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Understanding the Concepts in Probability of Pre-School and Early School Children

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In the Slovenian National Mathematics Curriculum the probability contents are first mentioned in the ninth grade of elementary school (at the age of 14), yet they are introduced informally, only in some first triad textbook sets. The researchers disagree as to the age of children at which they are able to deal with certain probability contents. In view of this fact our aim was to establish the age at which children are able to differentiate among certain, possible and impossible events, and predict the likelihood of various events. 623 pupils of the first three grades of elementary school participated in the study. We presumed that they were able to differentiate among certain, possible and impossible events, and compare the probability of various events, while only half of the children aged 4-5 years participating in the research were equally able. The major difference in their abilities was noticed between the children aged from 4-5 years and the first graders, but there were only slight gender differences. Children of all age groups encountered difficulties at predicting events with equal probability. The first graders can be taught the latter by applying the teaching approach, based on their concrete experience, and by mastering the technique for solving tasks with equal probability. When comparing the results with the opinions of the respondent teachers and pre-school teachers, it is evident that they are under the misconception regarding the children's abilities to solve probability tasks. The majority of the respondents stated that children were able to differentiate among certain, possible and impossible events, and compare the probability of various events not earlier than at the age of eight years; on the contrary, the findings of our research established that children were able to achieve both goals much earlier.
Keywords: Children, 5-8 Years, Probability, Probability Tasks, Equal Probability

INTRODUCTION

Probability is an old mathematical discipline dealing with calculating probability of various events. Out of 21 examined mathematics textbook sets in Slovenia for the first three grades of elementary school some of them also comprise the contents in probability related to various events, despite the fact that this topic is not part of the curriculum. When the new curriculum for elementary school was introduced in Slovenia in 1998 some topics related to statistics and probability were systematically examined for the primary mathematics (Cotič, Hodnik Čadež, 2002). The contents in probability are informally included in some teaching sets used in lower grade lessons. Cotič states about probability teaching in primary schools, as follows: »In elementary school probability teaching and learning is not explicit and formal, but a mere systematic acquisition of experiences, on the bases of which probability is delved into more effectively later (in high school); it is a very demanding topic from the teaching viewpoint, because high school students and university students often have misconceptions about it despite their attendance of formally unobjectionable lessons.« (Cotič, 1999, p.70.) According to Fischbein (1984) the reasons to introduce probability are 'dealing' with uncertain situations, predicting, deciding among

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State of the literature

- There was very intensive research conducted into the establishment of understanding the probability contents of the first graders and pre-school children in the 70-ies and 80-ies of the last century.
- The subject matter of understanding the probability contents with the youngest population is still topical, as it also being delved into nowadays.
- Many researchers contradicted their findings on abilities of children regarding their perception of the probability contents and argued the converse.

Contribution of this paper to the literature

- In our research it was established that children in the first three grades were able to differentiate among certain, possible and impossible events, and compare the probability of various events, while only half of the children aged from 4−5 years were able to do that.
- The information gathered in the research proved to be a good indicator for teachers and pre-school teachers among others, of the abilities of children of different age groups to solve the probability tasks, of to their potential difficulties to cope with and also form a solid basis for probability lesson planning.
- The teaching approach to teach equal probabilities was a certain experiment to determine the manner of teaching the first graders to correctly predict equal probability. We are aware that these results may not necessarily indicate the pupils' understanding of the probability concept, but prove familiarity with a certain technique to establish equal probability.

different possibilities (critical interpretation), problem solving (deliberate action-taking) and developing the thinking ability different from the deterministic one.

As evident, teaching contents in probability have numerous advantages, which other mathematical disciplines lack. Through dealing with the mentioned contents children learn to accept the fact that also negative situations can be encountered, which are not possible to be precisely predicted. The only thing to be done is to critically interpret all the possibilities and choose the one which is most likely to happen. In this way children gather experiences for real life situations, in which it is necessary to decide on the best option out of many on a daily basis. At the same time children have to accept the fact that some events are impossible to happen. So, it is necessary to act deliberately and solve the problem, whereby one should make use of his mode of thinking, different from the one applied at learning other mathematical disciplines.

THEORETICAL BACKGROUND

There was very intensive research conducted into the establishment of understanding the probability contents of the first graders and pre-school children in the 70-ies and 80-ies of the last century. These research works, which shall be presented in detail in continuation, were the key starting points of our research. It should be emphasized that the subject matter of understanding the probability contents with the youngest population is still topical, as it also being delved into nowadays. Let us shortly mention some such research works: Gelman and Glickman (2000) researched the importance of the demonstration and concrete experience with teaching probability contents and established that children better understood more difficult concepts if they actively participated in the corresponding demonstrations. Mills (2007) delved into the attitudes of teachers towards probability contents establishing their positive attitude to the statistics and probability contents, and their wish to be offered the possibility of suitable additional training. Ashline and Frantz (2009) dealt with the connection between proportionality and probability contents, while Chick (2010) was engaged in probability games played at lessons. Van Dooren et al. (2003) was interested in pupils' misconceptions pertaining to the probability contents.

As the most absorbing discussion on the understanding of the probability contents, as already mentioned, was conducted a long time ago among the scientists, such as, e.g.. Piaget, Inhelder, Fischbein, Davies and others, we shall focus more precisely on their findings, and on some conclusions of the most recent research in this field, which are topical for our research. The opinions of various researchers about the abilities of children with regard to solving probability tasks differ a lot. Piaget as well as Inhelder (1951) state that a child in a concrete-operational period is neither able to differentiate between certain and random predictions nor formulate predictions, taking into account his experiences form previous similar situations. In their opinions a child first encounters the concept of probability at the level of his concrete operations, at which time he starts to differentiate between a certain and a possible event (Piaget, Inhelder, 1951; Goldberg, 1966). They also note that the systematic understanding of probability starts not earlier than between the ages of 9 and 12 years and even during that period children solve problems intuitively, and not on the basis of formal reasoning (Piaget & Inhelder, 1951).

Many researchers contradicted their findings on abilities of children regarding their perception of the probability contents and argued the converse, among them Fischbein et al. (1970, 1984) and also Falk Ru. et al. (1980), Davies (1965) and Yost et al. (1962). Yost et al. (1962) criticized Piaget's research mainly because it was based on a child's verbal abilities; they developed »the decision making method«, which was not based on verbal abilities; using it children decide between two boxes (children draw out of the very box, from which they believe to extract the chip of the desired colour), but they do not need to verbalize the answers and do not need to use the expression "most probable", whereas in Piaget's research the box contained chips of two colours and children had to choose the colour, which was more probable to be extracted. (Yost et al., 1962). Thus, Yost et al. proved (1962) that already four-year olds possess some understanding of probability. Goldberg (1966) conducted similar research into the difference between Piaget's method and the »decision-making« method, developed by Yost et al. (1962). Goldberg (1966) minimized the differences between both methods and found out that despite the reduction of the differences children performed better using the »decision-making« method, but the difference was a slight one considering the research carried out by Yost et al. Among others she established that children chose the box containing more chips and not the box in which the proportion of the chips was higher in favour of the desired colour, and further, that the number of failures went up when the situation with chips was to reach equal probability for the desired colour of chips (Goldberg, 1966).

The two above mentioned researches, performed by Yost et al. (1962) and Goldberg (1966), were criticized by Falk et al. (1980), because children only compared the complement probabilities (e.g. four red balls and one blue one and four red balls and three blue ones), and the number of the desired colours was always the same in both boxes. In this way a child can compare only the number of elements of an undesired colour, and besides the winning box always contained fewer balls (Falk Ru. et al., 1980). Falk et al. conducted the research in such a manner that children aged from 4 to 11 years had to compare two different non-complement probabilities using different materials (boxes with balls, a spinner and a roulettes). They found out that younger children tended to choose the option containing a greater number of the desired elements as the winning one, and also that children at their pre-operational level, at which Piaget studied their conservation abilities, e.g. fluids, were not able to take into consideration many dimensions at the same time (children took into consideration only one prevailing characteristic feature at their decision making) (Falk Ru. et al., 1980). Apart from that the authors established that at the age of six years children started to seek the right solution systematically, hence it was crucial to introduce the probability topic already in the first grade of primary school (Falk Ru. et al., 1980).

Hoemann and Ross (1971) emphasize the fact that all the tasks in relation to probability do not presuppose knowledge on probability, as they believe that children decide on the basis of their misperception – they compare only the number of elements and not the relationships between the elements. The latter finding is crucial for the researchers with regard to solving probability tasks. At the same time they agree with Piaget's conclusion that children do not understand probability in their preoperational period.

Chapman (1975) also agrees to Piaget's theory and, the same as Falk et al. (1980) criticizes the research conducted by Davies (1965) and Yost et al. (1962) for two reasons, mainly; firstly, it supposedly provided for the minimum verbal explanation of probability concepts and, secondly, the mentioned researchers did not establish the manner of children's task solving. Further, Chapman (1975) also claims the knowledge Test applied by Davies (1965) and Yost et al. (1962), is not a valid test to establish proportionality, as in their research a child was only supposed to compare the number of elements (coins and balls, respectively), and not provide for solutions based on comparison of the relationships between elements. Consequently, Chapman (1975) concluded that children aged 10 and 11 years were not able to establish the relationships between elements, the fact which, in his opinion, was essential for understanding probability.

Ginsburg and Rapoport (1966) argue the converse stating that children (without ball counting) are able to determine the relationships between elements. They found out that six-year olds were able to estimate the relationships between balls of different colours by drawing lines of different length (Ginsburg, Rapoport, 1966).

Fischbein and his numerous colleagues also elaborated on teaching and learning the probability concepts, thereby concluding that it was possible to teach probability without any major efforts, which had a positive influence on the child's prejudices and misconceptions about the sequence of events and uncertain situations (Fischbein, Gazit, 1984; Fichbein, Pampu, Manzat, 1970). Among other things he found out that under certain conditions learning of probability concepts may have a negative impact (children taught probability topics performed worse at some tasks compared to those children who were not presented with these topics); nevertheless, Fischbein believes it would be possible to avoid this by presenting children with tasks including relationships calculations and probability estimations (Fischbein, Gazit, 1984).

Lately also Gürbüz et al. (2010) dealt with probability teaching and learning. They were trying to establish the effectiveness of the teaching approach based on pupils' active participation, whereby pupils were making numerous experiments pertaining to probability, and were discussing their findings among themselves in the follow-up. In the control group the pupils were deprived of this possibility at lessons (Gürbüz et al. 2010). In the research, in which 50 children participated, it was established that children who were provided with the teaching approach based on the discussion between pupils and teachers performed better than those children who were provided with lecturing lessons only (Gürbüz et al., 2010). Also Andrew (2009) stresses the importance of concrete experience, as he believes that pupils better understand probability contents if they perform experiments related to probability in advance. Thus, it is important that pupils gain experience also by drawing out, thus trying to determine the more likely event. Concepts in probability can be more readily understood if pupils are first exposed to probability via experiment. Performing probability experiments encourages pupils to develop understandings of probability grounded in real events, as opposed to merely computing answers based on formulae (Andrew, 2009). Andrew (2009) further states that pupils who have gained concrete experience in probability develop their understanding on this basis and wish to define the starting points to calculate probability of certain events.

Polaki (2002) also researched probability and suggested four levels of probabilistic thinking, the first being subjective, at which pupils predict the most/least likely event based on subjective judgement, e,g. pupils predict the extracted colour to be red, because this is their most favourite colour (Polaki, 2002). Transitional probabilistic thinking represents the second level, for which it is significant that students are able to predict the most and the least likely event based on quantitative judgement; which is often invalid, and besides, they may revert to subjective judgements (Polaki, 2002). For the third, informal quantitative probabilistic thinking level it is significant, that pupils correctly predict the most and least likely events, based on quantitative judgements and use numbers informally to compare probabilities (Polaki, 2002). At the fourth level of probabilistic thinking, which is a numerical one, (Polaki, 2002), pupils assign a numerical probability and make a valid comparison.

Polaki (2002) among others, was also into teaching contents in probability to pupils aged from 9 to 10 years, whereby he applied two different approaches; in the first one the emphasis was on 'analyses of small-sample experimental data and sample space composition as strategies for tackling probability problems' (Polaki, 2000), whereas the latter group of pupils was provided with a teaching approach at which pupils were challenged to make connections between large-sample experimental data (drawn from computer simulations) and sample-space composition after looking at small sample data and sample-space symmetry' (Polaki, 2000). Both approaches proved effective, as pupils of both groups achieved a higher level of probability understanding (Polaki, 2000). It should be pointed that with the mentioned teaching approaches equal probability was not taught, which was the case in our research.

With probability predictions one has to differentiate between random predictions and intuitive predictions (estimation, solution and outcome prediction, namely). The difference between them is that intuitive predictions are based on some piece of information and a mental operation, respectively, containing direct and global predictive functions, subjected to general behavioural changes (Fischbein, Grossman, 1997).

There are different opinions as to the ability of children regarding their perception of probability concepts. Piaget, Inhelder (1951), Heomann and Ross (1971), as well as Chapman (1975), agree that children are not able to understand probability tasks when they start elementary education. On the other hand, there are many researchers, i.e. Fischbein et al. (1970, 1984), Falk et al. (1980), Davies (1965), Goldberg (1966), Yost et al. (1962), Ginsburg and Rapoport (1966), and the recent ones, i.e. Andrew (2009) and Polaki (2002) who oppose these assumptions and claim that it is crucial to start learning probability topics already in the early school years as their research results show that children are able to perceive probability contents as early as at the level of concrete operations.

ESTABLISHING THE ABILITIES OF CHILDREN TO SOLVE PROBABILITY TASKS

Problem Definition and the Aim of Research

With regard to the mentioned research works revealing different findings about the child's ability to perceive probability concepts our aim was to establish the manner in which pre-school children (4- and 5-year-olds) and children of different age groups (5- and 8-year-olds) in Slovenia deal within these topics, bearing in mind that these topics are not part of the school curriculum, but are part of the pre-school curriculum. Further, we were interested at what age children were able to differentiate among certain, possible and impossible events at the level of graphic presentations, and predict the probability of various events. The abilities to achieve the set goals were established on the basis of their knowledge Test 1 performance (Appendix 1), with children aged 4-5 years, and with children of the first (5-6 years), the second (6-7 years) and the third (7-8 years) grade of elementary school. Further, we aimed at establishing any potential statistically significant differences among different age groups of children and between genders at solving individual tasks.

Upon determining that children were not able to predict the events with equal probability, we posed ourselves a question if children aged from 5-6 years (first grade) could be taught such predictions. For this purpose we developed a teaching approach to teach equal probability and establish its effectiveness with the first graders, which was done on the basis of their learning outcomes, that were examined with employing the knowledge Test 2 devised for the first graders (Appendix 2). In order to test the effectiveness of the teaching approach the first graders were selected, because we wanted to find out whether children are able to argue the basic principles of probability as early as in the first grade. Should this be the case, new possibilities regarding the decision on formal introduction of the selected contents in probability into the mathematics curriculum at the beginning of schooling would arose.

Research Questions

In the research the following three questions were posed:

1. How successful are children of different age groups at solving probability tasks, related to certain, possible and impossible events, and at the comparison of different probabilities of events at the level of graphic presentation (knowledge Test 1, Appendix 1)

2. Do any potential statistically significant differences emerge at individual tasks in knowledge Tests 1 and 2 on basic concepts in probability between genders and among different age groups?

3. Shall the teaching approach for teaching equal probability in the first grade of elementary school be effective?

METHOD

Participants

The sample to establish the ability of children to solve probability tasks was composed of 623 children of sixteen schools and kindergartens in Slovenia, whereby children attending both, urban and rural schools, who were granted the consent to participate, were included. The fewest children participating in the research were in the age group of 4-5 years, i.e. 110 or 18% of all the sampling participants. The share of other age groups was approximately equally large. The proportion of participants was almost equal regarding gender. The greatest deviation between the genders was with the youngsters (58%: 42%), in favour of the boys. The sample was purposeful.

The questionnaire to establish the opinions of teachers and pre-school teachers on probability contents was answered by 141 teachers and pre-school teachers of 18 Slovenian schools and kindergardens.

In order to establish the effectiveness of the teaching approach the sample was composed of four first grade classes (with 68 children already participating in the first test taking). Their performance on tasks checking the understanding of equal probability amounted to 14.9 percent. In the last hour of lessons employing the teaching approach to teach equal probability the first graders took the knowledge Test 2 (Appendix 2), the tasks of which were devised to test, whether pupils were able to pinpoint the examples in which probability of individual events was equal and, if they were able to draw conclusions that probability was equal when half of equally distributed elements in the box in which there were more elements, was equal to the number of elements in another box.

Instruments

The knowledge Test 1 (Appendix 1) comprised 6 tasks, in the majority of which it was required to circle the solution. In the third task children were supposed to continue the sentence, in the fifth task they were asked to finish the sentences and, in the last task the justification of both answers was required. The second, the third and the sixth task were different for younger children due to their lack of knowledge on numbers and colours; different sorts of animals and fruit were used instead. Objectivity was achieved through provision of standard instructions and anonymity.

The validity of the tasks of the knowledge Test 1 was checked with the questionnaire for the teachers and pre-school teachers, in which they indicated their level of agreement with the question whether the tasks of the Test 1 were in line with the set goals: a child should differentiate among certain, possible and impossible events and compare various probabilities of events at the level of their graphic presentation.. It was established that the tasks of the knowledge Test 1 were in line with the set goals, as Cronbach's Alpha Reliability Coefficient, providing for the reliability of the measuring instrument, was 0.624.

The effectiveness of the developed teaching approach for teaching equal probability in the first grade was determined on the basis of the knowledge Test 2, as well (Appendix 2), taken by the first graders in their last hour of lessons on equal probability. The knowledge Test 2 comprised four tasks. The test results proved either the effectiveness or non-effectiveness of the developed teaching experiment to teach equal probability. Objectivity was achieved through training teachers to teach equal probability.

Data Collection

Data was collected by presenting teachers of the second and third grades with the knowledge Test 1 (Appendix 1) on probability tasks. The instructions for the teachers were uniform. Also the pre-school teachers and the first grade teachers were given standard instructions, specifying the manner of questioning, the pictures accompanying a particular task and the place and manner of writing the children's answers. The knowledge Test 1 and oral questioning were being carried out anonymously, from November 2006 to March 2007. Pupils were being tested during their maths lessons, whereas the youngsters were being questioned in kindergardens after breakfast in the morning and after their afternoon rest. The responding child was not removed from his group, he was only temporarily placed at the separate desk, at which the questioning took place, together with his teacher and pre-school teacher, namely. The other children of the group did not hear the answers provided by an individual child.

The effectiveness of the developed teaching approach to teach equal probabilities in the first grade was established by training two first grade teachers to teach equal probabilities with employing this approach, whereas teaching of the other two first grade classes was undertaken by the researcher. Equal probability teaching was employed in two elementary schools and, as mentioned, in four first grade classes.

The acquired data were analyzed by the SPSS 12 computer programme, using descriptive statistics, T-test, Variance, Mann-Whitney U test (used to test the equality of medians between two groups), Kruskal-Wallis test (to compare more groups of sample data) and contingency tables with $\chi 2$ statistics.

RESULTS

In continuation the results of children of different age groups at solving tasks of the knowledge Test 1 are presented. More detailed results and findings are presented in Škrbec (2008).

Almost all the tasks relating to the children's differentiating among certain, possible and impossible events were correctly solved by more than half of the respondents, which is well evident from Figure 1. The only exception was the 3.b task, in which the sentence was supposed to be finished providing the answer as to the impossibility of extracting objects by Mojca out of her bag. The correct answers were considered, as follows: the colour of the kerchief or a toy that was not in the bag, and also other objects that were not listed to be in the bag. The percentage of the correct answers of the 3.b task was low, mainly due to answers provided by younger children. Only a small proportion (10.8%) of pre-school children and the first graders named some other toy that was not in the bag, whereas more second graders and third graders named the kerchief which was not in the bag. The difference in solving this task could be due to the different bag contents, namely the youngest age groups were presented with bags containing drawn toys, whereas the older age groups were presented with bags containing kerchiefs of different colours. Our assumption is that much younger children would name the other colour than a toy. This task requested from children to think about the objects that were not present in the bag. Children were namely considering which objects were least likely to be extracted by the girl and not which were impossible to extract.

Among others it was established that 4-5 – year-olds encountered most difficulties with the word »certain«, as both tasks, which were related to certain events, were correctly solved by 42.8% preschool children. These difficulties were also noticed in other age groups, but not to such an extent.

Also on the tasks relating to the comparison of probabilities of various events children performed well, as in only one of the tasks the average performance was below 60%, which is evident from Figure 2. In a very negative sense the answers to the 6.b task set out of the ordinary solutions, as only 16.6% of children responded that it did not matter from which box the girl should extract the objects. This was a rather difficult task for each age group, which indicates that children aged four to eight years are not able to predict the outcomes of events with equal probability. Children also had to justify their answers to the sixth task. It was established that almost everyone who answered the question in the 6.a task correctly (the probability was not equal) did not do this by merely guessing, as they properly justified their answers, which was done only by 1.8% of the research participants.

Figure 3 exhibits tasks solving from the point of view of both goals and individual age groups. It can be stated that 72% of all the respondents differentiate among certain, possible and impossible events (1st goal), i.e. the majority of the third graders (78.1%), and a smaller proportion of the second graders (77.3%). Also the first graders are good at differentiating among such events. (70.8%). Children in the last year of the kindergarden performed slightly worse, (53.8%), however it can be stated that more than half of them were able to solve the task.

From the Figure 3 it is also evident that children performed only slightly worse at comparing different events (2nd goal). In total 66% of the participating children predicted the outcomes correctly. Again, the third graders performed best with 73.2% of the correct answers, followed by the second graders (71.3%) and the third graders (65%). Only one half (49.9%) of the children aged from 4-5 years compared various events correctly. As already mentioned all age groups experienced most difficulties at predicting outcomes of events with equal probability. It was established that school children were able to predict various events.

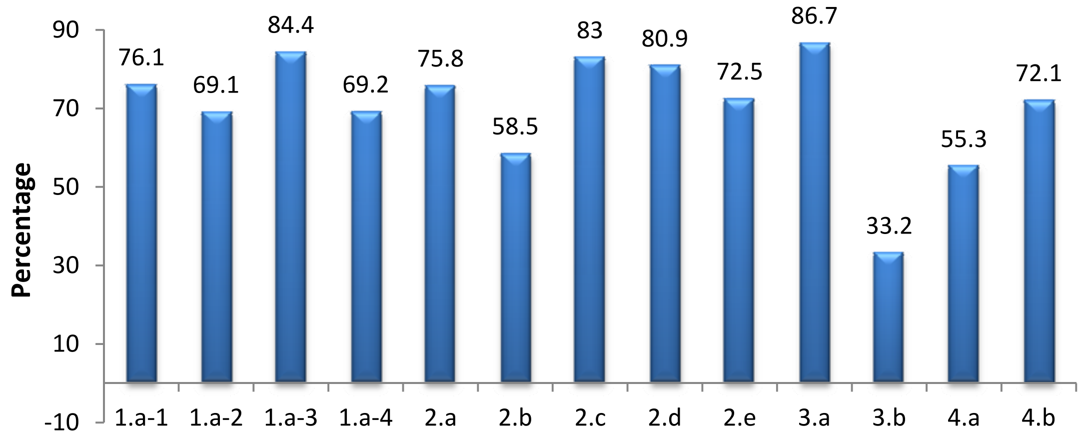

Figure 1. The percentage of correctly solved tasks relating to the children's differentiating among certain, possible and impossible events

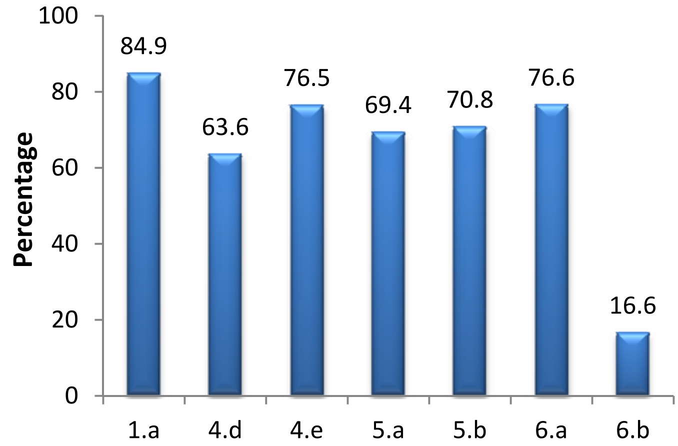

Figure 2. The percentage of correctly solved tasks relating to the comparison of probabilities of various events

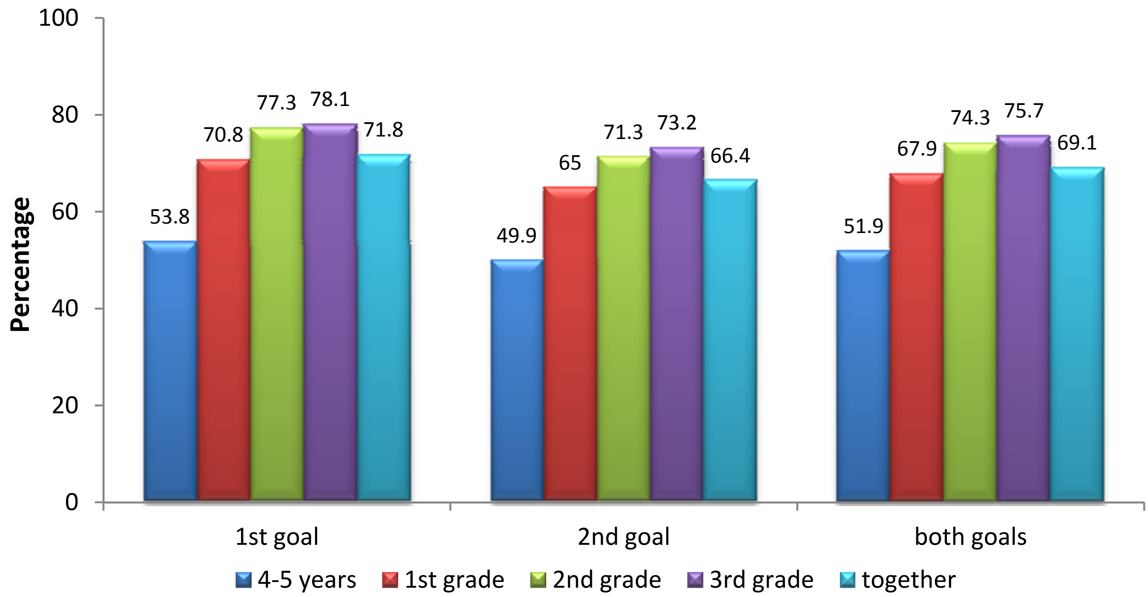

Figure 3. Average performance on task solving at the knowledge Test 1 with regard to the goals

Upon examination of the whole Test 1 it can be established that children of all age groups were able to solve probability tasks, as more than 50% of the tasks were correctly solved in all the groups. The youngest children performed worst (51.9%), whereas much better results were achieved by the first graders (67.9%), followed by the second graders (74.3%), and slightly better third grade performers (75.7%). The average performance of all the respondents at the whole knowledge Test 1 is 69.1%, which again proves the ability of children to solve probability tasks.

The greatest difference in performance was noticed between the youngest age groups, with the first graders performing statistically better than children aged from 4-5 years on as many as 15 tasks of 23 ones. The first graders performed statistically better than 4-5 – year-olds on tasks listed in Table 1.

The difference between these age groups can be accounted for by the developmental stage of the respondents, which according to Piaget, is the period of transfer from preoperational thinking to the concrete operational stage and due to the fact that the first graders are used to similar work (test taking, attentive listening, collaboration with a teacher, answering, longer concentration span).

A statistically significant difference between the first and the second graders was observed twelve times, with the first graders performing better than the second graders three times. The second graders performed statistically better than the first graders on the tasks in the Table 2.

Whereas the first graders performed statistically better on the following three tasks (Table 3):

This difference can be attributed to two different knowledge tests, as these two age groups were presented with a slightly different knowledge tests for reasons of the first graders' lack of knowledge on reading and writing and their potential ignorance of numbers or colours. Thus, the knowledge test for the first graders included toys and fruit instead colours and numbers. Besides, the teacher read out the tasks to the first graders, who responded orally, whereas the second graders were asked to circle or write down their answers.

A statistically significant difference between the second and the third graders was experienced in 5 tasks, with the third graders performing statistically better than the second graders on the tasks listed in Table 4.

In the task 4b ($\chi2=6.247$, $p<0.01$) the second graders attained statistically better results than the third graders. The task 3b, requesting from children to expand their depth of thinking taking into consideration the objects, not present among the elements, was solved better by the second graders; however, the difference was not statistically significant. Bad scores in these tasks and better performance of the younger ones indicate that schoolchildren are presented with few tasks developing their thinking ability other than the deterministic one. Apart from that children attending school for more years tend to indulge in deterministic thinking.

The influence of gender on solving probability tasks was noticed in five tasks. Girls performed better on three tasks (Table 5).

Boys were also better performers on three tasks. They achieved better results at the tasks listed in table 6.

Parallel to taking Test 1, 141 pre-school teachers and teachers expressed their opinions as to the appropriate age of children to reach both goals. When comparing the results with the opinions of the respondent teachers and pre-school teachers it is evident that they misperceived the children's abilities to solve probability tasks. The majority of the respondents (36.7%) stated that children were able to differentiate among certain, possible and impossible events, and compare probability of various events not earlier than at the age of eight years. 29.6 % of the respondents considered children were able to do all this at the age of seven. Also, as regards the age at which children are able to compare probabilities of various events, the majority (41.6 %) of the teachers and pre-school teachers indicated this age to be eight years, whereas 22 % of them believed this age to be seven years. On the contrary, the findings of our research established that children were able to achieve both goals much earlier.

As regards the first research question it can be concluded that more than half of the participating children correctly solved most of the tasks. Children performed better on differentiating among certain, possible and impossible events than on comparing different probabilities. The task with equal probability was most difficult for them to solve. Differences in successful task solving were observed among different age groups, but were less substantial with advancing age; they were also observed between genders to a lesser extent.

It can be concluded that our results are similar to the ones of the researchers, such as Fischbein et al. (1984), Falk et al. (1980), Davies (1965), Goldberg (1966), Yost, et al. (1962) and Ginsburg and Rapport (1966)), who believe that children are able to solve certain probability tasks as early as at the age of four and five years.

As children neither learn about these topics at school nor teachers introduce them directly, we can conclude that probability tasks are solved intuitively, based on the children's experience with predicting events. The issue is not about random estimating, but intuitive problem solving based on some piece of information and also on experiences with probability, which children possess, as they encounter probability in their everyday lives, mainly at various children's games.

Table 1. Tasks in which statistically significant difference emerged between 4-5 year-old children and first graders, in favour of the latter group

TASK	Chi2 test	Percentage of correctly solved tasks	
		4-5-year olds	First grade
Exercise 1 Task 1a	(χ^2=27.701, p<0.01)	45.5	68.4
Exercise 3 Task 1a	(χ^2=11.905, p<0.01)	81.8	90.3
Exercise 4 Task 1a	(χ^2=10.86, p<0.01)	46.8	65.4
Exercise 1 Task 2	(χ^2=23.059, p<0.01)	44.5	73.5
Exercise 2 Task 2	(χ^2=14.03, p<0.01)	40	63.4
Exercise 3 Task 2	(χ^2=20.725, p<0.01)	73.6	92.3
Exercise 4 Task 2	(χ^2=22.275, p<0.01)	67.3	89
Exercise 5 Task 2	(χ^2=23.092, p<0.01)	48.2	73.5
Task 3.b	(χ^2=23.185, p<0.04)	5.7	15.8
Exercise 1 Task 4	(χ^2=9.364, p<0.01)	34.5	53.5
Exercise 2 Task 4	(χ^2=7.102, p<0.01)	44	60.6
Exercise 3 Task 4	(χ^2=4.856, p<0.03)	62.7	75.3
Exercise 4 Task 4	(χ^2=33.342, p<0.01	38.2	73.7
Task 5.b	(χ^2=10.421, p<0.01)	52.7	71
Task 6.a	(χ^2=14.126, p<0.01)	54.5	74.2

Table 2. Tasks in which statistically significant difference emerged between the first graders and the second graders, in favour of the latter group

TASK	Chi2 test	Percentage of correctly solved tasks	
		First grade	Second grade
Exercise 1 Task 1a	(χ^2=13.784, p<0.01)	68.4	85.5
Exercise 2 Task 1a	(χ^2=70.681, p<0.01)	49.7	88.6
Task 3b	(χ^2=78.271, p<0.01)	15.8	55.8
Exercise 1 Task 4	(χ^2=4.185, p<0.04)	53.5	64.7
Exercise 2 Task 4	(χ^2=40.774, p<0.01)	60.6	91
Tasks 6a	(χ^2=9.235, p<0.01)	74.2	86.5
Explanation of Task 6.a	(χ^2=29.929, p<0.01)	72.1	86.2
Task 6.b	(χ^2=14.592, p<0.01)	14.2	21.4
Explanation of Task 6.b	(χ^2=31.966, p<0.01)	2.1	3.1

Table 3. Tasks in which statistically significant difference emerged between the first graders and the second graders, in favour of the former group

TASK	Chi2 test	Percentage of correctly solved tasks	
		First grade	Second grade
Exercise 3 Task 2	(χ^2=12.675, p<0.01)	92.3	77.9
Exercise 4 Task 2	(χ^2=6.448, p<0.04)	89	78.9
Task 3.a	(χ^2=41.447, p<0.01)	97.4	81.8

Table 4. Tasks in which statistically significant difference emerged between the second graders and the third graders, in favour of the latter group

TASK	Chi2 test	Percentage of correctly solved tasks	
		Second grade	Third grade
Exercise 1 Task 2	(χ^2=6.508, p<0.04)	82.6	90.5
Exercise 4 Task 4	(χ^2=6.478, p<0.04)	62.6	71.8
Exercise 5 Task 4	(χ^2=6.379, p<0.04)	75.7	82.2
Task 5.a	(χ^2=10.606, p<0.01)	68.9	80.4
Task 5.b	(χ^2=6.443, p<0.04)	73.4	79.5

Table 5. Tasks in which statistically significant difference emerged between boys and girls, in favour of the latter group

TASK	Chi2 test	Percentage of correctly solved tasks	
		Boys	Girls
Exercise 2 Task 1a	(χ^2=8.414, p<0.02)	64.1	73.6
Exercise 1 Task 2a	(χ^2=9.66, p<0.01)	70.7	81.5
Exercise 2 Task 4	(χ^2=8.091, p<0.01)	67.1	77.6

Table 6. Tasks in which statistically significant difference emerged between boys and girls, in favour of the former group

TASK	Chi2 test	Percentage of correctly solved tasks	
		Boys	Girls
Exercise 3 Task 2	(χ^2=9.473, p<0.01)	86.3	79.6
Exercise 4 Task 2	(χ^2=6.165, p<0.05)	84.8	76.8
Exercise 5 Task 4	(χ^2=6.133, p<0.05)	79.9	73.2

The presented research showed that children already exhibited some pre-knowledge on probability prior to their school enrolment. Furthermore, it showed that most difficulties were encountered at predicting events with equal probability.

Taking into account the different levels of probabilistic thinking that were identified by Polaki (2002), the majority of the students participating in the research reached the second level of probabilistic thinking, as particularly older children solved the tasks appropriately. The younger the students, the more they tend to reach the first level, which is characterized by subjective thinking.

Results of the Teaching Approach

At attempting a teaching approach to teach equal probabilities we drew on concrete experience, taking into account also the results of some research, e.g. Aspinwall and Shaw, (2000), Castro (1998), Gates (1981), Nilsson (2007; 2009), Polaki (2002), Pratt (2000), Tatsis et al. (2008) showed that students are more interested in probability contents, their perceptions and conceptual understanding if they are provided with concrete materials.

As already mentioned at maths lessons we tried to teach children to compare events with equal probability by employing the adopted teaching approach, including the following step-by-step goals: children are able to predict the probability of various events with unequal probability, children learn to establish equal probability, children are able to divide equally, children are acquainted with the task solving technique in case of equal probability. The teaching approach was based on learning and understanding the technique enabling to predict equal probability, which is based on dividing the contents of an individual box or bag containing more elements and on comparison with the contents of the box or bag, containing fewer elements. The probability is equal if the divided contents equal the contents of the second box or bag. Students established equal probability in such a case by prior drawing out and systematic note taking. The other key elements of this teaching approach were, as follows: students' motivation, concrete activities with which students tested their predictions by a concrete experiment, active participation and small group discussions. The latter activity proved to be crucial for learning probability contents, because a bi-valent logic "right – wrong" does not apply with them, or children tend to provide arguments for different solutions and event predictions. In the final part of this teaching approach the mentioned technique to establish equal probability was presented.

In continuation the results which were achieved by students employing the mentioned technique are presented. The effectiveness of the teaching approach was checked with the knowledge test (Appendix 2), composed of 4 tasks, each of them presenting two boxes with different number of balls in them. Upon examining the results of the knowledge Test 2 it was established that it was possible to teach 5-6-year-olds to predict equal probability correctly. The test 2 results are presented in Figure 4.

Children participating in the research achieved their goal and learned to correctly predict events occurring with equal probability. From Figure 4 it is evident that 79 % of children correctly solved the first and fourth task with equal probability, and the whole Test 2 was correctly solved by 83.2 % of the participating pupils.

When comparing the results of the task 1 of the knowledge Test 2 and the 6.b task (in which the probability was equal, too) of the knowledge Test 1, the difference is obvious and statistically different ($\chi2$=75.358, p<0.01), as only 14.9 percent of children participating in the presented research, as well, correctly solved the task with equal probability at first measuring. Also at comparing the task 4 of the knowledge Test 2

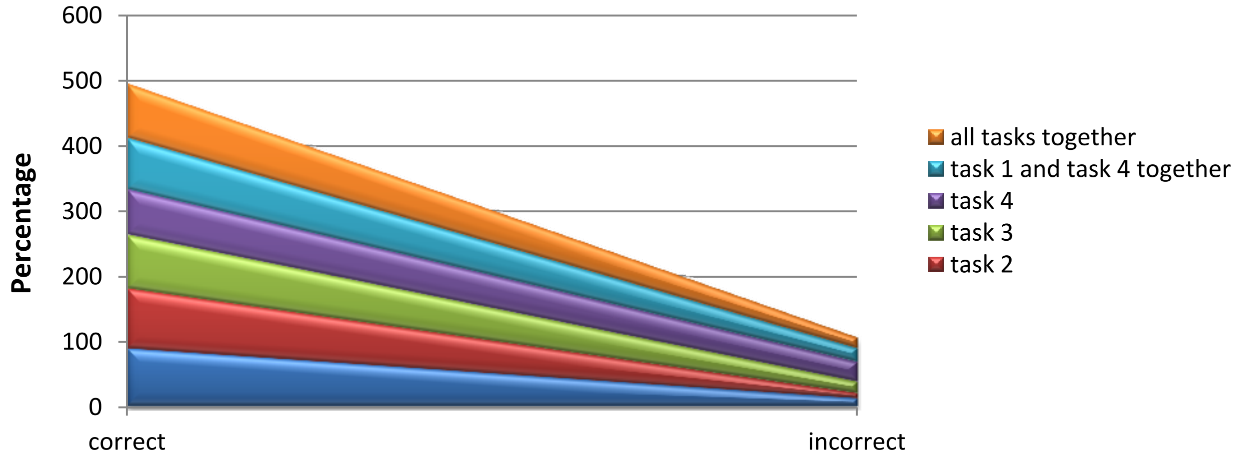

Figure 4. Knowledge Test 2 performance

and the task 6b of the knowledge Test 1 (in both cases the correct solution of the task is that it does not matter out of which box one extracts the objects) the difference is statistically significant ($\chi 2=42.802$, $p<0.01$). In both cases statistical significance is in favour of the knowledge Test 2, which means that after four hours of systematic learning of equal probability children performed better than on the knowledge Test 1 (prior to systematic learning of equal probability). It was also established that there are no statistically significant differences with regard to gender.

The presented results show that the adopted teaching approach was effective, especially created for this age group, taking into consideration the children's prior knowledge and abilities; they were motivated to learn, the method was adapted to time limitations as no lesson lasted more than 40 minutes. We started teaching already in the first hour of lessons. We proved it was possible to teach younger children certain probability topics. Consequently, it is recommended that younger children should be provided with useful experiences, teachers should choose the appropriate motivation method and adapt activities to their pupils' abilities. In this way they would acquire knowledge and stimulating experiences, exerting many positive impacts to assist them in their further education.

As regards the third research question we believe the teaching approach to be effective. There are no bigger differences between genders as regards the answers to the second research question. Girls performed better on three tasks and boys performed better on three tasks, too, so none of them prevailed.

Contrary to Ficshbein and Gazit (1984) we found out that following the adopted teaching approach it was possible for children to correctly predict the events with equal probability. Thus, we reached a similar conclusion as Gürbüz et al. (2010) and Polaki (2002), who also confirmed that it is possible to teach certain probability contents to pupils if they are provided with the appropriate teaching approach.

DISCUSSION AND CONCLUSION

In the research it was established that children in the first three grades were able to differentiate among certain, possible and impossible events, and compare the probability of various events, while only half of the children aged from 4−5 years were able to do that. Differences in gender influenced probability tasks solving only to a small extent. It was confirmed that children encountered the majority of problems at predicting events at which it did not matter from which box the extraction took place (when probability was equal).

On some tasks younger children performed better than the older ones, the difference being certainly due to two different knowledge tests, as the youngest two age groups took rather different knowledge tests for reasons of their lack of knowledge of reading and writing and their potential ignorance of numbers or colours. Besides, test taking was conducted in two ways (orally for the youngest age groups and in writing for the eldest groups). This, however, did not account for the better performance of the second graders compared to the third graders on solving one of the tasks, as they all took the same test and applied the same task solving manner. At this task children were required to predict the impossible event, the fact, which triggered a more open, divergent thinking mode in children.

The information gathered in the research proved to be a good indicator for teachers and pre-school teachers among others, of the abilities of children of different age groups to solve the probability tasks, of to their potential difficulties to cope with and also form a solid basis for probability lesson planning. This information is needed as the opinions of teachers and pre-school teachers on the abilities of children to solve probability tasks deviate to a large extent from the acquired results, namely, they underestimate the abilities of children they teach very much.

The teaching approach to teach equal probabilities was a certain experiment to determine the manner of teaching the first graders to correctly predict equal probability. We are aware that these results may not necessarily indicate the pupils' understanding of the probability concept, but prove familiarity with a certain technique to establish equal probability. It would be essential to establish the manner of pupils to justify their answers, further, whether they would be able to use this knowledge in different circumstances and transfer it to new situations.

Probability contents in the pre-school period and early school period are dedicated to (Threlfall, 2004): relating everyday statements to probability language, answering probability or likelihood questions about the provided data, answering probability or likelihood questions about a described situation, collecting and reflecting on empirical data.

In all the mentioned activity groups children predict, assess the likelihood of an event. The situations differ among themselves, they are related to everyday life, to common language, they are presented in different ways, they include mathematical concepts (the number 0, parts of a whole, a uniform line), and offer children many possibilities for discussion, assessment and arguing the likelihood of an event. Alongside the vocabulary development and familiarity with recording conventions, all of the different types of activity offered to primary aged children in mathematics lessons were supposed to bring with them some aspect of a mathematical perspective on the relationship between possibilities and probabilities. That is, after all, the main point of introducing probability into mathematics classes. The question to what extent it is possible to attribute mathematical understanding and mathematical knowledge of these concepts to children in the pre-school and early school period, remains open. On the basis of his research Threlfall (2004) believes probability contents should be explored only when children are able to deal with complex situations and not only simple ones, for which he expresses his doubt whether they prove mathematical understanding and thinking of children.

The author concludes (ibid.), that children should be taught probability in higher grades of primary school, whereas in the lower grades only those children should be presented with probability tasks that are able to deduce the complexity of simple situations from probability. We agree that we could hardly prove strict mathematical argumentations of children who are challenged by probability tasks; however, we believe that probability contents could be introduced within the context of mathematical literacy in the early school period, with the emphasis being on a child's active participation in the discussions on situations that are possible, impossible, likely, unlikely, less probable, equally probable; all this undoubtedly contributes to developing those competences that modern Man should possess in order to adapt rapidly to the world of today, which is of unpredictable situations and various challenges, and in order to foster the critical attitude towards 'numerical information', disseminated by media (Howson, Kahane, 1986). Deterministic thinking no longer suffices in order to comprehend certain branches of science; non-deterministic schemes of thinking are needed more and more and are also witnessed, e.g. in the field of genetics, biology, physics, economy. Nowadays, probability is also utilized in the areas close to everyday life of humans: in meteorology, at elections, in actuarial science, etc. Young children are taught the probability 'alphabet', because, as already Fischbein (1985) found out, the probability concepts and techniques need to be integrated in mathematics lessons as early as at the primary level, and not only in higher grades or even in high school, when the mindset of a human is already developed. If we want Man to develop a thinking mode that would be considerably different from deterministic schemes of thinking, we should start teaching probability at the level of concrete operations, if not earlier, or in the phase of the transition from the level of concrete operations to the level of formal operations at the latest (Fischbein, 1985).

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◈◇◈

Appendix 1: Test 1

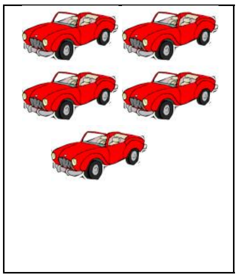
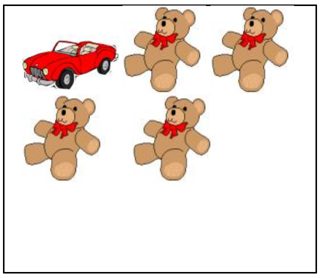
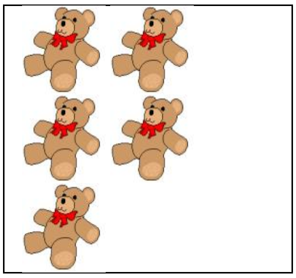
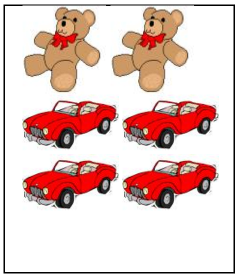
Circle **Grade 2** **Grade 3**

I Am A Boy **I Am A Girl**

Probability

1. In The Boxes There Are Teddy Bears And Cars. Imagine That You Close Your Eyes And Extract One Toy From Each Box.

A) Could You Extract A Car? Circle The Correct Answer.

Certainly	**Ceratinly**	**Certainly**	**Certainly**
Possibly	**Possibly**	**Possibly**	**Probably**
Impossibly	**Impossibly**	**Impossibly**	**Impossibly**

B) In Which Box Would You Reach, In The Second Or The Fourth One In Order To More Probably Extract A Car?

The Second **The Fourth**

2. Domen Is Throwing The Die, Containing Numbers From 1 To 6. Circle The Suitable Word For Each Sentence.

A) The Die Will Display The Number 6.	Certainly	Possibly	Impossibly
B) The Die Will Display The Number Below 7..	Certainly	Possibly	Impossibly
C) The Die Will Display The Number 7.	Certainly	Possibly	Impossibly
D) The Die Will Display The Number Above 7.	Certainly	Possibly	Impossibly
E) The Die Will Display The Number 3.	Certainly	Possibly	Impossibly

3. Mojca Has Six Coloured Kerchiefs In Her Bag, Of Which Two Are Of Yellow, Two Of Blue, One Of White And One Of Green Colour. She Extracted One Kerchief From The Bag Not Looking At Its Colour.

What Can You Say About The Kerchief That Mojca Extracted?

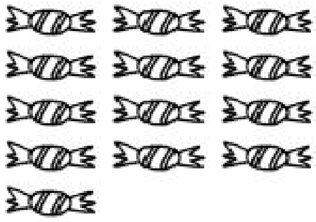
Continue The Sentences:

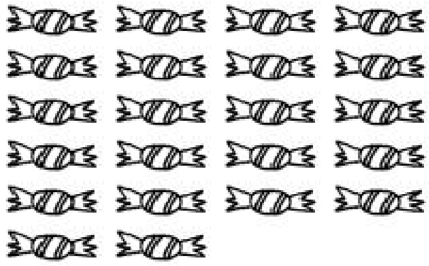
A) It Is Possible That She Extracted ____

B) It Is Impossible That She Extracted ________________________________

Appendix 1 cont.

4. Gregor Has Three Bags Wıth Candıes. In Each Of Them There Is One Chocolate Candy, Whereas The Other Ones Are Fruıt Candıes. Wıthout Lookıng He Is Tryıng To Extract The Chocolate Candy From The Bag. Cırcle The Correct Answer.





The Fırst Bag The Second Bag The Thırd Bag

A) Can He Extract The Chocolate Candy From Any Bag Already In Hıs Fırst Trıal?

Yes **No**

B) Is It Possıble For Hım To Extract The Mılk Candy From The Thırd Bag?

Yes **No**

C) Is It More Probable For Hım To Extract The Chocolate Or A Fruıt Candy From The Second Bag?

The Chocolate One **A Fruıt One**

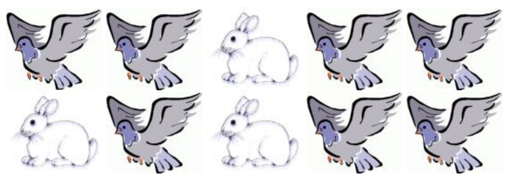
D) In Whıch Bag Wıll He Reach In Order To Most Probably Extract The Chocolate Candy?

The Fırst **The Second** **The Thırd**

E) In Whıch Bag Wıll He Reach In Order To Most Probablx Extract A Fruıt Candy ?

The Fırst **The Second** **The Thırd**

5. The Magıcıan Put 10 Rabbıts In Hıs Hat Bewıtchıng Them Into Pıgeons. Seven Rabbıts Were Turned Into Pıgeons, Whereas Three Rabbıts Remaıned The Same.



Jaka Randomly Extracted One Anımal.

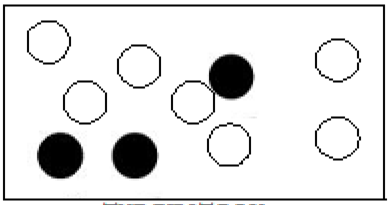
Fınısh The Sentences.

A) Jaka Wıll Most Probably Extract ________________________.

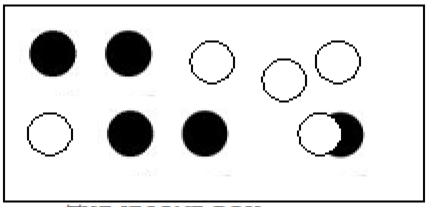
B) Jaka Wıll Least Probably Extract _______________________.

Appendix 1 cont.

6. a) In The First Box There Are 7 White And 3 Black Balls, Whereas In The Second Box There Are 5 White And 5 Black Balls. Sabina Will Get A Present If She Extracts A White Ball. From Which Box Should Sabina Extract The Ball? Circle The Correct Answer.



THE FIRST BOX



THE SECOND BOX

A) From The First Box.

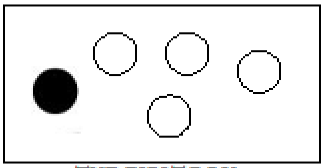
B) It Is All The Same

C) From The Second Box.

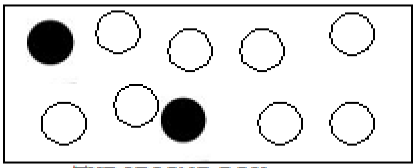
Why? ____

____

B) In The First Box There Are 5 Balls, Of Which 4 Are White And 1 Is Black. In The Second Box There Are 10 Balls, Of Which 8 Are Of White And 2 Of Black Colour. Sabina Will Get A Present If She Extracts A White Coloured Ball. From Which Box Should Sabina Extract A Ball? Circle The Correct Answer.



THE FIRST BOX



THE SECOND BOX

A) From The First Box.
B) It Is All The Same.
C) From The Second Box.

Why? ____

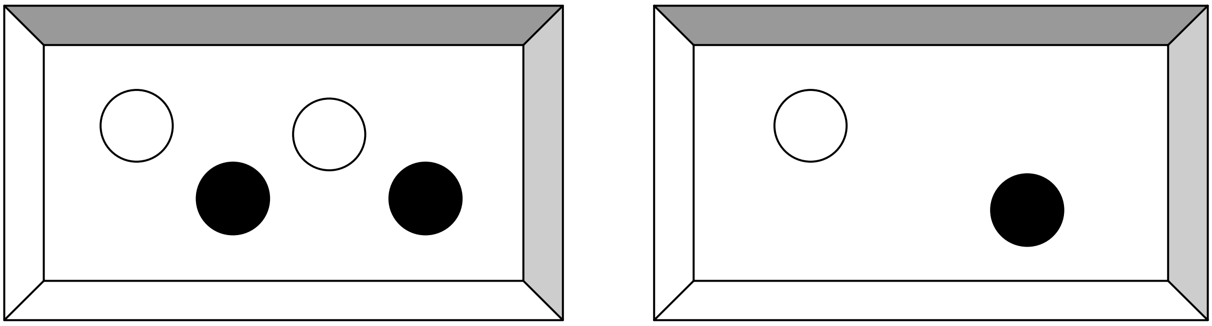
____

Appendix 2: Test 2

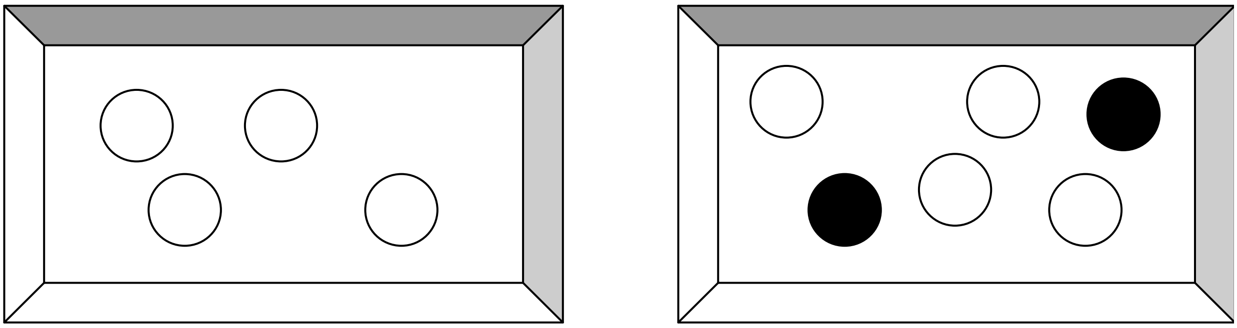
Exercıse

Cırcle The Box In Whıch The Boy Should Reach In Order To Extract A Black Ball.

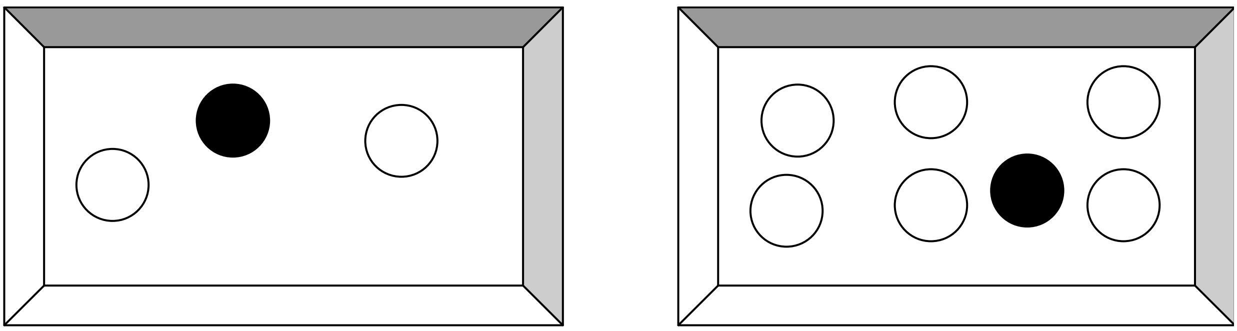
Task 1



Task 2



Task 3



Task 4