., Hayati, L., & Sariyasa, S. (2020). From less to more sophisticated solutions: A socio-mathematical norms to develop students' self-efficacy. In *Proceedings of the 1st*

Annual Conference on Education and Social Sciences (pp. 286-290). https://doi.org/10.2991/assehr.k.200827.072

Baki, G. O., & Kilicoglu, E. (2023). Social and socio-mathematical norms constructed by teachers in classes through the development of noticing skills. *International Electronic Journal of Mathematics Education, 18*(1), em0723. https://doi.org/10.29333/iejme/12649

Chuene, K. M. (2011). Socio and socio-mathematical norms: Traits of interaction in a first year undergraduate mathematics talking classroom. *South African Journal of Higher Education, 25*(8), 1497-1512.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*(3-4), 175-190. https://doi.org/10.1080/00461520.1996.9653265

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2010). Participating in classroom mathematical practices. In A.a Sfard, K. Gravemeijer, & E. Yackel (Eds.), *A journey in mathematics education research* (pp. 117-163). Springer. https://doi.org/10.1007/978-90-481-9729-3_9

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education*. Routledge.

DBE. (2011). *Curriculum and assessment policy statement grade 10-12 mathematics*. Department of Basic Education.

DBE. (2018). *Mathematics teaching and learning framework for South African*. Department of Basic Education.

Dini, D. F. R., & Maarif, S. (2022). Students' problem-solving ability: An analytical practice viewed from the aspect of socio-mathematical norm. *Desimal: Jurnal Matematika* [*Decimal: Journal of Mathematics*], *5*(1), 91-102. https://doi.org/10.24042/djm.v5i1.11491

Graven, M., & Stott, D. (2012). Conceptualizing procedural fluency as a spectrum of proficiency. In *Proceedings of the 18th Annual National Congress of the Association for Mathematical Education of South Africa* (pp. 146-156). North-West University.

Gülburnu, M., & Gürbüz, R. (2022). Investigation of effects of negotiations of socio-mathematical norms on mathematical process skills. *The Journal of Educational Research, 115*(2), 161-172. https://doi.org/10.1080/00220671.2022.2074949

Gülburnu, M., & Gürbüz, R. (2023). An investigation of socio-mathematical norms perceived by students regarding the legitimacy of solutions. *South African Journal of Education, 43*(3). https://doi.org/10.15700/saje.v43n3a2215

Güven, N. D., & Dede, Y. (2017). Examining social and socio-mathematical norms in different classroom microcultures: Mathematics teacher education perspective. *Educational Sciences: Theory & Practice, 17*(1), 265-292.

Kang, S. M., & Kim, M. K. (2016). Socio-mathematical norms and the teacher's mathematical belief: A case study from a Korean in-service elementary teacher. *EURASIA Journal of Mathematics, Science and Technology Education, 12*(10), 2733-2751. https://doi.org/10.12973/eurasia.2016.1308a

Kilpatrick, J., Swafford, J., & Findell, B. (2001). The strands of mathematical proficiency. In *Adding it up: Helping children learn mathematics* (pp. 115-118). National Academies Press. https://doi.org/10.17226/9822

Kim, J. H. (2015). *Understanding narrative inquiry: The crafting and analysis of stories as research*. SAGE. https://doi.org/10.4135/9781071802861

McClain, K., & Cobb, P. (2001). An analysis of development of socio-mathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education, 32*(3), 236-266. https://doi.org/10.2307/749827

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. Jossey-Bass.

Mokwana, L. L. (2021). *Sociomathematical constituted for promoting learners' proficiency in mathematics* [Doctoral Thesis, University of Limpopo, Polokwane, South Africa]. http://ulspace.ul.ac.za/handle/10386/4438

Morrison, S., Venkat, H., & Askew, M. (2021). Journeys towards socio-mathematical norms in the foundation phase. *South African Journal of Childhood Education, 11*(1), a927. https://doi.org/10.4102/sajce.v11i1.927

NCTM. (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Partanen, A. M., & Kaasila, R. (2015). Socio-mathematical norms negotiated in the discussions of two small groups investigating calculus. *International Journal of Science and Mathematics Education, 13*(4), 927-946. https://doi.org/10.1007/s10763-014-9521-5

Pi, J., Davis, C., Baki, Y., & Pantano, A. (2023). Emphasizing socio-mathematical norms in introductory proof-writing courses. *arXiv*. https://doi.org/10.48550/arXiv.2312.14332

Polkinghorne, D. E. (1995). Narrative configuration in qualitative analysis. *International Journal of Qualitative Studies in Education, 8*(1), 5-23. https://doi.org/10.1080/0951839950080103

Robertson, S. A., & Graven, M. (2019). Exploratory mathematics talk in a second language: A sociolinguistic perspective. *Educational Studies in Mathematics, 101*(2), 215-232. https://doi.org/10.1007/s10649-018-9840-5

Roy, G. J., Tobias, J. M., Safi, F., & Dixon, J. K. (2014). Sustaining social and socio-mathematical norms with prospective elementary teachers in a mathematics content course. *Investigations in Mathematics Learning, 7*(2), 33-64. https://doi.org/10.1080/24727466.2014.11790341

Sánchez, V., & García, M. (2014). Socio-mathematical and mathematical norms related to definition in pre-service primary teachers' discourse. *Educational Studies in Mathematics, 85*(2), 305-320. https://doi.org/10.1007/s10649-013-9516-0

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*(1), 20-26. https://doi.org/10.5951/at.26.3.0009

Van Zoest, L. R., Stockero, S. L., & Taylor, C. E. (2012). The durability of professional and socio-mathematical norms intentionally fostered in an early pedagogy course. *Journal of Mathematics Teacher Education, 15*(4), 293-315. https://doi.org/10.1007/s10857-011-9183-y

Wester, R., Wernberg, A., & Meaney, T. (2015). Students' perceptions of Norms in a reformed classroom. In *Proceedings of the 9th Congress of the European Society for Research in Mathematics Education* (pp. 3150-3156).

Widjaja, W. (2012). Exercising socio-mathematical norms in classroom discourse about data representation: Insights from one case study of a grade 6 lesson in Indonesia. *Mathematics Educator, 13*(2), 21-38.

Wood, M. B. (2016). Rituals and right answers: Barriers and supports to autonomous activity. *Educational Studies in Mathematics, 91*(3), 327-348. https://doi.org/10.1007/s10649-015-9653-8

Yackel, E. (2001). Explanation, justification and argumentation in mathematics classrooms. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (pp. 9-24). Freudenthal Institute.

Yackel, E., & Cobb, P. (1996). Socio-mathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*, 458-477. https://doi.org/10.5951/jresematheduc.27.4.0458

Yackel, E., Rasmussen, C., & King, K. (2000). Social and socio-mathematical norms in an advanced undergraduate mathematics course. *The Journal of Mathematical Behavior, 19*(3), 275-287. https://doi.org/10.1016/s0732-3123(00)00051-1

Yenmez, A. A., & Erbaş, A. K. (2023). Facilitating a sustainable transformation of socio-mathematical norms through mathematical modeling activities. *International Journal of Science and Mathematics Education, 21*(3), 761-785. https://doi.org/10.1007/s10763-022-10275-5

Zembat, I. O., & Yasa, S. A. (2015). Using classroom scenarios to reveal mathematics teachers' understanding of socio-mathematical norms. *International Journal of Education in Mathematics, Science and Technology, 3*(3), 242-261. https://doi.org/10.18404/ijemst.34754