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Understanding Prospective Teachers' Mathematical Modeling Processes in the Context of a Mathematical Modeling Course

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ABSTRACT
This paper investigates how prospective teachers develop mathematical models while they engage in modeling tasks. The study was conducted in an undergraduate elective course aiming to improve prospective teachers' mathematical modeling abilities, while enhancing their pedagogical knowledge for the integrating of modeling tasks into their future classroom practices. The participants of this study were six prospective teachers selected among the nineteen who enrolled in the course. Data was collected through five modeling tasks. The results showed that the prospective teachers went through five main stages during the modeling process: understanding the task, devising a solution plan, working out the plan, interpreting and verifying the model, and presenting the model. The nature of prospective teachers' modeling processes was rather result-oriented, consisting of a single cycle as they did not seek an improved solution by revising and refining their models.
Keywords: mathematical modeling; mathematics education; modeling process; prospective teachers

INTRODUCTION

Research in mathematics education emphasizes the role and importance of modeling tasks in students' interest and their attitude towards mathematics, reasoning, communication, and problem solving (Bracke & Geiger, 2011; Carlson, Larsen, & Lesh, 2003; Kaiser, Schwarz, & Buchholtz, 2011). While engaging in modeling tasks, students try to mathematize meaningful

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State of the literature

- Researchers agree that mathematical modeling should be the important part of mathematics education. However, teachers' lack of knowledge on mathematical modeling and inexperience with modeling activities are the main shortcoming of using modeling activities in the classrooms.
- Researchers argued that modeling is learned and developed through practice with modeling tasks. Thus, modeling should also be part of mathematics teacher education programs.
- To be able to develop and improve courses to teach mathematical modeling skills and to provide support, there is a need for research to investigate how prospective teachers work on the modeling activities and how they would develop mathematical models while they engage in modeling activities.

Contribution of this paper to the literature

- The results revealed that the prospective teachers' modeling processes consist of five stages: understanding the task, devising a solution plan, performing the plan, interpreting and verifying the model, and presenting the model.
- Working on the modeling tasks with a result-oriented focus, the prospective teachers went through a single-cycle modeling process and did not seek a better model by reflecting on and improving their models.
- When faced with difficulty at a point in the solution process, the prospective teachers sometimes ignored the most relevant or main variables, or made assumptions that supported their intuitive answers to be able to reach a solution; in other cases, they considered their solutions as valid, with no need for further verification.

situations by organizing, selecting, sorting, or quantifying information in order to reach a solution to realistically complex problems (English & Sriraman, 2010; Lesh & Doerr, 2003). While engaging in modeling tasks, students see the need for mathematization and the need to use mathematical concepts together (Lesh & Doerr, 2003; Lesh & Zawojewski, 2007).

As students engage in multiple modeling cycles involving testing, refining, and revising their current ways of thinking, they develop or rebuild their current understanding of mathematical concepts (English, 2006; Johnson & Lesh, 2003). In the process of solving modeling problems, students develop both their communication skills and gain insight into the function and power of mathematics in order to understand and formulate problems from different subject areas (English & Sriraman, 2010; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007). Therefore, they improve their competency to solve real-life problems, in addition to developing their understanding of mathematical concepts (Blomhøj & Kjeldsen; 2006; English & Sriraman, 2010; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007).

In the broadest sense, mathematical modeling could be defined as a complex process including the application of mathematics to solve realistic problems (Berry & Houston, 1995; Brown, 2002; Edwards & Hamson, 1990; Maaß & Gurlitt, 2010; Schaap, Vos, Goedharhart, 2011; Verschaffel, et al., 2002). Processes involved in modeling are perceived differently depending on the research purpose, modeling view held, and modeling tasks used (Borromeo-Ferri, 2006; Kaiser & Sriraman, 2006). Even though they differ in terms of the number and names of the stages, various researchers have proposed that mathematical modeling is a cyclical process involving some basic steps: understanding, structuring and simplifying the problem situation, mathematizing, working mathematically, interpreting the mathematical results, validating and then improving the results (see Berry & Houston, 1995; Blum & Leiß, 2007; Galbraith, 2012; Lesh & Doerr, 2003 for further details about the modelling process). In general, as shown in **Figure 1**, the modeling process begins with the specification of the problem through the construction of a simplified and idealized version of the problem by identifying the critical components of the model and by making assumptions. Then, the problem is translated into a mathematical model consisting of variables and mathematical expressions depicting the relationships among the variables. The problem is then analyzed and solved in order to obtain mathematical results. The results are further interpreted in terms of the simplified real-world situation. Finally, the solution generated for the simplified version of the problem is verified in the context of the original situation with the aim of answering the problem originally presented. If the solution is found to be unsatisfactory, or did not yield an answer to the initial problem, the process starts all over again.

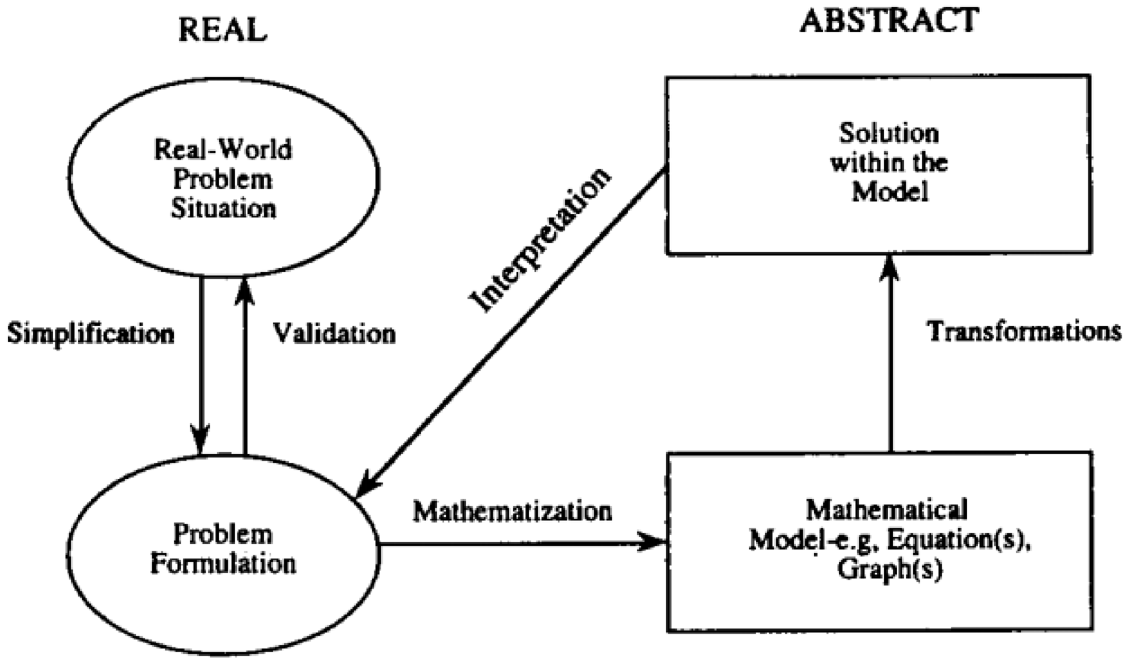

Figure 1. Mathematical modelling process (NCTM, 1989, p. 138)

According to Blum and Borromeo-Ferri (2009), in addition to representing the essential stages of modeling, frameworks describing the modeling process could also help to identify the cognitive barriers of students working on modeling problems. However, students have various difficulties while working on modeling tasks (Blum & Borromeo-Ferri, 2009; Galbraith & Stillman, 2006; Maaß, 2006, Soon, Lioe, & McInnes, 2011). Understanding and constructing, simplifying, and validating stages involved in the modeling process are particularly identified as problematic for both students and teachers (Berry, 2002; Blum & Borromeo-Ferri, 2009; Galbraith & Stillman, 2006; Sol, Gimenez, & Rosich, 2011). Students also have considerable difficulties in understanding and structuring the correct problem situation and making related assumptions in order to simplify the situation so as to make it workable (Schaap et al., 2011; Verschaffel, Greer, & De Corte, 2002). Accordingly, students have difficulty in moving from the real world to the mathematical world and do not usually check the reasonableness and appropriateness of their solutions (Berry, 2002; Sol et. al, 2011). Aiming to identify the difficulties and advances in mathematical modeling for in-service and prospective mathematics teachers, Biembengut and Faria (2011) reported that most of the participants faced difficulties not only in reading and understanding the modeling task, but also in remembering the mathematical knowledge needed to be able to solve it. Likewise, in investigating prospective teachers' professional knowledge of modeling competencies, Kaiser, Schwarz, and Tiedemann (2010) found that although prospective teachers (PTs) had knowledge of modeling to different degrees, most of them did not consider applying the validation phase. Similar findings reported by Julie and Mudaly (2007) who pointed out an absence of the processes of returning to the real problem situation and checking for the appropriateness of the proposed solution. Such research findings indicate that it is not only the students, but also the in-service and pre-service teachers who had difficulties with certain stages of the modeling process (i.e., understanding the problem, structuring and simplifying the problem, making appropriate assumptions, making connections between real-world problem situation and mathematical representation, interpreting the model in terms of real-life problem situations, and validating the obtained model).

Although various researchers suggest that school and university mathematics curricula should place more emphasis on modeling tasks in order to develop students' modeling skills and to solve real-life problems (e.g., Berry, 2002; Lesh & Doerr, 2003; Niss, Blum, Galbraith, 2007; Verschaffel, Greer, & De Corte, 2002), teachers' lack of knowledge and inexperience regarding mathematical modeling and its pedagogy are the main shortcomings when modeling tasks are integrated into the classroom (Antonius, et al., 2007; Biembengut & Faria, 2011; Blomhøj & Kjeldsen, 2006; Çiltaş & Işık, 2013). In modeling activities, although students themselves experience being an active participant in the modeling process, when students tackle a point in the modeling process they could not pursue, the teacher should identify their difficulties and ask them to justify their conjecture or other approaches to the solution (Galbraith, 2012). It is important and yet hard to maintain a balance between the students' independence, as emphasized in the modeling perspective, and the teacher's support and guidance (Borromeo-Ferri & Blum, 2010). Various studies document in-service and PTs' difficulties in solving modeling tasks (Biembengut & Faria, 2011; Julie & Mudaly, 2007; Kaiser, 2007), understanding different approaches to modeling (Borromeo-Ferri & Blum, 2010), and teaching (through) modeling (Lingefjärd & Holmquist, 2005; Stillman & Brown, 2011). Thus, various researchers have emphasized that teacher education programs should provide opportunities to learn and experience various dimensions of the modeling process

(Biembengut & Faria, 2011; Blum et al., 2002; Borromeo-Ferri & Blum, 2010; Burkhardt, 2011; Kuntze 2011; Maaß, 2006; Niss et al., 2007). Verschaffel et al. (2002) claim that the courses received during their training had the power to shape the PTs' disposition towards certain aspects of modeling. In particular, long-term engagement with modeling activities and the solving of real-world problems would offer significant opportunities for the professional development of PTs. (Berry & Houston, 1995; Burkhardt 2011; Maaß, 2006). PTs should therefore be more knowledgeable in the field of modeling and have actual first-hand experience with modeling activities (Blum et al., 2002; Niss et al., 2007; Swetz & Hartzler, 1991).

As to how PTs construct mathematical models when they engage in modeling tasks is an area that requires further research (Berry, 2002; Borromeo-Ferri, 2006), the purpose of this study is to investigate and document the basic thought processes of PTs when constructing mathematical models, while attempting to solve a variety of modeling tasks. In particular, we wanted to learn about PTs' thought processes involved in understanding the task, devising a plan for producing a solution, working out/implementing the plan, interpreting and verifying the model they constructed, and presenting their models. The following research question guided the study:

How do prospective teachers construct mathematical models when they engage in modeling tasks in the context of a mathematical modeling course?

METHOD

Participants and research context

The participants of this study were 19 prospective teachers (fifteen females and four males) who enrolled in a mathematical modeling course designed for PTs at a public university in Turkey. While 18 of them were students in the department of elementary mathematics education (EME) with a focus on middle school mathematics, one was from the department of computer education and instructional technology (CEIT). Given the fact that the PTs had taken courses such as calculus, discrete mathematics, introductory probability and statistics, they were considered as having the necessary mathematical background and capability of tackling the modeling tasks within the course.

The PTs worked in groups of three or four (formed by themselves) throughout the whole semester. For the current study, the data gathered from two of the six groups selected for in-depth analysis as they better represented the modeling processes of all the groups and provided relatively rich information about the groups' modeling processes. In choosing these groups, we ensured that both undergraduate and graduate students including the only participant from CEIT program were involved in the study, and that each gender was represented in the groups. Thus, Group-A consisted of one male (PTA1) and two female students (PTA2 and PTA3) who were in their 3rd year of a four-year EME program. On the other hand, Group-B consisted of one 4th year male student (PTB1) from the CEIT program and two female master's students (PTB2 and PTB3) from the EME program. The PTs in these groups and other groups reported that they had not previously taken any courses on mathematical modeling.

The course schedule and the mathematical concepts involved in the modeling tasks used throughout the course are presented in the Appendix-1. The class met for four hours each week over a 14-week semester. During the first week, course expectations and objectives were reviewed. In the succeeding weeks, one modeling task was implemented each week. In each class, PTs were required to work individually on a modeling task for about 20-30 minutes, to read and understand the task on their own, to think about the possible solution without being affected by each other's ideas and hence ensure the diversity of the ideas. Then, they worked on the task as a group for a further 90-100 minutes where they were asked with writing in detail about their approach and solution. If PTs wanted to start the group work earlier, they are allowed to do so. The final 30 minutes of the class looked at group presentations of their approaches and solutions. While the PTs were working on the task, the instructor who was also one of the researchers listened to the PTs' ways of thinking and tried not to intervene while letting them construct their own models and evaluate their own strategies. Possible mathematical concepts behind the activities were not provided, nor were they implied in the explanations provided to them in order not to limit the PTs' solution approaches. However, whenever a PT or a group requested assistance about what to do for the next step, or when they really struggled and clearly asked for help, they were asked probing questions in order to help them refocus on their thinking. Examples of these probing questions include "Why do you say that? How do you know it? What would be another way to look at it?" In some cases, they were also asked to draw sketches.

The initial modeling tasks used in the course were open in terms of procedures and the outcomes (Berry, 2002). PTs could produce a solution to these tasks by using elementary mathematics, while becoming familiar with the nature of modeling tasks. During implementation of the first two modeling tasks ("The Postman" and "Bus Stop"), PTs were given no instruction about the modeling process or approaches to see the details of their thoughts. In the fourth week, the PTs were provided the opportunity to discuss their understanding of models, mathematical models, and the modeling process. The PTs were also given sample solutions for the first two activities, based on the modeling cycle described by NCTM (1989, p.138). Starting from the fourth week, during the first 20-30 minutes of each lesson, PTs were asked to critique and discuss the mathematical models that the groups had presented in the previous week. The PTs were also provided with sample solution approaches for the activities solved in the previous week in order to see/examine more of possible approaches for the same task. The explanations related to the sample solutions were provided for each stage involved in modeling (i.e., understanding the problem, choosing variables, making assumptions, solving the equations, interpreting the model, verifying the model and criticizing and improving the model). The aim of this activity was to reinforce/encourage PTs use mathematics in order to solve open-ended, real-life problems, and to provide a supportive mental structure for students to both learn and perform the modeling process (Galbraith, 2012). After presenting sample solutions for the task based on these stages, the strengths and weaknesses of PTs' own models and of the sample model were shared, compared, critiqued, and discussed in the class.

Data collection and analysis

In the study, the data was collected through multiple sources. Each of the two groups of prospective teachers' were audio and videotaped while working on the modeling tasks. Both individual and group solution papers were collected after each modeling task. PTs were also asked to write short papers reflecting their modeling experiences for each task. Furthermore, in-depth semi-structured interviews were conducted with the PTs after each modeling task, but before the following class began. These task based interviews eased data collection procedure by allowing face-to-face interaction in order to be able to thoroughly understand the solution processes of the participants by asking what they did and why they did. Some sample interview questions are as follows:

"What was the problem situation that you focused on? What was your aim in this problem? What was the first solution method that came to mind in order to solve the problem? While formulating the problem, what were the assumptions and situations about the problem that you took into consideration? Did you encounter any difficulties while working on the task? How did you use different kinds of representations (graphs, tables, drawings, etc.) in the modeling process? When you could not arrive at a solution, what did you do? Did you check/discuss the validity and usability of your solution?" During the interviews, which each lasted about 30-45 minutes, both individual and group solutions were provided to the PTs and they were asked to describe their experience with the modeling process; explaining what they did and why they did it to achieve the solution, and providing as much detail and as many examples as possible.

The fact that some of the modeling tasks required PTs to use high level mathematical concepts like derivatives and integrals, or procedures like statistical simulations, might have prevented PTs from fully engaging in the modeling tasks. Therefore, five modeling tasks that allowed PTs to produce a solution using elementary mathematics were selected for detailed analysis: "The Postman, Bus Stop, How to Store the Containers? Let's Organize a Volleyball Tournament! and Who Wants 500 Thousand?" (see Appendix-2).

In the study, the data were analyzed using qualitative data analysis techniques (Miles & Huberman, 1994). Firstly, all the transcribed data including the videotaped classroom sessions and interviews were read, and the participants' solution papers were examined. During the first analysis, the aim was to understand the individual participants' and the groups' solution approaches to each of the problems. After observing the general solution approaches of each participant, the main points of the participants' solution approaches to the same activity were summarized. To do this, the framework suggested by NCTM (1989, p.138) was initially used as a basis for analyzing PTs' modelling process as it included the basics of the modelling process without getting into so much details. During the coding process, other frameworks (e.g., Blum & Leiß, 2007; Borromeo-Ferri, 2006) representing modeling process were also used to identify and name the codes. Thus, five initial categories (stages) were identified as simplification, mathematization, transformation, interpretation, and validation. Moreover, presentation of the solutions suggested for the task was considered to be a part of the modeling process since the PTs' presentations at the end of each modelling activity expected to play a role in both revising and developing their model and help to better understand their thought processes and what they focus on.

During the second data analysis, transcribed videotape and interview sessions were coded together with the solution papers in order to assist in identifying the sub-processes for these stages of the modeling process for each of the PTs across the modeling tasks, including both their individual and group work. This process was carried out for each participant across all of the tasks, and included both their individual and group work. To ensure coding reliability the researchers coded the data by sharing and discussing individual codings with each other until resolving all disagreements (Strauss & Corbin, 1990).

Continuing analysis led to four main conclusions in refining codes and categories. First, the individual and group works were interrelated and thus have to be considered together. Second, mathematization, the construction of a mathematical model of the problem, the resulting transformation, and the identification of solution by working with the mathematical entity were considered interrelated and thus combined under a new stage named "working out the plan" (i.e., performing the plan to formulate an appropriate model and to reach a solution). As PTs did not utilize any mathematics in some cases, instead of calling it "working mathematically", it was redefined less specifically as "working out the plan". Third, the interpretation and validation stages were also considered interrelated and grouped together. While interpreting is considered to be the evaluation of the reasonableness of the solution in terms of the real-life situation, verifying means to check the appropriateness of the model for the original problem situation and the correctness of the computations.

Finally, "identification and simplification of the problem situation" was considered under two stages: understanding and planning. Understanding (i.e., to grasp the meaning of the problem text and to identify the problem situation) consists of the processes of reading, summarizing and evaluating in terms of first noted factors. Planning included seeing the essential features of the problem, devising solution approaches accordingly, and deciding on a way to reach a solution. Consequently, data analysis suggested five categories of the modeling process: (*i*) understanding, (*ii*) planning, (*iii*) working out the plan, (*iv*) interpreting and verifying, and (*v*) presenting.

RESULTS

Understanding the modeling task

The analysis of data revealed that in this stage, the prospective teachers generally read and reread the problem to ensure that they understood the task. In general, PTs underlined the words or phrases in the problem text including critical mathematical information like numbers. During this stage, PTs evaluated the problems in terms of *mathematical structure, mathematical content, difficulty level,* and *contextual setting*. Regarding the mathematical structure, most of the time, PTs categorized the tasks as mathematical or non-mathematical (i.e., could be solved through logical reasoning). For instance, PTA3 categorized "Let's Organize a Volleyball Tournament!" as one that could just be solved through logical reasoning because it did not require any mathematical work. Some of the PTs also evaluated the problems as those requiring a single correct answer or open to alternative solutions. For example, PTB3 categorized "Let's Organize a Volleyball Tournament!" as open to alternative solutions. For the mathematical content, PTs evaluated the problems in terms of the mathematical concepts involved and attempted to fit the problem into a mathematical concept. For instance, PTA1 explained that the (key) words in "Who Wants 500 Thousand?" informed him that the problem was related to probability. The results indicated that PTs looked for certain clues or expressions to decide on the concepts involved in the tasks. Thinking that these problems would be solved using particular concepts, PTs narrowed the problem into a routine problem requiring only a certain set of calculations using these concepts. In "How to Store the Containers?", where the students were expected to find out the maximum number of cylindrical containers that fit into each storage area, four of the PTs (PTA1, PTA2, PTA3, and PTB3) did not consider the rearrangement of the containers to ensure the lowest cost. They did not show any signs of understanding what they were expected to do. Similarly, for "Who Wants 500 Thousand?" except for PTA1 and PTB2, the PTs responded that the probability of a contestant winning 500 thousand as asked in the second part of the problem was $(1/4)^{15}$. As they did not make any calculations that took the difficulty levels of the questions into account, it was obvious that the PTs considered the task as one involving routine probability calculations.

The findings revealed that PTs evaluated the modeling tasks for its level of difficulty. For example, for the "The Postman" and "Bus Stop" problems, PTA2, PTA3, and PTB3 stated that they perceived these problems to be easy ones. PTs also evaluated the problem context and looked for similarities among the ones solved on the course.

PTs tried to summarize the information given to understand "How to Store the Containers?", "Let's Organize a Volleyball Tournament!" and "Who Wants 500 Thousand?" either writing down the key points or using drawings, since their texts were longer compared to the others. For instance, in the case of "How to Store the Containers?" PTA1, PTA2, PTA3, and PTB1 summarized the information by using drawings. While PTB1 drew a picture representing the container and introducing its radius and height, PTB3 summarized the information by listing the key quantities and formulas (see **Figure 2**).

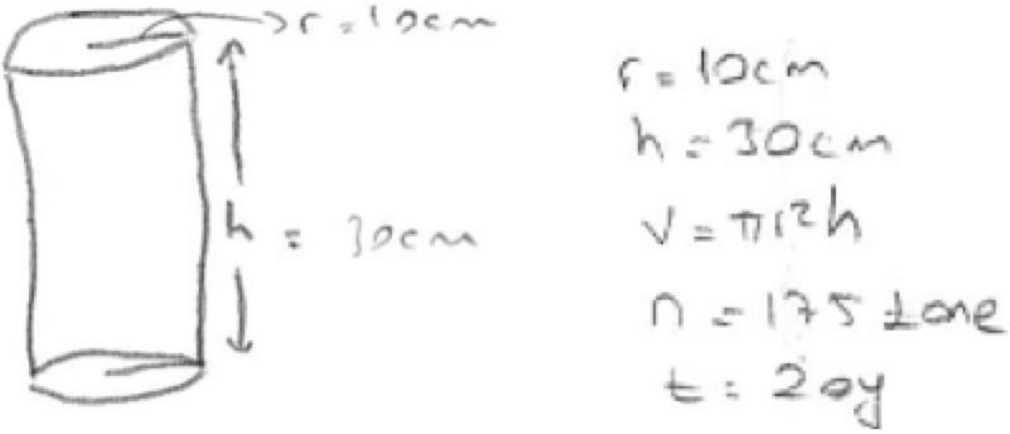

Figure 2. PTB1's and PTB3's initial representations for "How to Store the Containers?"

Planning a solution

The data revealed three processes of planning: imagining and representing the situation in the mind, devising a plan, and deciding on a solution approach. In general, at the beginning of the planning stage, PTs attempted to imagine and represent the problem situation in their minds by imagining themselves in the problem situation, or visualizing the situation with or without considering the context.

> PTA1: Then I thought of myself in the competition. I thought that I would have used the joker immediately [in Who Wants 500 Thousand?].

> PTA2: I just imagined a normal street and thought how a postman would deliver the post on that street [in The Postman].

Furthermore, PTs attempted to relate the problem to ones they had previously experienced and considered similar in order to find an approach to a solution and/or to devise a plan. The following statements of PTA2 reflect most of the PTs' experiences.

> PTA2: We used to solve questions similar to "How to Store the Containers?" at high school. For instance, when we were asked to find how many of a particular item would fit into a certain volume, we divided the volume by that of the particular item. Even, we solved a question with a round ball. To think of airspace while solving this question resulted from that question at high school. Because of that, I used the volume in solving the problem.

> PTA2: We had compared two situations in The Postman. I thought that I should initially compare two situations for this problem [Bus Stop] as well. Afterwards, I

> thought about how to do it, and decided to compare the two situations by taking constants like x and a. It was the Postman problem that originally influenced me.

While devising plans to solve the problems, PTs' assumptions fell into three groupings: making simplifying assumptions in order to solve the problem more easily, making supportive assumptions in order to strengthen the correctness of their intuitive response, and making assumptions based on their knowledge about the problem context.

For some of the problems PTs made simplifying assumptions in order to be able to solve the problem more easily, for example:

> PTB3: There was so much uncertainty in the problem [The Postman]. I immediately thought that I had to imagine such a street in order that it would simplify the problem; there would be an equal number of houses on both sides of the street. In other words, I want to simplify it as much as possible.

PTs made some supportive assumptions in order to strengthen the correctness of their intuitive response. In "The Postman", assuming that the pedestrian crossing should be at the beginning and at the end of the street in order to prevent crossing the road, PTB2 used the "U method" (i.e., delivering mail to one side of the street without crossing back and forth and then delivering to the other side).

PTs also made assumptions based on their prior experience/knowledge about the problem contexts. For example, in "Let's Organize a Volleyball Tournament!", thinking that every player should play in every position in a volleyball game, PTA1 stated that weights of the variables derived from the tryouts (i.e., the height of the player, vertical leap, 40-meter dash etc.) should all be considered as equal.

When devising a plan, in some cases PTs drew a "situation model" or "real model" in line with their assumptions. The term "situation model" is described by Borromeo-Ferri (2006) as the "mental representation of the situation" and the "real model" as the simplified and structured version of the "situation model" which is then converted to the "mathematical model" (Blum & Leiß, 2007). In "The Postman", after finishing their group work based on intuitive response (see **Figure 3a**), PTs were asked to show the appropriateness of their answer and convince the instructor of its correctness. As they considered that their answer was correct, they drew a real model by making some simplifying assumptions (see **Figure 3b**).

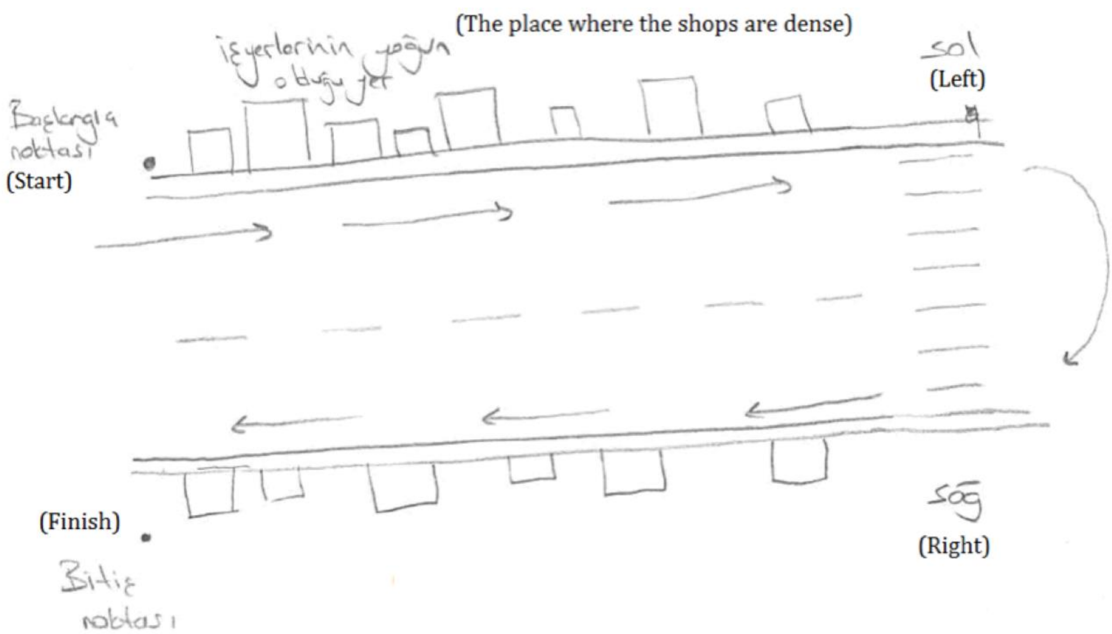

Figure 3a. Initial model (U-method) produced by Group-A for "The Postman"

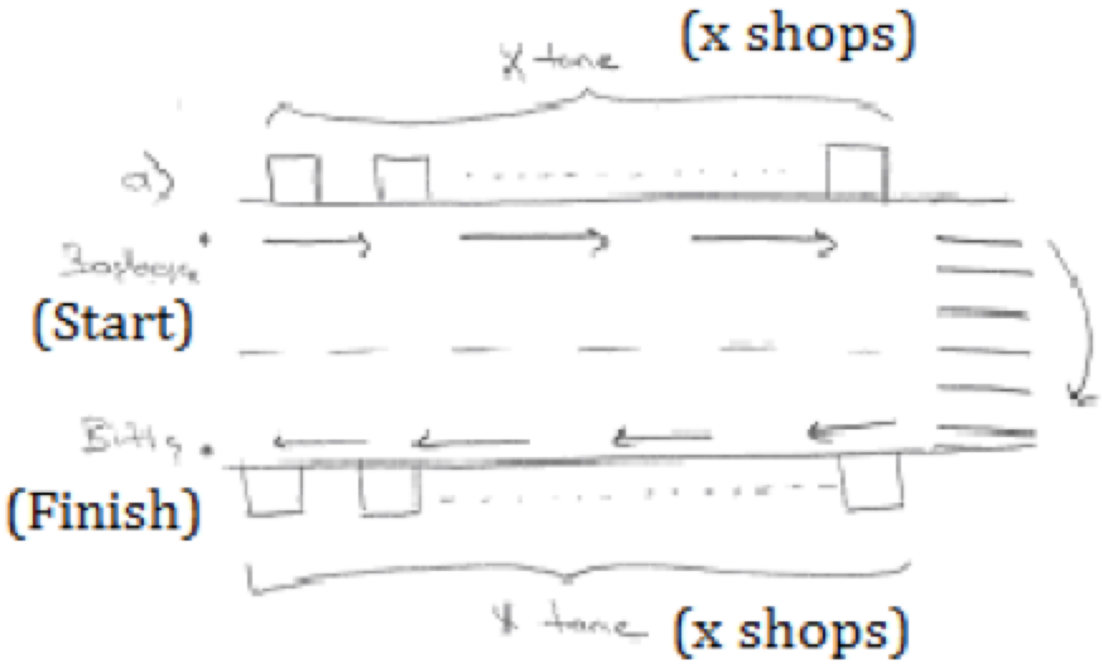

Figure 3b. Revised model produced by Group-A after considering mathematics in their solution

For the other problems, in general, PTs did not draw a situation model. Drawings made during this stage did not include an exact situation model. For instance, in "How to Store the Containers?" PTB1 considered a different arrangement to place more containers, but did not draw a detailed situation model (see **Figure 4**).

Figure 4. Part of PTB1's work for "How to Store the Containers?"

When devising a plan, PTs searched for the mathematical concepts that they are familiar with and could be used for solving the problem. In "The Postman", as PTs finished their work based on their intuitive response, the instructor asked them to mathematically reason the correctness of their answers. Therefore, they tried to devise a new plan as a group. They went through their mathematical knowledge and wrote out some formulas that could be used to solve the problem. In some cases, to identify the mathematical concept they could use, they tried to remember all the formulas related to the concept. For instance, in "The Postman", PTB3 wrote down the formula for work $W = F \cdot x$ and PTB1 wrote down the formula for distance $x = V \cdot t$ in order to calculate the total distance travelled by the postman. Similarly, PTA1 explained that in general, he employed a mathematical concept that he already felt comfortable with and used it in solving the problems: "*I generally use the concept that I know best; for example, derivative.*"

PTs solution approaches to the modeling tasks were more intuitive than mathematically based or planned. They usually aimed to construct a plan that would enable them to prove their intuitively based answers, and end up with a formula or a mathematical expression. By making assumptions, PTs aimed to create a situation in which they could prove their solutions through algebra. For example, in "The Postman", it was only after they were told to justify their answer mathematically that they began to use algebra. However, as they thought that their intuitively based answer was correct, they also attempted to show the correctness of their initial answer. Similarly, PTB3 explained that she generally simplified the problems in order to solve them more easily: "To tell the truth, I just look at the question, and start to write the easiest solution; for example, the one that comes into my mind first." In "How to Store the Containers?", PTA1 explained that their group decided to choose the most convenient method because, in their school life, they solved routine problems in which they applied the easiest method without looking at the problem from different perspectives or thinking about different alternatives.

PTA1: I don't know how to express it, but there is something like background. We grew up in such a way that we always think about the easiest one. You see that the containers will be set and there is storage unit. Find the volume, divide it and get the result. As we were taught that way, to place and rearrange them is very difficult for us. You will set it and labor on it; it's really hard.

...

PTA1: As I said before while solving the problem, I use the method that I know best, so I used the one which is the easiest for me.

Researcher: Is it volume, which comes to your mind?

PTA1: It's easy. I don't want it to be complex.

Related to deciding on a solution approach, class observations showed that, in general, especially for the individual work, all PTs decided on a solution approach immediately after the initial reading of the problem. This did not differ from group to group, except for Group-B in "How to Store the Containers?" and "Who Wants 500 Thousand?" where although they carried discussion for deciding on a solution, PTs inclined to accept the first solution idea that occurred to them instead of developing multiple alternatives.

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PTA1: Let's write now? Ok, let's start. Shall we start like this, then we can use x [How to Store the Containers?]

...

PTA1: Come on! Let's be quick.

...

PTA2: Just a minute, do not make a mistake here! [to PTA1]

PTA1: Do not stop! Please keep writing. Come on!

PTA2: You are in a hurry; first of all, let's think for a while [to PTA1] [smiling]

PTA1: Write something [to PTA2], we did not get an answer in hand [Who Wants 500 Thousand?]

PTA2: There is no point to write something without finding the right way?

PTA1: We are thinking, but we should write something.

Data from multiple sources confirmed that, after the initial reading of the problem, PTs devised a general plan to solve the problem that they followed to the end. Furthermore, after working on the problem individually, PTs shared their ideas at the beginning of the group work. They chose one of these ideas for the solution and attempted to make a few improvements on this solution in the group work. They did not think about or work on the possible alternatives that had the potential to yield better models. In general, PTs produced similar approaches to the problems. However, when they had different approaches to the problems, they tried to persuade each other. As a group, they adapted the approach of the group member who insisted the most strongly that his/her approach was the best. However, when they had no idea about the mathematical concepts or the approaches to be used, they worked together to identify the concepts and approaches to solve the problem.

Working out the plan

Analysis of data revealed two processes in working out the plan: performing the plan based on an intuitive decision (no mathematization), and performing the plan using mathematics (mathematization). For "The Postman" and "Who Wants 500 Thousand?" PTs performed their plans based on intuitive decisions. For instance, before attempting any mathematical work, PTA2 intuitively concluded that the postman should begin from the closest part of the street, deliver the mail, cross the road, and deliver the other mail. Similarly, for the part (a) of "Who Wants 500 Thousand?" PTs chose to answer the problem intuitively without producing a mathematical model. Four of the PTs explained that they would leave the contest without risking the prize, because even if they used their lifelines, the probability of losing was still fifty-fifty (50%).

During the modeling process, the PTs tried to mathematize the situation by using mathematical notations. For example, below is PTB3's explanation during the group work for "The Postman".

PTB3: We will generate a formula now…Let x be the number of shops with even numbers, and y be the number of shops with odd numbers. Let's call the duration of delivery by the postman for each shop as t.

PTs applied various strategies to progress mathematically. For "Bus Stop", for instance, PTA3 assumed the distance between the houses as 100 meters and tried to reach an answer based on this specific case. In "How to Store the Containers?" Group-B used circular objects to better visualize the problem and to see whether rearranging the boxes would allow the fitting in of more containers (see **Figure 5**). PTB2 argued that she had a similar experience before, and she therefore tried to convince others in the group that they could fit more containers into the second storage unit by rearranging.

Figure 5. A snapshot from Group-B's work while working with manipulatives on "How to Store the Containers?"

Similarly, PTA2 changed the shape of the given containers. While working mathematically, she considered the shape of the cylindrical containers as square prisms (see **Figure 6**). Thus, she eliminated the computational problems aroused by the gaps among the containers. Using prisms instead of cylinders in their group solution, PTs did not feel the need for other possible arrangements of the containers.

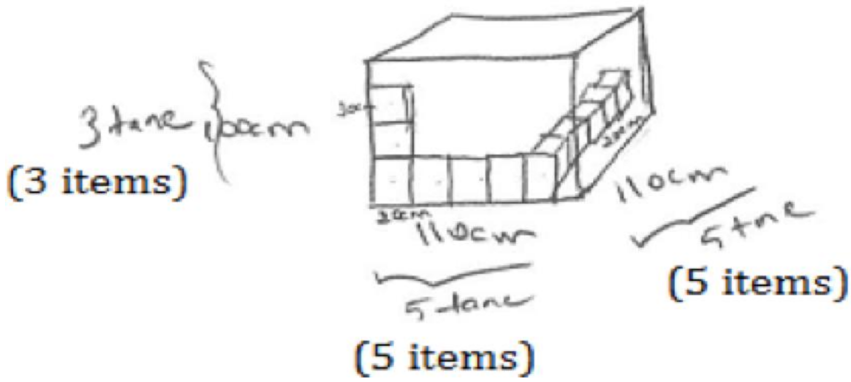

Figure 6. Group-A's approach to "How to Store the Containers?"

In "Let's Organize a Volleyball Tournament!", as PTA1 decided to combine the scores of each player and find a total score, he made informal weightings (rescaling) to make the effects of the variables closer by changing the values of the variables in an unsystematic way. During the interview, he explained that as the heights of the players ranged from 1.55 to 1.85 meters, and vertical leaps ranges roughly from 40 to 60 centimeters, and serve results ranges from 5 to 10, he tried to make the values closer to each other by dividing each score by a number such as 10 or 100. Moreover, he explained that he found his approach unreasonable and did not share it in the group work. It was a method that could have yielded a better model if they had thought about constructing it in a systematic way. He exemplified this case on his solution paper for the first player on the list. PTs also made guesses about unknown variables by comparing them with known variables. When comparing the "U-method" and "crossing the street back and forth method" in "The Postman", Group-A guessed that the postman would walk $2{\cdot}l$ (where l is the length of the street) distance for the crossing the road back and forth method in addition to walking distances for crossing the road. As the postman would walk on two sides of the street which is $2{\cdot}l$ long for the "U-method", they thought that the postman would also walk the same distance for the "crossing the street back and forth method" in addition to the walking distances for crossing the road. Both Group-A and Group-B considered that crossing the street back and forth would always produce greater walking distance than the "U-method". The analysis of Group B's solution papers also supported this case (see **Figure 7**). At first, they wrote the corresponding mathematical expressions for these two methods as $2nx+y$ and $2nx+ny$ (x: the distance between the houses, n: the number of houses, y: the width of the street). Only after they were asked to justify the correctness of their answer, PTs drew the situation model (see **Figure 7**) in detail and realized that the postman would walk only one side of the street in crossing the street back and forth method not two sides.

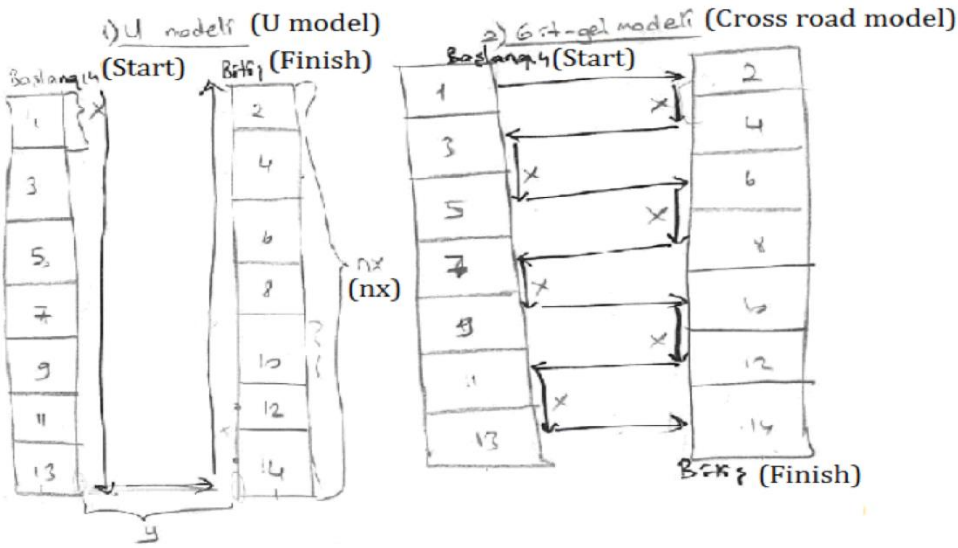

Figure 7. Part of the Group-B's work showing the U-method vs. crossing the road back and forth for "The Postman"

Similarly, in "How to Store the Containers?", based on her prior experience with a similar problem, PTB2 thought that all the containers could fit into the second storage unit through rearrangement. However, she could not find the related mathematical concept (i.e., the Pythagorean Theorem) that would help her to reach a solution individually.

On the other hand, to find a result easily, PTs ignored some of the relevant variables in some cases. For example, in "The Postman", as PTA1 asked his group to ignore the distance between the houses because the time required to deliver all of the posts would be the same for two paths, they decided to ignore a relevant variable in order to be able to more easily solve the problem. A similar situation occurred in "How to Store the Containers?" While sharing ideas in the group work, although PTA2 offered to render the shape of the cylindrical containers as prisms as a way to take the gaps/spaces between the containers into account, PTA1 again wanted to ignore the gaps. Although it would be the case, he claimed that it would not cause a significant change in the results because they ignored this for all storage units. Similarly, while considering to combine the scores for each player in "Let's Organize a Volleyball Tournament!" PTB3 suggested adding the players' heights in centimeters and vertical leaps in meters. She explained that as she disregarded the type of the measurements for all players, she thought that this would not create a difference in the result. She also explained that they ignored the amount of differences between the scores of the players while ranking. She thought that they would not reach a mathematical conclusion without ignoring certain points or variables.

To combine the mathematical expressions, they obtained in reaching a final answer, PTs used several strategies. They tried to make a comparison among the mathematical expressions they found through making assumptions, using logical arguments, and writing the expressions in a comparable form. Regarding the first assumption, in "The Postman", for example, by eliminating the same terms in the equation, PTs in Group-B reached the following inequality: $nx + y > ny$. To be able to show its correctness and conclude that the way they believed would yield the shorter walking distance, they thought about the assumptions that could yield this conclusion. In doing so, PTB3 explained that x should be smaller than y. They also thought about the assumption that y (the width of the street), is much greater than x (the width of each shop), in order to ensure the correctness of their intuitive response. PTB1 also explained that while solving the problem, he based his assumptions on an imaginary street in his mind after reading it. Regarding the second approach, PTs combined the mathematical expressions by making logical mathematical statements. For example, in her individual work on "Who Wants 500 Thousand?" PTA2 analyzed the options of risking 7,500 TRY (Turkish Lira currency) for walking away with 16,000 TRY, and thus winning 8,000 TRY more, or not. She explained that, in case of using the lifeline (joker) and reducing the number of choices from four to two, the probability of correctly or incorrectly answering the tenth question would remain the same (i.e., 50%), but the money that could be won would be greater than the money that could be lost (i.e., 8,000 versus 7,500). Therefore, she concluded that answering the tenth question by using the remaining lifeline (joker) would be the most logical thing to do for the contestant. In the group work, they also used this argument as their final answer. For this problem, while Group-A answered the problem by using logical arguments, Group-B faced difficulty in combining the probability of correctly answering the problems and the corresponding amount of prize money won. In the interview, PTB2 explained that they mainly focused on the probabilities, and so they could not combine the amount of money earned and the corresponding probabilities.

Regarding the third approach, PTs tried to write the expressions in a comparable form. For example, in "Bus Stop", after writing the mathematical expression for the bus station located at the middle of the street for the case of odd number of houses/shops, although it was not necessary, PTs tried to liken the general term of the second sequence (i.e., the one for the even number of houses) to the first one. Instead of thinking through the correct expression, they used trial-and-error to form the general term of the second sequence after deriving the general term of the first sequence (see **Figure 8**). PTB2 realized the correct expression while thinking about the problem for several minutes individually. After PTB2's explanations, others in the group realized that they worked unnecessarily to derive the expression by trying to write them out in a comparable form.

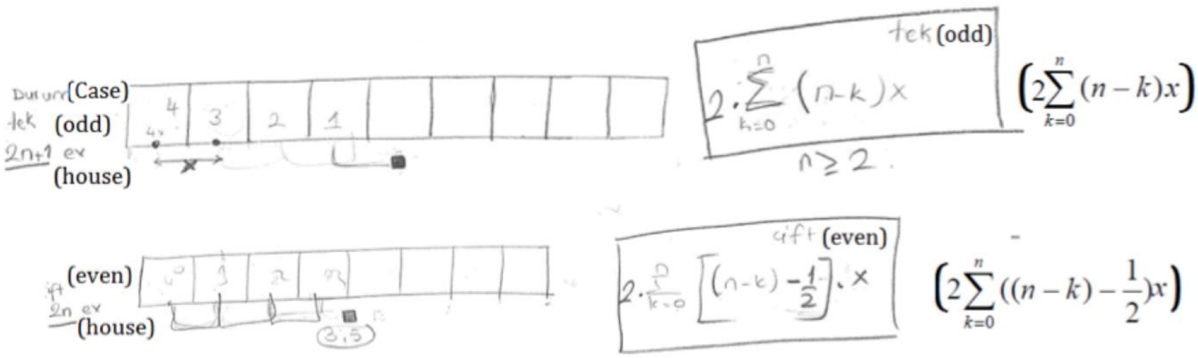

Figure 8. Part of Group-B's work for the "Bus Stop"

Interpreting and validating the model

The analysis of the data shows that in general, PTs did not verify their solution approaches and/or test their models, because they either believed in the correctness of their model, or considered their solution as a proof. For example, regarding "The Postman" believing strongly in the correctness of their intuitive responses, PTs considered that the most efficient way of the delivery would be the "U-method" and there was no need to consider different ways of delivery. Furthermore, PTA1 and PTA2 stated that they offered the "crossing the street back and forth" to prove the correctness of their intuitive response:

> PTA2: When we looked at the problem, we said that the result is obvious. We thought that the postman would walk, then cross the street, and then walk back again to the starting point. We never thought that it was such a different solution. We only tried to justify it. We believed in the U-method from the start.

Although PTs did not feel the need to interpret the results they reached in most cases, they did try to interpret the findings occasionally as illustrated in the following excerpt regarding "Who Wants 500 Thousand?"

> Researcher: Have you ever thought that your result of 1/2 to 30 is too big (the probability of correctly answering fifteen questions); I mean did you interpret it?
>
> PTA2: I thought that this value was so low at that moment. Accordingly, I thought that nobody could ever win five hundred thousand.

Similarly, regarding "How to Store the Containers?" PTB3 interpreted the fractions (obtained by dividing the total volume of the storage unit to the volume of a can) in the results as gaps.

> PTB3: When I saw that fraction, I thought that I should not include it. It should be an integer, and the fraction shows the spaces in the storage.
>
> Researcher: For example, you assumed that it is 17.2?
>
> PTB3: It means that 17 cans fit into the container. So can we think of 0.2 as space?
>
> Researcher: Do you think so?
>
> PTB3: It does not seem logical, but it is also not illogical to think so.

To verify the correctness or reasonableness of their answers, PTs attempted to interpret them in real-life. For instance, regarding "How to Store the Containers?" PTA1 argued during the group discussion that "It is always better to take something in large quantities. I mean wholesale". Moreover, they also used specific values for the variables to check the correctness of their solution/equation as exemplified in the excerpt from the group dialogs in "The Postman".

> PTB3: Now I say that I am taking n multiple of y and n multiple of x. Is this correct? Would $n \cdot y$ is certainly bigger than $n \cdot x$?
>
> PTB1: Look, if you give values to them, it is easier to visualize in your mind. Give 100 for y, 1 for x, 5 for n. So, this side becomes 500 and the other becomes 105. Why is this side bigger [referring to the right hand side of the inequality], because we took this [pointing out $y > x$], if we take this assumption then the result becomes so [PTB1 is writing all these on the paper at the same time]

In some cases, PTs sought the approval of the instructor as the authority or from other group members for the correctness of their solutions as illustrated in the following excerpt from "Who Wants 500 Thousand?"

> PTB2: Hey, can one of you [to the teammates] ask this to the instructor?

PTB1: We will ask a question.

PTB2: It becomes eight fold; should we ask the instructor whether we are on the right track or not; otherwise, we will find something that does not make sense and not related to the solution.

Presenting the model

After working on the modeling task, all of the groups were asked to orally present their solutions/models and the reasoning behind them using the whiteboard in about five minutes. After the presentations, although we can expect revisions in their models, in this study our aim was to better understand PTs' thought processes and how they present their models and what they focus on during the presentations. For giving PTs time to reflect on and revise their models, discussions were left to the next week. Moreover, even though the PTs wrote reflection papers on each modeling activity presented throughout the semester, they did not reflect much on the other presentation and only focused on revising their own models.

In general, in their presentations, PTs explained the assumptions made and the notations used in their models. Groups usually began their presentations by drawing the situation model on their solution papers. For some tasks, they presented the mathematical symbols used for the variables on their drawings and mentioned details about the assumptions they had made. However, this was not always the case. For example, although, in the presentation of "Bus Stop", PTA2 drew the situation model including the mathematical notations like in their group solution paper, she did not verbally mention either the assumptions or the meanings of the symbols. In general, PTs' presentations for "Bus Stop" and the "How to Store the Containers?" were result oriented. For instance, as the presenter from Group-B, PTB3 did not mention the computational procedures they carried out for "Bus Stop" in an effort to try to keep the presentation simple. She presented the case of the even number of houses as part of their solution. After drawing the situation model, she directly wrote the mathematical expression reached in the solution.

On the other hand, in some cases (e.g., "The Postman"), both groups explained the procedures and the computations in detail. Similarly, after "Let's Organize a Volleyball Tournament!" PTA2 presented the procedures step-by-step along with the reasons for the choices made as follows.

PTA2: We thought that it would be a bit unclear to conceive the height of the players and their vertical leap differently because each player has a different leap and height. For that reason, we took the total vertical leap distance found by adding these two values into consideration. Then, we wrote down the players' 40-meter dash, their successful serves, and spikes. Additionally, we transferred them to an Excel spreadsheet table and ranked them from the highest to the lowest.

DISCUSSION AND CONCLUSION

The analysis of the findings indicated that the prospective teachers presented similar approaches when working with the modeling tasks. In general, PTs approached to the tasks in a result-oriented way and their modeling process consisted of a single cycle. This finding is consistent, in the broadest sense, with Berry (2002) whom found that students favored result-oriented simple solution approaches and did not spend much time analyzing or clarifying the given tasks. In general, PTs did not spend enough time to reflect on the problem and the way they approached to the problem. Thus, in some cases they spent time needlessly working on inappropriate solutions and ended up with inadequate models. As Schoenfeld (1985) explained, good control is needed to overcome the loss of time and energy caused by incorrect perceptions during the solution process, and which will result in lower performance.

According to Van den Heuvel-Panhuizen (1989), plausible scenarios could be produced especially in familiar contexts and that the students would prefer to work on the problem based on those scenarios because the context could provide an environment for students to "imagine themselves in the situation" (p.136) and inspire their solution strategies. During the understanding stage, upon reading and understanding the problem situation, imagining themselves in the problem situation was central to PTs approach to certain modeling tasks in this study (e.g., for "The Postman", "Bus Stop", and part (a) of the "Who Wants 500 Thousand?"). PTs also attempted to produce solutions based on their previous knowledge, experience, or intuition related to the context. On the other hand, PTs also tried to visualize a specific context based on their experiences and began to think in that context. The dependency on thinking about real-life contexts, particularly in the first activity, "The Postman", led to context-dependent visualizations. Thus, PTs spent most of their time making effective assumptions that would lead them to a solution. This finding supports Busse and Kaiser (2003) who concluded that the context could be motivational for students. However, activating a broad range of real-life knowledge related to the context also caused confusion. Although PTs mostly identified the variables correctly and developed appropriate assumptions, in some cases they faced difficulty in developing appropriate assumptions to structure the problem due to focusing on the context. Especially during the first weeks of the study, it took a great deal of time for PTs to simplify and identify the task because of working on the details of the context, rather than working on the problem. As simplifying and constructing are cognitively demanding steps in solving ill-structured problems, because they require identifying the constraints and the multiple possible states of the problem (Blum & Leiß, 2007; Jonassen, 1997; Spandaw, 2011), PTs in this study oversimplified the assumptions to easily reach a result, which, in the end, produced simple models in some cases. This is an important finding that has not been highlighted in the related literature.

While trying to develop a solution plan, PTs tried to remember a previously solved similar problem and tried to apply the same solution method for the problem at hand. According to Polya (1973) and Schoenfeld (1985), this is usually the first method people use when trying to solve a problem. The results showed that PTs attempted to solve "Bus Stop" just like they did in "The Postman", mainly because they thought that there were comparable similarities between the contexts of the problems. After reading "How to Store the Containers?" and part (b) of "Who Wants 500 Thousand?" that are tasks "closed in terms of outcomes" in Berry's sense (2002), PTs tried to fit the problems into a mathematical content they are familiar with so that they could directly employ the solution methods of routine tasks/problems related to that specific content. In other words, they attempted to solve/approach these problems just as they would solve any other routine problem. In "Let's Organize a Volleyball Tournament!" and part (a) of the "Who Wants 500 Thousand", for example, when PTs found out that the mathematics behind the problems was not obvious and did not prompt a specific mathematical concept through keywords in the problem text, they inaptly chose some mathematical concepts/tools through which they believed they could solve the problem. In some cases although they realized that other mathematical tools/concepts (e.g., ratio and proportion, standard deviation) would help the solution process and might yield better models, they did not even try to use them because they believed that it would be more difficult to reach an answer. Instead, they chose basic mathematical concepts with which they felt more comfortable in order to carry out the computations that would lead them to a solution.

On the other hand, PTs in this study did not want to engage in problems dictating a specific model or requiring them to use complex mathematical concepts in order to quickly reach an answer. They preferred to work on open-ended tasks that allowed them the freedom to set conditions and decide upon a model. This finding is contrary to Kuntze (2011), who found that PTs preferred tasks asking for a single correct solution instead of tasks allowing for different solutions. As Kuntze (2011) explained, this was a result of PTs' views and fear of incompatibility of the modeling tasks with the mathematical exactness. PTs in this study also did not attempt to build a mathematical model when they thought the mathematics to be beyond their level of knowledge, even though they might have the sufficient background knowledge and skills to carry out all the related procedures, they could not transfer mathematical knowledge to the modeling situation. However, these results are contrary to those of Berry (2002), who found that problem solvers attempted to develop a model where the selected mathematics was beyond the modelers' knowledge or competence level.

When problem solvers adopt intuitive approaches instead of a systematic approach, they are more likely to fail to consider the role of relevant information or even ignore it completely (Tyre, Eppinger, & Csizinszky, 1993). PTs in this study developed intuitive answers (particularly in the first two activities, "The Postman" and "Bus Stop") and were unable to take some of the relevant variables into consideration when they need to provide mathematical solutions to certain modeling tasks. This might be due to the problem contexts that included familiar situations, allowing PTs to come up with an intuitive response. On the other hand, there might be drawbacks of solving daily-life problems by taking mathematical knowledge into account. For instance, regarding the "The Postman", PTB3 stated that a postman in real life would just choose any convenient path for delivering the post without any mathematical justification/reasoning for selecting the best available path. Galbraith and Stillman (2006) describe this dilemma as a blockage occurring during the transition from the mathematical solution to the real world meaning of the solution. However, our findings indicate that this dilemma could also be an obstacle during the planning stage, from the real world to the mathematical expression.

On the other hand, as also reported by Maaß (2006), the PTs in this study lost track of their progress in some cases and needed to re-examine their purpose in the problem. Although the PTs correctly understood the problem and decided on a solution method, in some cases, they subconsciously shifted their thoughts to the real life context. This might also be the result of being distracted by their experience/knowledge regarding the real-life/realistic context of the problem (Busse & Kaiser, 2003).

After deciding on a general solution approach to be used, PTs quickly moved on to start the quantitative work. They simply accepted and proceeded with the very first solution that they thought of and did not seek multiple alternatives. Instead of thinking about the situation in more detail, some of the PTs were particularly eager to get the task done faster and to reach an answer as quickly as possible. This finding is consistent with those from other studies (e.g., Berry 2002; July & Mudaly, 2007; Verschaffel et al., 2002). Furthermore, the reason for the PTs general tendency to not draw accurate or appropriate situation models might be their willingness to immediately start the quantitative work in order to reach an answer. Thus, they could not gain insight from spending time on accurate drawings (Schoenfeld, 1985). Upon sharing their ideas about the solution of the problem based on their individual work, the PTs generally decided to use one of their individual solution approaches in the group work and began to write the report. For the undecided points in the selected solution approach, they tried to combine the ideas of other individuals in the group. However, planning the solution should be an interactive and social process in which the group members discuss and brainstorm as well as assess the solution options available (Tyre et al., 1993). In general, similarities among the individual solution approaches and the desire to reach a solution as soon as possible might be the factors leading to such action. As a result, group problem solving was not successful except for Group-B on "How to Store the Containers?" and "Who Wants 500 Thousand?" While working on the solutions, PTs made and proceeded with guesses about hidden variables and the relationships among them without calculating the correct value or testing the correctness of the relationships. In a similar vein, when faced with difficulty at a point in the solution process, PTs sometimes ignored the most relevant or main variables that were crucial to reach a solution. Disregarding the main variables for the sake of getting a model has not been mentioned in the related literature.

Prospective teachers in this study interpreted their intermediate or final results occasionally. Supporting the findings of Busse and Kaiser (2003), in some cases, they used their knowledge of the task context to verify their results. However, also supporting the findings of Bukova-Güzel (2011), they did not interpret the solution in a real-life context in some cases. As

Zbiek and Conner (2006) stated, the modelers might also not explicitly present interpretation in the solution process, since it could be considered a subliminal act.

The results suggested that PTs followed a linear progression toward a solution rather than a cyclic and nonlinear process as argued in most theoretical models explaining mathematical modeling process (Blum & Leiß, 2007; Galbraith, 2012; Lesh & Doerr, 2003; NCTM, 1989). Acting on the tasks with a result-oriented focus, PTs went through a single-cycle process and did not seek a better model by reflecting on and improving their models. Even if they found their solution unsatisfactory, they did not return to the previous stages to revise or restart the modeling process.

The results indicated that, except for some cases, PTs believed in the correctness of their solution and did not feel the need to test it. When they arrived at intuitive answers, they used those as supportive assumptions at the beginning of their solutions. In other cases, they considered their solutions as a proof, with no need for further verification. In fact, PTs did not even generally check for computational mistakes, or validate the reasonableness of their solutions for the given problem situation. Such results indicated that the verifying stage was particularly weak or absent in the PTs' modeling process although they can check and self-assess their findings. As this was the case, they were not able to refine their solutions to develop better models. This is consistent with the findings of other studies reporting that (prospective) teachers did not reflect further on their solution processes (e.g., Blum & Borromeo-Ferri, 2009; Blum & Leiß, 2007; Galbraith & Stillman, 2006; Hodgson, 1997; July & Mudaly, 2007; Kaiser et al., 2010; Maaß, 2006; Sol et al., 2011).

Related to the modeling instruction, the role of different teaching styles or strategies (Berry, 2002) and the role of the instructor to execute the modeling activities for different class settings need also further attention (Ikeda, 1997). The research on this area could help the development of correct modeling instruction. Furthermore, these kind of methodological choices used in this study might have an impact on the results of the study and therefore might not be appropriate to identify certain processes.

The results suggested that as well as in the school curriculum, mathematics teacher education programs should integrate mathematical modeling courses into their curricula in order to improve PTs' modeling competencies (Maaß, 2006) and longer engagement of PTs in modeling activities are needed (Blum & Ferri, 2009; Kaiser, 2007). Engaging in, working on, developing, and implementing mathematical modeling tasks may also provide PTs with opportunities to develop better understanding of related mathematical concepts. This also implies that PTs should be provided with modeling activities starting as early as possible. However, our findings suggested that modeling tasks that are open in terms of procedures but rather closed in terms of outcomes could be better for beginners. Only after the internalization of such tasks, the problems open in terms of outcomes might be introduced. In doing so, rather than finding exact numerical solutions, PTs should be encouraged to reflect on the problem situations more deeply before they proceed with the approaches developed after the first reading of a problem (Berry, 2002). At the end of each modeling activity, PTs should also be given enough time to reflect on the problem and possible solution approaches in order to develop appropriate problem schemas (Jonassen, 1997). Therefore, PTs should be given enough time not only to develop better mathematical models, but also to reflect on the overall solution process and think more about their own work.

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APPENDIX

Appendix-1. Course schedule with modeling tasks and the mathematical concepts involved in the solutions

Weeks	Course content and modeling tasks	Mathematical concepts involved in the solutions
Week 1	Introduction	-
Week 2	"The Postman"	Manipulating equations and inequalities
Week 3	"Bus Stop"	Constructions of functions and inequalities
Week 4	• Presentation on models and modeling perspective	
	• "Gardening Job"	Construction of equations, derivatives
	• "How many computers to be manufactured?"	
Week 5	"How to Store the Containers?"	Pythagorean theorem, ratio and proportion
Week 6	"Let's Organize a Volleyball Tournament!"	Weighted averages, ranking, scoring, re-scaling
Week 7	"Forest Management"	Derivative, integral, limit, sequences (convergence), power functions
Week 8	"Who Wants 500 Thousand?"	Expected value, probability
Week 9	"Drug Therapy"	Exponential and logarithmic functions, derivative (rate of change), limit, integral
Week 10	"The Cashier"	Probability, random number generation (simulation)
Week 11	"Dentist Appointment"	Probability, random number generation (simulation)
Week 12	"Traffic Lights"	Probability, (simulation)
Week 13	Presentations of modeling tasks PTs developed*	-
Week 14	Presentations (continued) Overall evaluation of the course	-

* As part of course grading PTs were expected to develop a modeling activity.

Appendix-2. The modeling tasks reported in the study

The Postman [adapted from Swetz, F., & Hartzler, J. S. (1991). *Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises*. Reston, VA: NCTM. (p. 38)].

A postman needs to deliver the mail to the shops on both sides of a street. To do this, the postman could use different methods in terms of the order of delivery. The postman needs to make a decision from where to begin and which way to follow. For you, which method would be better?

Bus Stop [adapted from Swetz, F., & Hartzler, J. S. (1991). *Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises*. Reston, VA: NCTM. (pp. 29-30)].

There is a need to decide the placement of the school-bus stop for a group of students living along a road. Determine where the station should be located so that the total distance the students will have to walk is the minimum amount.

How to Store the Containers? [adapted from Swetz, F., & Hartzler, J. S. (1991). *Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises*. Reston, VA: NCTM. (p.12)].

A small company in the production of canned goods needs to find short-term storage for some cylindrical containers. The company wants to do this with minimum expense. The containers are right circular cylinders with a radius of 10 cm and a height of 30 cm. The company plans to store 175 containers for two months.

Storage units are available for rent in three sizes. The unit sizes of the storage units, each of which has a height of 100 cm, and the rental costs are given in the table below.

Width (cm)	Length (cm)	Rent cost for a month (TRY)
110	110	100
110	220	150
110	330	200

Note: The cans must be stored in an upright position for security purposes.

Let's Organize a Volleyball Tournament! [adapted from Lesh, R., Yoon, C., & Zawojevski, J. (2007). John Dewey revisited: Making mathematics practical versus making practice mathematical. In R. A. Lesh, E. Hamilton, & J. Kaput (Eds.), *Foundations for the future in mathematics education* (pp. 315-348). New Jersey, NJ: Lawrence Erlbaum Associates].

The organizers of a volleyball camp want to have more competition in the camp's tournament. Thus, they need a way to fairly divide the campers into teams. They have compiled information about some of the players from tryouts and from the coaches regarding the coach's comments about each player and the data for the player's heights, vertical leap, 40-meter dash, number of serves successfully completed, and spike results.

Based on the data, your task is to split the players into three equal teams and to write a report explaining how you created your teams. The organizers will use your process for the next camp when they need to split a large number of players into equal teams. Thus, you need to make sure that your process for creating teams will also work for a very large number of players.

Who Wants 500 Thousand?' [adapted from Quinn, R J. (2003). Exploring the probabilities of 'Who wants to be a Millionaire?' *Teaching Statistics*, 25(3), 81-84].

Consider what you would do in the scenarios given below.

(a) A contestant has correctly answered the first nine questions, but has no idea what the correct answer to the 10th question is. The contestant has already used the 'ask the audience' lifeline and the 'phone a friend' lifeline. Therefore, his options are to leave with 8,000 TRY or to make a guess among the answers and take the risk of leaving with 500 TRY. If, however, the contestant guesses correctly, he will not finish with less than 16,000 TRY and he will have a further, risk-free, chance of increasing their winnings to 32,000 TRY.

Question	Money Won (TRY)
1	50
2	100
3	200
4	300
5	500 (guaranteed)
6	1,000
7	2,000
8	4,000
9	8,000
10	16,000 (guaranteed)
11	32,000
12	64,000
13	125,000
14	250,000
15	500,000

If you were in such a position, what would you do? Do you think the contestant should continue to compete or not? Examine possible alternative situations and discuss which alternatives are more reasonable using the mathematical data.

(b) What is the probability of correctly answering all 15 questions?

(c) To be able to have the right to play the game in each episode, the contestant must win the fastest finger competition. In this competition, 10 contestants are asked to place four items in order. The order might be chronological, east to west, largest to smallest, etc. The contestant who correctly orders the choices in the shortest period is the winner. Therefore, in order to be successful in the 'fastest finger' competition, the contestant should use some strategies. When the contestant sees the question and has no idea about the order of the second item what should he do: Immediately entering an answer based on a random guess, or thinking through the correct order while wasting valuable time?