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Understanding Students' Mathematical Thinking for Effective Teaching: A Comparison between Expert and Nonexpert Chinese Elementary Mathematics Teachers

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ABSTRACT

It is widely believed that teachers' knowledge of students' thinking has a significant impact on teachers' teaching and students' learning. However, there is far less research on how teachers acquire their knowledge of students' thinking before, during, and after lessons. This study is designed to compare the differences between expert and nonexpert mathematics teachers on their behaviors and perceptions related to understanding students' mathematical thinking. Based on 554 Chinese elementary mathematics teachers' responses to a survey, the study found that teachers took actions to understand students' thinking more often when students were learning new topics or encountering difficulties, and they were more likely to do so before lessons than during or after lessons. The comparison revealed that significantly more expert elementary mathematics teachers attempted to understand students' thinking from a variety of perspectives before making the necessary adjustments to their predetermined teaching plans than did nonexpert teachers. Significantly more expert teachers also relied on their own teaching experiences to understand students' thinking. In contrast, significantly more nonexpert teachers claimed that they did not rely on prior teaching experiences because they "did not know how to."

Keywords: expert teachers, nonexpert teachers, students' mathematical thinking, elementary mathematics teachers, China

INTRODUCTION

There is universal agreement that teachers' understanding of students' thinking has a substantial impact on their classroom instruction and, hence, upon students' learning (Cai, 2004, 2005; Gardner, 1999). The field of mathematics education continues to investigate how teachers acquire an understanding of students' mathematical thinking. The purpose of this study is to investigate how expert and nonexpert Chinese elementary mathematics teachers attempt to understand their students' thinking before, during, and after lessons. Expert teachers are defined as those who hold advanced rankings in the Chinese educational system whereas nonexpert teachers are defined as those who do not. This study is designed to address the following three research questions via a comparative analysis of expert and nonexpert teachers:

1. How do Chinese expert and nonexpert elementary school mathematics teachers view their own understanding of students' thinking? (Perceptions)
2. How do Chinese expert and nonexpert elementary school mathematics teachers attempt to understand students' thinking before, during, and after their lessons? (Behaviors)
3. What are the differences in expert and nonexpert teachers' behaviors and perceptions related to understanding students' thinking across the three time periods (before, during, and after lessons)? (Time Period Effects)

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Contribution of this paper to the literature

- This study developed a survey of mathematics teachers' behaviors and perceptions related to understanding students' mathematical thinking before, during, and after lessons.
- This study investigates the ways in which teachers understand students' mathematical thinking via a comparative lens (i.e., expert vs. nonexpert mathematics teachers).
- The survey revealed several significant differences between expert and nonexpert teachers that support earlier research about successful teachers' understanding of students' mathematical thinking.

THEORETICAL BASIS FOR THE STUDY

Importance of Understanding Students' Thinking

Understanding students' thinking is important because teaching needs to build on what students already know. This idea is not new and in fact can be dated back to 500 B.C. to Confucius' teaching in accordance with students' aptitudes (因材施教; Huang, 1997). Only in the past several decades, however, have researchers empirically investigated the impact of teachers' knowledge of students' thinking on their students' learning.

Carpenter, Fennema, Peterson, Chiang, and Loef's seminal work, *Cognitively Guided Instruction* (CGI; 1989), is one such exemplary line of research in this area. CGI is a professional development program that is intended to increase teachers' understanding of the knowledge that students bring to the learning process and how they connect that knowledge with formal concepts and operations. The program aims to help teachers develop not only a knowledge base for students' conceptions and problem-solving strategies that they can use in planning their instruction but also skills for listening to students and interpreting their thinking. Carpenter et al. (1989) found that CGI students scored significantly higher than control students on complex addition and subtraction items as well as on problem-solving items.

Similarly, Clarkson and Presmeg (2008) noted that teachers can make use of the knowledge they have gained from understanding students' thinking to guide their decision making about subsequent instructional steps. In particular, such understanding can help teachers determine which activity should be planed next, which piece of content should be used to best support students' development of foundational understanding, and which questions should be asked to help students develop a richer understanding. Henderson (2003) referred to teachers as implicit psychologists because teachers regularly make judgments about their students' thinking processes based on students' behaviors in and out of the classroom as well as their own personal psychological theories.

Listening to and understanding students' thinking has been widely promoted and supported in the education community. For instance, the U.S. *Standards* (NCTM, 1991) highlights analysis of students' thinking as one of the key tasks of mathematics teaching because it promotes a learning environment that is conducive to and respectful of students' own sense making and intellectual autonomy (Crespo, 2000; Davis, 1996; Kamii, 1989; Perry, MacDonald, & Gervasoni, 2015). Although it is believed that teachers who are effective know how to access students' thinking (see Moulthrop, Calegari, & Eggers, 2006), Ball (1993) reminds that doing so is difficult, particularly when students' ideas diverge from what teachers have prescribed. Correspondingly, Ball (1997) commented that teachers need a rich and complex understanding of content and students in order to understand students' thinking and appraise their work.

Understanding Students' Thinking Before, During, and After Lessons

Although clear evidence demonstrates the importance of understanding students' thinking for effective teaching (e.g., Beswick, 2008; Cai, Ding, & Wang, 2014; Cai, Kaiser, Perry, & Wong, 2009; Confrey, 1990; Davis, 1997; Thompson, 2002), there is limited research on how teachers acquire their understanding of students' mathematical thinking. Rather, greater attention seems to have been paid to in-class teaching. For example, professional noticing has mainly been framed as a metacognitive process with a focus on classroom events and teachers' corresponding perceptions (Carter & Amador, 2015; Mason, 2011). Star and Strickland (2008) pointed out that teachers may notice a variety of elements, such as environment, classroom management, tasks, content, and communication. Distinguishing from other types of noticing, Jacobs, Lamb, and Philipp (2010) highlighted professional noticing of students' mathematical thinking to include attending to students' thinking, interpreting this thinking, and then deciding how to respond on the basis of this noticing within classroom contexts.

Bishop's (1976) advice of "buying time" for teachers when students' thinking remains unclear is another example of how teachers can approach their understanding of students' in-class thinking. Specifically, Bishop (1976) proposed that teachers may "pause, smile, repeat the statement [a student made] . . . [and] ask the children

'What does anyone else think about that?'" (p. 45). Using this technique, teachers can acquire additional information about students' thinking and how that thinking fits within the work of the class, information which helps them to move the class forward (Borko, Roberts, & Shavelson, 2008).

Clarkson and Presmeg (2008) noted that although many of the decisions teachers make are based on students' thinking as it occurs during class time, teachers should also develop such understanding in contexts outside of class time. It is believed that effective teaching involves thoughtful and intentional planning, in-the-moment decision making, and post-lesson reflection (Cai et al., 2009; Cai & Ding, 2017). All of these activities ought to work complementarily in order to promote thoughtful and intentional planning for subsequent lessons. Therefore, it is important for researchers and educators to consider all three of these time periods. In this study, we investigate how teachers attempt to understand students' thinking before, during, and after lessons.

Understanding Students' Thinking in the Chinese Context

Chinese students' strong mathematics performance in several international comparative studies (e.g., TIMSS, PISA) has aroused many researchers' interests, both locally and internationally. In recent years, many researchers, educators, and policymakers have sought the reasons for Chinese students' mathematics success. Such efforts have included visiting mathematics classrooms in China, importing Chinese mathematics textbooks and practical books, and inviting Chinese mathematics teachers to teach local students in other countries.

Some researchers have investigated Chinese students' mathematical understanding from a cognitive perspective; however, such studies are relatively limited. For instance, Cai (2000) compared Chinese and U.S. students' mathematical thinking while they were working on routine and nonroutine problem-solving tasks. He found that on routine tasks, Chinese students outperformed U.S. students on execution, translation, and planning components but not on the integration component; meanwhile, on nonroutine tasks, Chinese students performed better on constrained-process tasks whereas U.S. students performed better on open-process tasks. In addition to analyzing students' correctness on tasks, Cai (2000) examined the strategies that students applied to further understand students' mathematical thinking and reasoning. This analysis revealed that U.S. students were much more likely than Chinese students to use drawing strategies and representations whereas Chinese students were much more likely to use numerical or symbolic representations. Similarly, in another study, he found that a higher percentage of Chinese students used generalized strategies than did U.S. students. Furthermore, after excluding the Chinese students who had used generalized strategies from the analysis, the success rates between the two groups of students were nearly identical. These findings suggest that Chinese students' mathematics success may be due to their preference for generalized strategies (Cai, 2004).

An, Kulm, Wu, Ma, and Wang (2006) investigated U.S. and Chinese mathematics teachers' understanding of students' mathematical thinking at the middle school level, finding that both groups of teachers valued the importance of such understanding in students' mathematics learning. Differences, however, were revealed in the ways that teachers gauged the students' understanding: Whereas 56% of the U.S. teachers evaluated students' thinking based on the students' explanations and discussions, about 72% of the Chinese teachers did so. Meanwhile, whereas 63% of the Chinese teachers approached students' thinking via checking students' homework and talking with them, only 19% of U.S. teachers did so. In addition, An et al. reported that the U.S. teachers adopted a variety of approaches for promoting students' thinking abilities, such as cooperative learning, asking different questions, engaging students in various activities, using inquiry and creativity, and using problem-solving strategies. Forty-four percent of Chinese teachers, meanwhile, used different levels of practice compared to only 7% of U.S. teachers who did so.

During one interview, Darling-Hammond (1994) remarked that teachers who are effective know how to access students' thinking, including what the students understand and how they learn and perceive (see Moulthrop et al., 2006). According to Darling-Hammond, good teachers are able to plan a curriculum based on where students are in their understanding, promote them to where they need to go, and plan the ideas and the corresponding scope and sequence that students need to encounter in order to help them deepen their understanding and enhance their proficiency.

In China, there is a three-tiered ranking system for teachers consisting of second grade (or junior grade), first grade (or middle grade), and senior grade (or high grade). In order to pass from one grade to the next, teachers must prove that they have contributed something tangible to the teaching community, including mentoring new teachers, writing for publication, winning awards, and so on. Whereas first grade teachers are only evaluated at the school level, the other two grade levels must be evaluated externally at the district level (Isaacs, Creese, & Gonzalez, 2015). In addition, although not included in the official ranking system, "master teacher" is an honorary title that is also widely used to recognize teachers with outstanding performance. This title is commonly perceived to surpass the level of senior grade teacher.

Table 1. Profiles of Participating Expert and Nonexpert Teachers

	Expert teachers (n = 242)	Nonexpert teachers (n = 312)
Gender		
Female	206 (87.7%)	260 (83.6%)
Male	29 (12.3%)	51 (16.4%)
Highest education level		
Technical secondary school or college	67 (27.7%)	69 (22.1%)
Undergraduate or higher	175 (72.3%)	243 (77.9%)
Location of schools		
City	213 (88.8%)	214 (69.0%)
Town or village	27 (11.3%)	96 (31.0%)
Years of teaching	19.85 years (SD = 6.42)	7.16 years (SD = 5.03)

The present study aims to examine whether expert teachers, as suggested by Darling-Hammond, are different from nonexpert teachers in their perceptions and behaviors related to understanding students' thinking before, during, and after lessons in China. This investigation contributes to our theoretical knowledge about teachers' understanding of students' thinking for effective teaching. It also contributes by identifying feasible techniques that teachers can use to discover what students already know and what students think about mathematics before, during, and after lessons.

RESEARCH METHODS

Participants

This study aims to investigate the approaches that teachers use to understand their students' thinking before, during, and after lessons as well as the reasons behind those approaches. In particular, this study aims to understand whether expert teachers have significantly different behaviors and perceptions than their nonexpert colleagues related to understanding students' thinking. A group of 602 elementary school mathematics teachers from one city in China were invited to participate in the study; 554 (92%) of this group provided valid responses. Of this final sample, 43.7% held a senior title or higher and were regarded as expert teachers whereas the remainder were regarded as nonexpert teachers.

Table 1 lists the 554 participating teachers' profiles. The majority of the teachers were females; however, there was no significant difference between the expert and nonexpert groups in their gender distributions. More than 70% of the participating teachers received an undergraduate education or higher, with the distributions between the two groups again similar. It can be seen that significantly more expert teachers were working in city schools than were nonexpert teachers, χ^2 (1, N = 550) = 30.291, p < .001, φ = 0.24. Not surprisingly, expert teachers had significantly more years of teaching experience, $t(433)$ = 24.824, p < .001, d = 2.23.

Instruments and Data Collection

This study included designing a survey, which consisted of two parts. Part I, containing seven items, focused on participants' background information (e.g., gender, highest education level, location of schools, etc.). Part II comprised five subsets of items that mainly covered teachers' perceptions (what do teachers think) and behaviors (what do teachers do) related to understanding of students' thinking. The three time periods (i.e., before, during, and after lessons) were another aspect investigated in the survey. Correspondingly, Items 1–4 addressed teachers' general behaviors and perceptions related to understanding students' thinking. Items 5–7 asked teachers about their behaviors related to understanding students' thinking before lessons, including the length of time spent, types of lessons, and approaches adopted. Items 8–12 asked teachers about relevant behaviors during lessons, and Items 13–16 concerned post-lesson behaviors. The last two items, Items 17 and 18, invited teachers to provide open-ended responses to illustrate how their behaviors related to understanding students' thinking influenced their present as well as future teaching. All but the last two items were multiple choice. Because the questionnaire was of the self-report type, caution related to the respondents' potentially giving socially desirable responses should be taken into account when interpreting the results.

A pilot test of the survey was conducted in a teacher professional development workshop involving 35 elementary school mathematics teachers. The initial version of the survey was modified based on the results of the pilot survey. In addition, a group of experts consisting of mathematics education professors, school principals, and teacher leaders was invited to ensure high content validity of the survey items.

Table 2. Uses for Understanding Students' Thinking

	Expert teachers	Nonexpert teachers
Setting up achievable learning targets†	184 (76.7%)	214 (69.7%)
Understanding students' starting points in learning new topics	175 (72.9%)	208 (67.8%)
Predicting students' difficulties in learning new topics	195 (81.3%)	238 (77.5%)
Selecting instructional methods aligned with students' cognitive levels	170 (70.8%)	208 (67.8%)
Designing and organizing teaching activities from students' perspectives**	189 (78.8%)	210 (68.4%)
Adjusting instruction according to students' learning	175 (72.9%)	215 (70.0%)
Others	0 (0.0%)	3 (1.0%)

† $p < .1$, * $p < .05$, ** $p < .01$

As reported earlier, the study's response rate was 92%. To further establish the stability of participants' responses, the entire sample of 554 teachers was first split into halves randomly following which a comparison between the two subsamples was conducted using Chi-square tests. The results showed that the responses from the two subsamples were similar on all 16 multiple-choice items, with p values ranging from .19 to .90.

Data Process and Analysis

The data from the questionnaire were analyzed using quantitative methods. Descriptive statistics (e.g., frequency and percentages) were used to describe participating teachers' overall perceptions and behaviors related to understanding students' thinking. Chi-square tests were used to examine potential differences between expert and nonexpert teachers on all 16 multiple-choice items in Part II. When significant differences were observed, follow-up Chi-square tests at the option level were then carried out. Moreover, effect sizes (φ or Cramer's V[1]) were calculated when significant differences were detected.

RESULTS

As described above, Part II of the survey consisted of five subsets of items. Because the last two items were open ended, the results of their analysis are not included in the present report. The main findings reported in this study correspond to each of the four subsets of Part II items followed by a comparison of common items across different instructional time periods. A brief summary of this data set has been included in Yu and Cai (2017) in Chinese. The results reported in this paper are based on the same data set but with expanded analysis and presentation.

Teachers' General Behaviors and Perceptions Related to Understanding Students' Thinking

When asked to identify the occasions on which they would take action to understand students' thinking, teachers responded with "learning a new topic" with the highest frequency (52.7%) followed by "lessons which may cause students to have difficulties" (43.1%) and "the first lesson in each unit" (42.6%). Although the expert teachers cited these three reasons more frequently than the nonexpert teachers, the overall differences between the two groups of teachers were small.

All of the suggested uses for understanding students' thinking were frequently selected by participating teachers, with the percentages ranging from 69.1% for "selecting instructional methods that are aligned with students' cognitive levels" to 79.2% for "predicting students' difficulties in learning new topics." Further analyses revealed a significant difference between the two groups of teachers in terms of their recognition of the various functions related to understanding students' thinking, χ^2 (7, N = 2381) = 16.947, p = .018, Cramer's V = 0.08 (see **Table 2**). In particular, considerably more expert teachers believed that understanding students' thinking was helpful in setting up achievable learning targets suitable for their students, χ^2 (1, N = 554) = 3.733, p = .053. In addition, significantly more expert teachers responded that such practices would help them to design and organize teaching activities from the students' perspectives, χ^2 (1, N = 554) = 7.877, p = .005, φ = 0.12.

A greater number of teachers (79.3%) responded that they attempted to understand students' thinking before lessons than they did during (53.2%) or after (55.2%) lessons. For all three time periods, more expert teachers indicated that they would attempt to understand students' thinking than did nonexpert teachers, with the difference reaching a significant level, χ^2 (4, N = 1039) = 11.479, p = .022, Cramer's V = 0.11 (see **Figure 1**). In fact,

[1] The phi coefficient, φ, is used to check the strength of the association of a 2 × 2 contingency table, whereas Cramer's V is used for tables with more than two rows or more than two columns. According to Cohen (1988), values greater than .10 indicate small effect sizes, values greater than .30 indicate medium effect sizes, and values greater than .50 indicate large effect sizes.

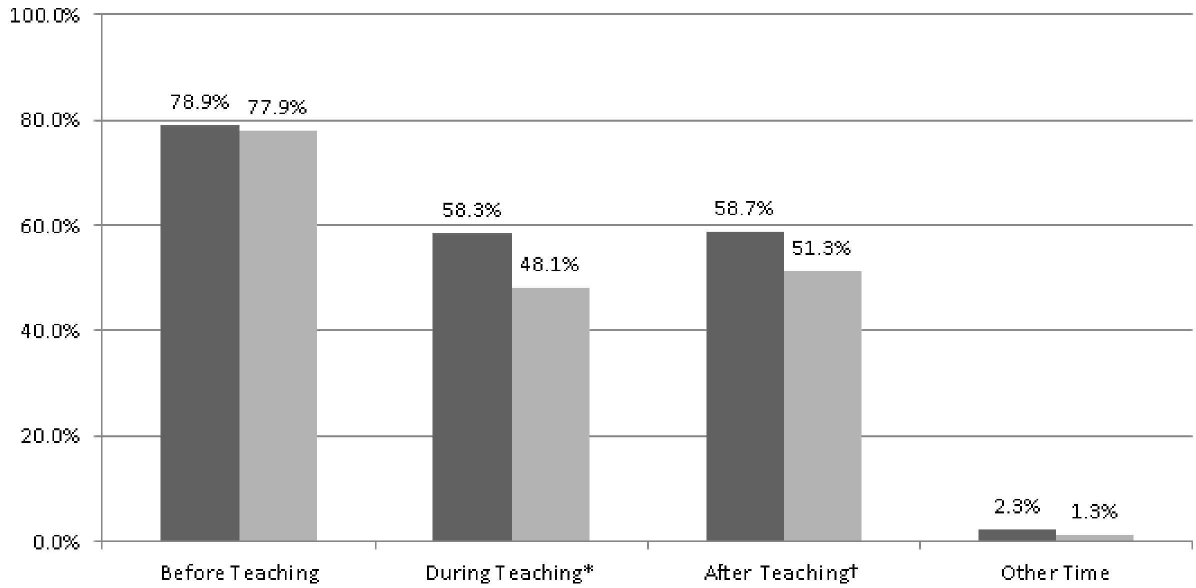

Figure 1. Time periods during which teachers indicated that they attempted to understand students' thinking. Dark bars represent expert teachers and gray bars represent nonexpert teachers; † $p < .1$, * $p < .05$

Table 3. Length of Time Spent Understanding Students' Thinking before Teaching

	Expert teachers	Nonexpert teachers
0–5 minutes	73 (30.2%)	97 (31.1%)
5–10 minutes	53 (21.9%)	59 (18.9%)
15 minutes	3 (1.2%)	4 (1.3%)
Dependent on the type of lesson**	148 (61.2%)	152 (48.7%)
Dependent on the extent to which teachers knew students	115 (47.5%)	145 (46.5%)
Others	1 (0.4%)	1 (0.3%)

** $p < .01$

the significant difference was observed during teaching, χ^2 (1, N = 554) = 5.672, p = .017, φ = 0.11, with considerably more expert teachers opting to do so after teaching, χ^2 (1, N = 554) = 3.006, p = .083.

Teachers who indicated that they never or seldom attempted to understand their students' thinking during their lessons were asked to provide the reasons behind this response. Among these teachers, more than 70% gave responses, with "hard to find time" being the most frequently cited reason (68.0%). About 28.3% of these teachers reported that attempting to understand students' thinking was too difficult, and 23.2% reasoned that they already understood their students' thinking. A comparison between the two groups of teachers revealed a significant difference, χ^2 (6, N = 389) = 15.308, p = .018, Cramer's V = 0.20, with the follow-up Chi-square tests revealing that this significant difference was due to the reason "don't know how to," χ^2 (1, N = 554) = 11.839, p < .001, φ = 0.15 (4.5% of expert teachers vs. 13.1% of nonexpert teachers).

Understanding Students' Thinking before Lessons

The second subset of items in Part II of the survey addressed teachers' behaviors related to understanding students' thinking prior to carrying out lessons. Based on the results, 54.8% of the teachers claimed that the length of time they spent attempting to understand students' thinking depended on the types of lessons they were conducting; 47.5% believed that the extent to which they knew their students was another important factor. Teachers also estimated the time that they spent attempting to understand students' thinking before lessons, with 31.1% of the teachers reporting that they spent 0–5 minutes, 20.5% reporting that they spent 5–10 minutes, and only 1.3% reporting that they used approximately 15 minutes to do so. Whereas there was no overall significant difference between the two groups of teachers on the amount of time spent attempting to understand students' thinking before lessons, significantly more expert teachers (61.2%) related such attempts to the types of lessons involved than did nonexpert teachers (48.7%), χ^2 (1, N = 554) = 8.494, p = .004, φ = 0.12 (see **Table 3**).

Regarding the types of lessons, 81.2% of the teachers suggested that they would attempt to understand students' thinking before the lesson when they planned to teach a new topic. Specifically, about one quarter of the teachers would do so for both exercise and revision lessons. In comparison, teachers indicated that they would do so less

Table 4. Adjusting Predetermined Teaching Plan According to Students' Learning

	Expert teachers	Nonexpert teachers	χ^2	p	φ
Adjusted plan	190 (78.5%)	183 (58.7%)	24.434	< .001	0.21
Considered adjusting plan	71 (29.3%)	140 (44.9%)	13.945	< .001	0.16
Did not consider adjusting plan	6 (2.5%)	23 (7.4%)	6.576	.010	0.11

Note. df =1, N = 554

frequently if the lessons were dealing with a discussion of test papers. There was no overall significant difference between expert and nonexpert teachers with respect to the relationship between teachers' behaviors in understanding students' thinking and the types of lessons.

Among the various approaches that teachers indicated that they used to understand their students' thinking prior to lessons, "posing questions before lessons" was the most popular (82.9%). The next two most popular approaches were "selecting some students to interview" (57.8%) and "communicating with colleagues" (55.5%). In comparison, understanding students' thinking via "students' rehearsal" was the least selected approach (7.1%). A Chi-square test revealed that there was an overall significant difference between expert and nonexpert teachers on the selection of approaches, χ^2 (7, N = 1598) = 19.608, p = .006, Cramer's V = 0.11. A further investigation showed that significantly more expert teachers (56.2%) indicated that they relied on their own teaching experiences to understand students' thinking before lessons than their nonexpert colleagues (48.6%), χ^2 (1, N = 554) = 10.047, p = .002, φ = 0.14.

Understanding Students' Thinking during Lessons

During lessons, more than two thirds of the teachers claimed to pay attention to their students' learning whenever possible and make adjustments to their predetermined teaching plan accordingly. Only about 5.2% of the teachers claimed that they seldom engaged in this approach during their teaching and, furthermore, did not consider changing their predetermined teaching plan. Around 0.9% of the teachers claimed that they would be satisfied if they completed the predetermined teaching plan, such that they tended to pay little attention to their students' learning during their lesson. Significantly more expert than nonexpert teachers indicated that they would adjust their predetermined teaching plan based on their observations of students' in-class learning experiences, χ^2 (3, N = 613) = 45.075, p < .001, Cramer's V = 0.27 (see **Table 4**). No teacher claimed that they never paid attention to students' thinking during their lessons.

When asked how sure they were that they would adjust their predetermined teaching plan according to students' in-class performance, 46.0% of the teachers claimed they would "certainly" make adjustments, 47.5% claimed that they would "sometimes" make adjustments, and 10.9% claimed that they would "occasionally" make adjustments. Whereas significantly more expert teachers claimed that they would "certainly" make adjustments during teaching based on how well their students were learning, χ^2 (1, N = 554) = 18.540, p < .001, φ = 0.18, significantly more nonexpert teachers claimed they would "occasionally" do so, χ^2 (1, N = 554) = 7.919, p = .005, φ = 0.12. These differences led to an overall significant difference between the two groups of teachers, χ^2 (3, N = 576) = 28.204, p < .001, Cramer's V = 0.22. In fact, out of the 544 teachers, only one nonexpert teacher claimed to make such adjustments during lessons.

Similar to the results for teachers before lessons, most teachers indicated that they posed questions to students in order to understand their thinking during their lessons (94.2%). Engaging in "observation of students' performance in activities" was another popular approach that teachers indicated that they used to understand their students' thinking during their lessons (85.9%). Although "choosing students to talk with" and "relying on their own teaching experiences" were not as popular as the two aforementioned approaches, more than 40% of the teachers claimed to adopt these two approaches during their lessons. In fact, although there was no significant difference between expert and nonexpert teachers on the selection of the two most popular approaches, significantly more expert teachers also used the other two methods to understand their students' thinking during lessons (see **Figure 2**; "choosing students to talk with": χ^2 (1, N = 554) = 6.485, p = .011, φ = 0.11; "relying on teachers' own teaching experience": χ^2 (1, N = 554) = 11.004, p < .001, φ = 0.14). Overall, there was a significant difference between the two groups of teachers in terms of the selection of different methods to understand students' thinking during their lessons, χ^2 (5, N = 1486) = 22.134, p < .001, Cramer's V = 0.12.

When selecting students to answer posed questions during lessons, the majority of the teachers indicated that they did so based on whether or not the students were raising their hands (82.1%) or based on students' in-class learning (82.8%). Two other important factors were "the extent to which students paid attention to learning" (76.1%) and "students with learning difficulties" (68.1%). Students' genders (2.7%) and distances from the teacher (1.8%) were the least considered factors. Some teachers (15.4%) also selected students to answer questions based on who learned well. Overall, both the expert and nonexpert teachers indicated that they adopted similar criteria in selecting students to answer questions in class, χ^2 (8, N = 1834) = 7.455, p = .49.

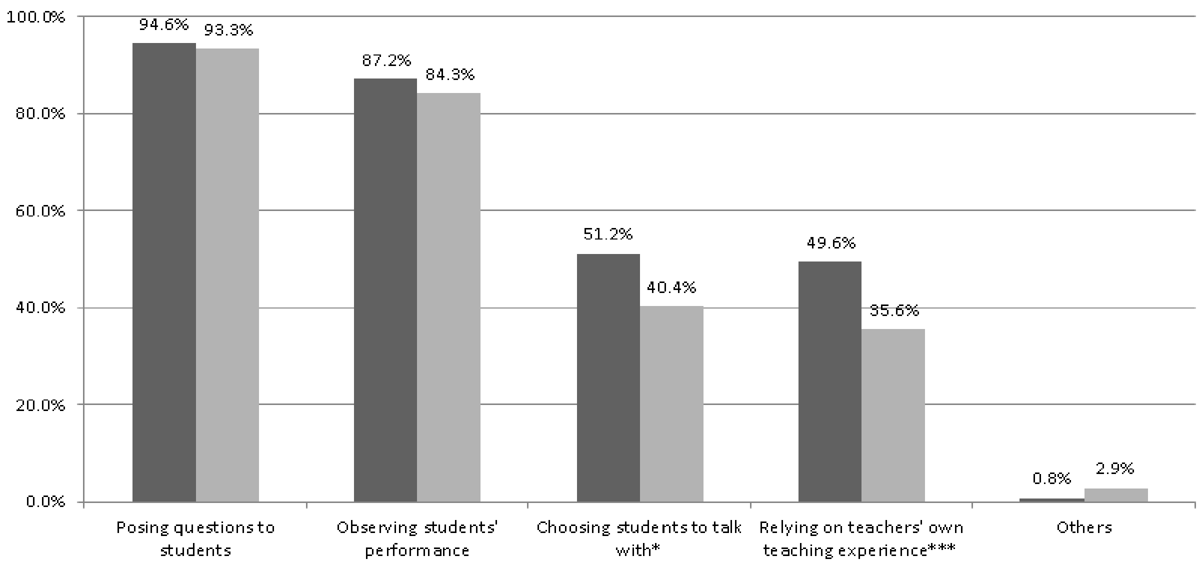

Figure 2. Teachers' selection of approaches for understanding students' thinking during lessons. Dark bars represent expert teachers and gray bars represent nonexpert teachers; * p < .05, ** p < .01, *** p < .001

Table 5. Adjusting Predetermined Teaching Plans According to Students' Learning

	Expert teachers	Nonexpert teachers	χ^2	p	φ
Students' existing knowledge and life experiences	167 (69.0%)	199 (63.8%)	1.660	0.198	--
Students' learning difficulties	214 (88.4%)	273 (87.5%)	0.111	0.739	--
Students' preferred learning styles	115 (47.5%)	101 (32.4%)	13.148	< 0.001	0.15
Correctness of students' answers	129 (53.3%)	165 (52.9%)	0.010	0.922	--
Correctness of students' exercises	117 (48.3%)	123 (39.4%)	4.420	0.036	0.09
Students' posed questions that they didn't understand	107 (44.2%)	103 (33.0%)	7.266	0.007	0.12
Students explained their thinking about new learned knowledge in their own words	120 (49.6%)	122 (39.1%)	6.089	0.014	0.11

Note. df =1, N = 554

Teachers responded that they paid attention to a number of issues related to students' thinking during lessons. "Students' difficulties in learning new topics" (88.2%) and "students' existing knowledge and life experiences related to new learned topics" (66.3%) were the top two issues that concerned the teachers. Although on these two issues, there were no significant differences between expert and nonexpert teachers, the two groups of teachers had an overall significant difference on the various issues related to students' thinking that they paid attention to during lessons, χ^2 (7, N = 1486) = 33.060, p < .001, Cramer's V = 0.13 (see **Table 5**). In general, for all possible issues, more expert than nonexpert teachers responded that they paid attention during their lessons to four out of the seven suggested issues, with the differences reaching a significant level but the corresponding magnitudes being small.

Understanding Students' Thinking after Lessons

Regarding the strategies teachers indicated that they used to understand students' thinking after lessons, students' performance on assignments was the most popular choice (96.4%). Students' in-class behaviors were another important aspect that teachers often referred to (students' involvement: 59.1%; students' reactions to the teachers' instruction: 54.4%). Although there was an overall significant difference between expert and nonexpert teachers in terms of the adopted methods, χ^2 (7, N = 1697) = 35.309, p < .001, Cramer's V = 0.14, the differences mainly occurred due to the choice of "students' involvement," χ^2 (1, N = 554) = 5.139, p = .023, φ = 0.10, "students' reactions to the teachers' instruction," χ^2 (1, N = 554) = 7.934, p = .005, φ = 0.12, and "relying on the teachers' own teaching experiences," χ^2 (1, N = 554) = 13.654, p < .001, φ = 0.16 (see **Figure 3**).

As reported, most teachers used students' performance on assignments to understand their thinking after lessons. Most often, they responded that they would analyze items on which many students made mistakes (97.4%) followed by comparing and analyzing the numbers of correct assignments versus incorrect ones (67.8%) and analyzing students' unique mistakes on assignments (62.1%). Some teachers indicated that they would also talk to students about their assignments (45.1%). It was found that there was an overall significant difference between

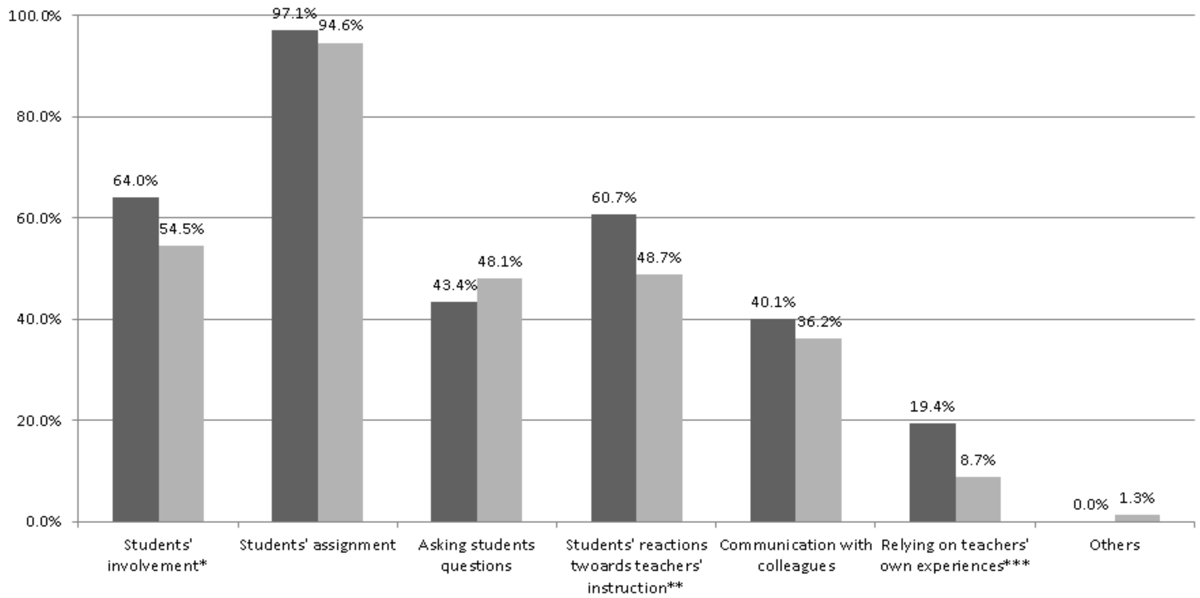

Figure 3. Teachers' stated strategies for understanding students' thinking after teaching. Dark bars represent expert teachers and gray bars represent nonexpert teachers; * $p < .05$, ** $p < .01$, *** $p < .001$

expert and nonexpert teachers on the ways in which they said they used students' assignments to understand relevant thinking, χ^2 (4, N = 1479) = 20.071, $p < .001$, Cramer's V = 0.12. In fact, significantly more expert teachers (72.5%) responded that they would look into students' unique mistakes on assignments to understand their thinking than did nonexpert teachers (54.1%), χ^2 (1, N = 554) = 17.429, $p < .001$, φ = 0.18.

It appears that teachers' decisions to take action to understand students' thinking after lessons were dependent on the kinds of lessons that were conducted. Although no teachers claimed that they never took action to understand students' thinking after lessons, lessons in which "students had learning difficulties" (72.4%), lessons "about problem solving" (64.5%), and "newly taught" (62.7%) lessons were the top three types of lessons after which teachers claimed that they would attempt to understand their students' thinking. Although there was a considerable difference between expert and nonexpert teachers, χ^2 (9, N = 2047) = 16.797, p = .052, significantly more nonexpert teachers (66.7%) said that they would do something to understand students' thinking after teaching "newly taught" lessons than expert teachers (56.6%), χ^2 (1, N = 554) = 5.865, p = .015, φ = 0.10.

Fewer teachers indicated that they would do something to understand their students' thinking "after teaching" (55.2%) than "before teaching" (79.3%). The most frequently cited reason for not doing so was that it was "hard to find time" (72.2%). Slightly more than one fifth of the teachers claimed they did not do so because "they already knew students' learning processes" (20.9%) or "they didn't know how to" (21.9%). Furthermore, about 9.1% of the teachers related their behavior to the difficulty in doing so and 2.7% commented that they never thought to do so before. There was an overall significant difference between expert and nonexpert teachers, χ^2 (5, N = 1479) = 20.071, $p < .001$, Cramer's V = 0.22, with the significant difference observed based on the reason "they didn't know how to", χ^2 (1, N = 554) = 6.699, p = .010, φ = 0.11 (expert teachers: 4.1% vs. nonexpert teachers: 9.9%).

Comparison of Teachers' Behaviors and Perceptions Related to Understanding Students' Thinking across Different Instructional Time Periods

The survey contained some common items that addressed different instructional time periods, including reasons for not attempting to understand students' thinking during and after lessons; strategies adopted before, during, and after lessons; and types of lessons in which they would try to understand students' thinking before and after lessons.

More teachers reasoned that they took no action to understand students' thinking during lessons than afterwards because it was "hard to find time" (33.4% vs. 24.4%) and because it was "too troublesome" (13.9% vs. 3.1%). For both time periods, significantly more nonexpert teachers claimed that they didn't know how to understand students' thinking. Furthermore, five nonexpert teachers "never thought of doing so" after teaching whereas no expert teachers reported this.

Regarding strategies used to understand students' thinking before, during, and after lessons, some commonalities can be seen from **Table 6**. It is not surprising that more teachers responded that they attempted to

Table 6. Teachers' Strategies to Understand Students' Thinking Before, During, and After Lessons

	Before	During	After
Paper and pencil tests	35.4%	--	--
Posing questions to students	82.3%	93.9%	--
Students' rehearsal	7.0%	--	--
Talk with selected students	57.4%	45.1%	46.0%
Communication with colleagues	55.1%	--	37.9%
Relying on teachers' own teaching experiences	48.6%	41.7%	13.4%
Students' in-class performance or engagement	--	85.6%	58.7%
Students' performance on assignments	--	--	95.7%
Students' reactions to teachers' instruction	--	--	54.0%

understand students' thinking via asking questions and observing students' in-class performance or engagement during lessons rather than during other time periods. During all three of the time periods, about half of the teachers said they talked to selected students, and more teachers said they did so before lessons. Although their own teaching experiences were an important resource that the teachers stated they used to understand their students' thinking, much fewer teachers said they relied on these experiences after lessons. Furthermore, across all three of the time periods, significantly more expert teachers responded that they relied on their own teaching experiences to understand students' thinking than did their counterparts.

The data show that teachers' stated behaviors related to understanding students' thinking had some relationship to the types of lessons being conducted. The survey gave a list of lesson types for teachers to choose from. Three types were common to the before and after instructional time periods. Whereas more teachers said they tried to understand students' thinking before lessons focusing on new topics than after, fewer said they did so before teaching exercises or revision lessons than after. Moreover, significantly more nonexpert teachers claimed that they took action to understand students' thinking after lessons focusing on new topics; such a difference was not observed before such lessons.

DISCUSSION AND IMPLICATIONS

This study surveyed 554 elementary school mathematics teachers in China to understand what they think and what they do related to understanding students' thinking before, during, and after lessons. Furthermore, these teachers were classified as experts or nonexperts according to their professional ranks so as to compare the potential differences between the two groups of teachers.

It was found that teachers responded that they attempted to understand students' thinking more often when students were learning new topics or encountering difficulties. No differences were revealed between expert and nonexpert teachers in this respect. However, whereas most teachers agreed that understanding students' thinking had multifunctional purposes, significantly more expert teachers indicated that they used this understanding to design and organize their teaching activities. In fact, significantly more expert teachers said they tried to access students' thinking from a variety of perspectives to make necessary adjustments to their predetermined teaching plan accordingly. In contrast, significantly more nonexpert teachers responded that they merely considered performing such adjustments. In general, students' learning difficulties as well as existing knowledge and life experiences were the two main factors considered by both groups of teachers in understanding their students' thinking.

Concerning the three instructional time periods, more teachers said they tried to understand students' thinking prior to lessons followed by after lessons. "Hard to find time" was many teachers' concern. Although not knowing how to do so was not a popular reason, significantly more nonexpert teachers gave this reason, which might indicate nonexpert teachers' weaker abilities in this area.

When eliciting students' thinking, most teachers indicated that they posed questions before and during lessons. Students' performance on assignments was often said to be used after lessons to understand students' thinking. Although expert and nonexpert teachers were similar in their selection of strategies for understanding students' thinking, it is not surprising that more expert teachers indicated that they relied on their own teaching experiences to access students' thinking during all three of the instructional time periods. Furthermore, when both groups of teachers referred to students' assignments, the expert teachers claimed to look at not only students' common mistakes but also their unique mistakes, whereas more nonexpert teachers looked at the former. Such a difference might imply that expert teachers' approach is more comprehensive, that is, they pay attention to the majority of students as well as individual students.

In addition, it was revealed that the type of lesson had a greater impact on teachers' chosen after-lesson behaviors aimed at understanding students' thinking than the other two instructional time periods. Consistently, the corresponding amount of time that teachers said they spent on this instructional practice largely depended on the types of lessons and also how well the teachers thought they knew the students. Significantly more expert teachers related this approach before lessons to the former factor than did the nonexpert teachers.

Although understanding students' thinking is not an explicit criterion for promoting teachers to senior ranking in China, this study did reveal that there were significant differences between expert and nonexpert teachers in terms of their behaviors and perceptions related to understanding students' thinking. In short, expert teachers' knowledge, practice, and strategies related to understanding students' thinking appear to be more comprehensive. This finding largely supports Darling-Hammond's (1994) argument that teachers who are effective know how to access students' thinking.

As previously mentioned, although teachers' knowledge of students' thinking helps to promote effective teaching, research on how teachers acquire such knowledge is limited (Cai & Ding, 2017). Furthermore, existing studies on this topic have largely focused only on in-class teaching. The survey instrument designed in this study not only covers all three of the major instructional time periods (i.e., before, during, and after lessons) but also investigates teachers' understanding of students' thinking on both the conceptual and practical levels. Thus, this study makes a methodological contribution to the field. Furthermore, the findings of this study increase the field's understanding of teachers' instructional behaviors as well as their underlying reasons for them. The comparison between expert and nonexpert teachers provides a first step towards the identification of feasible techniques for teachers to access students' thinking before, during, and after lessons in order to promote effective teaching.

Future research efforts could move from identifying feasible techniques to identifying effective techniques for understanding students' thinking and then further explore ways to help teachers use appropriate techniques at appropriate times to gain a better understanding of students' thinking. In addition, extending from the present within-system comparison, cross-system comparisons, by extending our knowledge in a systematic way, could provide other insightful information about how teachers from different systems learn to understand students' thinking for effective teaching. In sum, more research on this topic is necessary in order to fully understand the mechanism of teachers' behaviors related students' learning and to further explore ways of fostering high quality teaching.

REFERENCES

An, S., Kulm, G., Wu, Z., Ma, F., & Wang, L. (2006). The impact of cultural differences on middle school mathematics teachers' beliefs in the U.S and China. In F. K.-S. Leung, K.-D. Graf, & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions – A comparative study* (pp. 449–464). New York, NY: Springer.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, *93*, 371–397.

Ball, D. L. (1997). What do students know? Facing challenges of distance, context, and desire to hear children. In B. J. Biddle, T. L. Good, & I. F. Goodson (Eds.), *International handbook of teachers and teaching* (Vol. 2, pp. 769–818). Dordrecht, the Netherlands: Kluwer.

Beswick, K. (2008). Improving middle school students' proportional reasoning. In J. Vicent, R. Pierce, & J. Dowsey (Eds.), *Connected maths*: *Proceedings of the 45th annual conference of the Mathematical Association of Victoria* (pp. 25–39). Brunswick, Australia: Mathematical Association of Victoria.

Bishop, A. J. (1976). Decision-making, the intervening variable. *Educational Studies in Mathematics, 7*(1/2), 41–47.

Borko, H., Roberts, S. A., & Shavelson, R. (2008). Teachers' decision making: From Alan J. Bishop to today. In P. Clarkson & N. Presmeg (Eds.), *Critical issues in mathematics education: Major contributions of Alan Bishop* (pp. 37–70). New York, NY: Springer.

Cai, J. (2000). Mathematical thinking involved in U.S. and Chinese students' solving process-constrained and process-open problems. *Mathematical Thinking and Learning, 2*, 309–340.

Cai, J. (2004). Why do U.S. and Chinese students think differently in mathematical problem solving? Exploring the impact of early algebra learning and teachers' beliefs. *Journal of Mathematical Behavior, 23*, 135–167.

Cai, J. (2005). U.S. and Chinese teachers' knowing, evaluating, and constructing representations in mathematics instruction. *Mathematical Thinking and Learning, 7*(2), 135–169.

Cai, J., & Ding, M. (2017). On mathematical understanding: Perspectives of experienced Chinese mathematics teachers. *Journal of Mathematics Teacher Education, 20*(1), 5–29.

Cai, J., Ding, M., & Wang, T. (2014). How do exemplary Chinese and U.S. mathematics teachers view instructional coherence? *Educational Studies in Mathematics, 85*(2), 265–280.

Cai, J., Kaiser, G., Perry, R., & Wong, N-Y. (2009) (Eds.). *Effective mathematics teaching from teachers' perspectives: National and international studies*. Rotterdam, the Netherlands: Sense.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499–531.

Carter, I. S. W., & Amador, J. M. (2015). Lexical and indexical conventional components that mediate professional noticing during lesson study. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(6), 1339–1361.

Clarkson, P., & Presmeg, N. C. (2008). *Critical issues in mathematics education: Major contributions of Alan Bishop*. Dordrecht, the Netherlands: Springer.

Cohen, J. (1988). *Statistical power and analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Confrey, J. (1990). What constructivism implies for teaching. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning mathematics* (pp. 107–122). Reston, VA: National Council of Teachers of Mathematics.

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education*, *3*, 155–181.

Darling-Hammond, L. (1994). Performance-based assessment and educational equity. *Harvard Education Review, 64*(1), 5–30.

Davis, B. (1996). *Teaching mathematics: Toward a sound alternative*. New York, NY: Garland.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, *28*, 355–376.

Gardner, H. (1999). *The disciplined mind: What all students should understand*. New York, NY: Simon & Schuster.

Henderson, B. (2003). Teacher thinking about students' thinking. *MountainRise: The International Journal for the Scholarship of Teaching and Learning, 1*(1). Retrieved from http://mountainrise.wcu.edu/index.php/MtnRise/article/download/43/85

Huang, C. (1997). *The analects of Confucius (Lun Yu): A literal translation with an introduction and notes*. New York, NY: Oxford University Press.

Isaacs, T., Creese, B., & Gonzalez, A. (2015). *Aligned instructional systems: Shanghai*. Center for International Education Benchmarking (CIEB). Retrieved from http://www.ncee.org/wp-content/uploads/2015/09/Shanghai-Jurisdiction-Report.pdf

Jacobs, V. R., Lamb, L. L., Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*, 169–202.

Kamii, C. (1989). *Young children continue to reinvent arithmetic: Second grade*. New York, NY: Teachers College Press.

Mason, J. (2011). Noticing: Roots and branches. In M. G. Sherin, V. Jacobs, & R. Philipp (Eds.), *Mathematics teacher noticing* (pp. 35–50). New York, NY: Routledge.

Moulthrop, D., Calegari, N. C., & Eggers, D. (2006). *Teachers have it easy: The big sacrifices and small salaries of America's teachers*. New York, NY: Perseus Books.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Perry, R., MacDonald, A., & Gervasoni, A. (Eds.). (2015). *Mathematics and transition to school: International perspectives*. Singapore: Springer.

Star, J., & Strickland, S. (2008). Learning to observe: Using video to improve perspective teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*, 107–125.

Thompson, P. W. (2002). Didactic objects and didactic models in radical constructivism. In K. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolising, modeling, and tool use in mathematics education* (pp. 191–212). Dordrecht, the Netherlands: Kluwer.

Yu, W., & Cai, J. (2017). Survey of senior-ranked teachers and non-senior-ranked elementary mathematics teachers' knowledge of students [in Chinese]. *Journal of Mathematics Education*, *26*(2), 6–13.