Eurasia Journal of Mathematics, Science & Technology Education, 2015, **11**(4), 793-802

Understanding the Primary School Students' van Hiele Levels of Geometry Thinking in Learning Shapes and Spaces: A Q-Methodology

Tan Tong Hock, Rohani Ahmad Tarmizi, Aida Suraya Md. Yunus & Ahmad Fauzi Ayub
Universiti Putra Malaysia, MALAYSIA

Received 19 January 2015; accepted 3 March 2015

This study was conducted using a new hybrid method of research which combined qualitative and quantitative designs to investigate the viewpoints of primary school students' conceptual understanding in learning geometry from the aspect of shapes and spaces according to van Hiele theory. Q-methodology is used in this research to find out what factors (level) of geometry understanding in van Hiele theory contributed to the geometry learning of primary school children. Q-methodology involved creating a concourse - Q set which was developed through a literature review and the statements were later refined by experts and then conducted to 30 participants aged 12 at a primary school. The respondents expressed their understanding through Q sorting, by ranking strategies according to their geometry understanding. The results indicated that students' van Hiele Levels of Geometry Thinking are at the factor (a) Level 1 - Analysis (or Descriptive); and factor (b) Level 0 – Recognition (or Visualization). This common outcome revealed that the students' geometry understanding according to van Hiele theory is at the lower level. Significantly, the findings indicate that the deficiency of van Hiele Levels of Geometry Thinking appears to be global in nature, crossing the boundaries of educational practices and curriculum.

Keywords: van Hiele level, recognition/visualization, analysis/descriptive, informal deduction.

INTRODUCTION

Geometry is an important topic in mathematics and in any school curriculum, it is among one of the basic skills to be mastered (Effandi & Abdul Halim, 2012). In Malaysia, geometry is taught early in the primary level and further emphasized at secondary level. About forty per cent of the forty two topics in secondary mathematics curriculum are comprised of geometry content (Abu, Ali, & Tan, 2012; Ministry of Education Malaysia, 2010) Therefore, an in-depth conceptual understanding of geometrical properties at the primary level is not only important for the students in learning about geometry; it will also determine whether they can cope with its continuation when they later proceed on to secondary level.

Correspondence to: Tan Tong Hock,
Institute of Mathematical Research,
Universiti Putra Malaysia,
43400 UPM Serdang, Selangor, Malaysia.
E-mail:kenny7028@gmail.com
doi: 10.12973/eurasia.2015.1439a

STATEMENT OF PROBLEMS

In Malaysia, the teacher-centred learning of mathematics has been reported and students are not given much chance to develop their own thinking in teaching and learning environments (Noraini, 2007). This situation causes students to become passive information receivers and do not result in conceptual

ISSN: 1305-8223

State of the literature

- Teachers cannot adapt their geometry teaching to their students' levels and this is considered as the most critical gap in the learning process.
- Throughout the learning, the teacher has to play various roles such as task planning, directing students' attention to geometry properties of figures, introducing terminology and engaging students in discussions using appropriate terms and encouraging explanations.
- The van Hiele's model enables insight into why many students encounter difficulties in their geometry courses. As far as the learning of shapes and spaces are concerned, many studies have reported that generally students have difficulty in attaining the higher levels in the manner prescribed by the van Hiele's development of geometry conceptual thinking.

Contribution of this paper to the literature

- Malaysian students' abilities according to van Hiele's level of geometry thinking are consistent with worldwide finding, and also in relation to prior research.
- To identification of the perception of geometry understanding according to van Hiele level of geometry thinking that are traits of the primary school students in Malaysia. This study sought to provide a means of understanding why performance in geometry is not satisfactory in in international assessments such as TIMSS and PISA.
- To highlight positive aspects of learning in geometry especially those related to shapes and spaces at the primary level. This positive aspects focus in design and arrangement of teachers' instructions that emphasizes the need to study how positive outcome can occur.

understanding. For example, it was acknowledged that Malaysian students faced difficulty in identifying a square as a rectangle and this phenomenon has also been extensively reported in the literature (Clements & Battista, 1992). Most of the students cannot understand their mathematics teachers' instruction especially on the topic of geometry. Finishing the syllabus and focusing on examinations are the priority of teaching and learning for most of the teachers in Malaysian schools (Noraini, 2007). There is a need to provide suitable guidance which can lead to sufficient attainment in geometry that is consistent with learning processes and students' cognitive development and cater also to individual differences. In response to this need, a lot of technology-related applications have been supported to ease such learning difficulties across all levels of learning. For example, while using the Geometer's Sketchpad (GSP), teachers are encouraged to try counselling especially on teaching processes, guidance methods and making sure that sufficient time is allocated for the students to make use of the programme (Noraini, 2007).

There is a famous model which is called van Hiele Model of geometry thinking in the learning of geometry. This model has been proposed by Dina van Hiele-Geldof and Pierre van Hiele and many mathematics educators associate this model with the development of geometric thinking. They have identified five different levels of thinking which students must progress sequentially from one level to the next level of thinking without skipping any one level (Abu et al., 2012). Method and content of instruction are the priority of progression (van Hiele, 1986). Since then, researchers worldwide (including those in Malaysia) have been found to be interested in applying this model in investigating students' levels of geometric thinking (Ding & Jones, 2006; Noraini, 2007; Usiskin, 1982; Wu & Ma, 2005a). However, research in this area mostly centred on quantitative investigation throughout the world. So far there is a paucity of study employing Q-methodology. It is a gap in terms of Mathematics education; therefore, this study was initiated and Q-methodology is used to confirm the thoughts of Malaysian primary students about their geometry thinking.

Malaysian Mathematics text books introduce ideas in the van Hiele theory before students start learning 'Shapes and Spaces'. However, many practitioners strongly felt that a large number of Malaysian primary school students experienced learning difficulties similar to those encountered by learners throughout the world as mentioned earlier. In fact, an analysis of students' performance on the Malaysian Primary School Achievement Test - Ujian Pencapaian Sekolah Rendah (UPSR) in 2008 and 2010 showed that there is low performance in 'Shapes and Spaces' for the subject Mathematics, especially in the context of symmetry line, perimeter and area (MOE, 2010). As far as the conceptual understanding about geometry goes, students must watch, listen, jot down the notes and think about what the teacher said (Özerem, 2012). According to van Hiele's model, basically students must go through the hierarchical sequence exhibited through the students' ability to talk, draw or write about their conceptual understanding as prescribed by the van Hiele's development of geometry thinking (Monaghan, 2001). Hence, many practitioners feel that there is a need to reveal the thoughts of primary school students about their conceptual understanding of geometry especially in shapes and spaces in order to clarify the phenomenon that occurs in our learning environment based on van Hiele Levels of Geometry Thinking for the primary school curriculum.

The purpose of this study was to reveal the primary school students' conceptual understanding in learning geometry from the aspect of shapes and spaces by using the ideas in van Hiele theory. The research questions were:

1. What is the students' perception of geometry understanding and how can it be described according to van Hiele Levels of Geometry Thinking?
2. What is the relationship between the identified factors?

This study aimed to reveal the primary school students' understanding about van Hiele Levels of Geometry Thinking in learning shapes and spaces based on the Malaysian Integrated Primary School Curriculum - Kurikulum Bersepadu Sekolah Rendah (KBSR). The study relied heavily on students as the primary source of data. The data consists of individuals' understanding, experience, opinion and encounter with the attainments of geometry concept. As there is only a limited number of participants, this study is limited in its findings, and cannot be generalised to students in other areas. It is quite difficult to determine if what these participants revealed would be similar to other students in the whole country.

RELATED LITERATURE AND THEORIES

The lack of understanding in learning geometry will lead to poor attainment in geometry and discourages students from learning mathematics from elementary level to middle level. Several prevailing factors have been identified to explain why learning geometry is considered as a difficult task. These factors include geometry language, visualization abilities and ineffective instructions (Noraini, 2007). Students begin learning geometry in primary school, and if they were not equipped with the necessary achievements, this may cause them to lag behind others throughout their school life. Such difficult experiences may cause them to lose their confidence and result in them failing in mathematics achievement test.

The Third International Mathematics and Science Study (TIMSS) was conducted since 1999. Malaysia was one of the 38 participating countries from 1999 to 2011 (NCES, 2013). The Malaysian Form Two (equivalent to 8th Grade) students' performance in TIMSS achieved mean scores of 519(1999), 508(2003), 474(2007) and 440(2011). The scores have steadily declined over the years and compared with other countries, we have achieved slightly lower when compared to the TIMSS average of 500. However, among the Asian countries, we were far behind the countries of Singapore, Korea, Chinese Taipei, Hong Kong and Japan. In geometry, Malaysian students performed with mean scores of 497(1999), 478(2003), 477(2007) and 432(2011). These statistics show that over the years, Malaysian Form Two students' geometry performance gradually exhibited less competence in the international arena. When it comes to the analysis of benchmark cut point, Malaysian ranked at category of "Intermediate" which means students can only apply basic mathematical knowledge, simple algebraic forms and two-dimensional drawing. They can interpret basic notions and construct graphs and tables in the simple form (NCES, 2013).

Malaysia has also participated in the Programme for International Student Assessment (PISA) in 2009. The PISA mathematics literacy test evaluation is mostly related to solving problem in real life situation, which is different from TIMSS. Students must have a wide range of mathematical content knowledge to solve such problem whereas TIMSS measures more in traditional method of curriculum attainment, for example, an understanding of calculation in basic form of concept (OECD, 2012). Malaysia ranked at 57 out of 74 with a score of 404 in 2009, and ranked at 52 out of 65 with a score of 421 in 2012. PISA was administered to students with ages between 15 years and 3 months and 16 years and 2 months at the beginning of the assessment period (OECD, 2012), which is equivalent to form 2 students in Malaysia. It has a mean of 500 and a standard deviation of 100. Malaysian students' performance was at the bottom third of all the participated countries. On the other hand, Shanghai, Singapore, Hong Kong, South Korea and Taiwan were the top five countries among those that participated in PISA. Malaysian students' understanding in real life problem is again questioned in this assessment.

When it comes to geometry learning, van Hiele claims that students must pass through several levels of reasoning about geometry concepts. His assertion was derived from classroom observation. Consequently, the van Hiele Levels of Geometry Thinking was developed, and later it evolved into van Hiele theory. The theory suggests that students pass through numerous levels of geometry thinking merely from recognizing geometrical shapes to construction of a formal geometry proof (Abu et al., 2012; van Hiele, 1986 & 1999). The van Hiele theory enables us to explain why many students encounter difficulties in their geometry courses. The theory also offers educators a teaching model to apply and practice in order to promote their students' levels of geometry thinking (Fuys et al., 1988; van Hiele, 1986). The van Hiele theory originally posited five sequential and hierarchical discrete levels of geometry thinking (Senk, 1989; Usiskin, 1982). Descriptions of these levels are as follows: Level 0: Recognition (or Visualization). At this level, students recognize geometrical shapes as a whole (Burger & Shaughnessy, 1986). Level 1: Analysis (or Descriptive). The student at this level is able to reason about a geometry shape in terms of its properties but they do not yet understand the relationships between these properties and between different figures (Mason, 1998; Van Hiele, 1986). Level 2: Informal Deduction (or Theoretical). At this level, the student can logically order the "litany" of properties of figures previously identified, and begins to perceive the relationships between these properties and between different figures. Level 3: Deduction. At this level, deduction becomes meaningful. The student understands the significance of deduction on and the role of postulates, axioms, theorems and proof. Level 4: Rigor. At this level, students can reason formally about mathematical systems. The necessity for rigor is understood and abstract deductions can be made (Usiskin, 1982).

According to van Hiele, students must pass through these five levels in a sequential order assisted by any appropriate instruction method without skipping any level. The priority in teaching and learning is to enable learners' progress through the various level based on the content and method and not the age. Instruction should happen at the same level of the students so that the learning can occur (Crowley, 1987). Therefore, the teachers play an important role in the teaching and learning of geometry.

This study is conducted with an aim to reveal the perception and thoughts of primary students about the attainments of learning geometry concepts in early curriculum, particularly in Shapes and Spaces.

Q-METHODOLOGY

Q-methodology is a method which is suitable for the observation of perception either of an individual or a group of participants. In Q-methodology, data will be gathered through each individual on multiple issues of interest. Opinions of a group of individual will be clustered based on their similarity and these clusters later form the themes of these issues. Typically, an individual will be asked on a variety of different questions to reveal their perceptions on a variety of different constructs and then identify how those constructs of individuality affect their perceptions. Finally, the patterns will be formed to reveal the problem (Brown, 2004).

If the focus of a research is to obtain rich data in order to answer the research questions instead of making generalizations, Q-methodology is the preferred method of study as it utilized both qualitative and quantitative research methods to provide a systematic study of foundation (Job van Exel, 2005). Typically, there is a sample of statements called Q-set in a Q methodology study. P-set means statements ranked by respondents according to their opinions and they are arranged in a quasi-normal distribution pattern. The final step is called Q sorting in which respondents have to reveal their subjective viewpoint. Q methodology is used to reveal the experience of respondents (students) in line with the aim of the study. It has helped the researcher in this current study, which was to understand students' view of the learning process in geometry by the experiences and thoughts that were related to 'Shapes and Spaces'. It has also helped to confirm whether they have achieved the targeted level as proposed by van Hiele theory in learning geometry.

METHODOLOGY OF RESEARCH

The aim of this Q study is to check if Malaysian elementary school students are performing at the level targeted by van Hiele theory. Worldwide research has revealed that most elementary school students are not performing at the level proposed by van Hiele theory. The concourse of this study is derived from a review of the articles that has used van Hiele theory as the theoretical framework. There were 34 statements which consist of 7 statements of van Hiele's level 0 (Visualization), 14 statements of van Hiele's level 1 (Analysis) and 13 statements of van Hiele's level 2 (Informal Deduction). For each level of the statements, there are both positive and negative arguments to examine the understanding of the participants. In Q-methodology, the statements are interpreted by participants and any individual can be considered as a valid source of information, therefore it removes the view of the researcher and the issue of validity. In this study, the 34 statements which became the Q-Sample were validated for its content and grammar by an expert who is the Associate Professor of Mathematics Education in Universiti Putra Malaysia and a teacher expert in Mathematics. Table 1 shows the examples of positive and negative statements according to van Hiele Levels of Geometry Thinking.

The study was conducted in one of the girls' school in Batu Pahat, Johor, Malaysia. 30 participants were invited to attend this study with the consent of the principal of the school. 30 sets of the Q-Sample were prepared and administered to each participant for them to rank-order statements and record their viewpoints. Participants took about one hour to complete the task according to the instructions and ranked the statements into "Higher Understanding" (+4) to "Least Understanding" (−4). Participants also gave the reasons why they chose the extreme two of the +4 and−4 statements.

DATA ANALYSIS

The data collected was analysed using PQ Method 2.33 software programme to calculate the correlation matrices of the Q-sorts accordingly. Q-methodology

Table 1. Q-Samples

Category	Statements	No.
Van Hiele Level 0 of Geometry Concepts	P: The shape of a square looks like a perfect box.	10
	P: Rectangle has the shape of a long box.	5
	P: Roof of a house is normally in the shape of a triangle.	22
	N: A rectangular container without its cover is still known as a cuboid.	29
	N: If a corner is cut from a cube, it can still be recognized as a cube.	16
	N: Triangle must have three lines but they are not necessarily jointed together.	32

P: Positive Statement N: Negative Statement

Table 2. Statement Scores by Factors/Opinion Types

Statements	Factors	
	1	2
1. If four sides of a quadrilateral are equal in length, the figure is a parallelogram	4	1
2. A quadrilateral in which the diagonals are perpendicular to each other must be a square.	-1	-2
3. A rectangle is a quadrilateral which has 2 longer sides and 2 shorter sides.	4	2
4. A rectangle is a special kind of quadrilateral.	2	1
5. Rectangle has the shape of a long box.	3	1
6. There are many ways to cut a square into two exact halves	3	3
7. A square is a rectangle with equal sides.	0	0
8. A square is a subset of a cube.	0	-3
9. A square is not a parallelogram because parallelograms are slanted.	-4	0
10. The shape of a square looks like a perfect box.	2	3

*Item ranking: +4 = highest level of understanding; 0 = ambivalent; −4 = least level of understanding

showed the results of similarities, differences, correlation, factor analysis and examination of factor analysis scores by which they sort the statements in the Q-sample. In this correlation analysis, each participant was statistically correlated to a 30 X 30 matrix which demonstrates the relation of each participant (n = 30) subjectively arranging the 34 statements of the Q-sort to reveal their own geometry understanding. PQ Method 2.33 can be used to calculate the correlation matrices of the Q-sorts (Schmolck, 2002).

A result of correlation matrices score will show the +1 to −1 range in which +1 indicates a perfect positive relationship, −1 indicates a perfect negative relationship and it is completely opposite to +1, 0 indicated no relationship at all between the sorts. The significance level of a correlation value was identified using the formula of standard error (SE) (Schmolck, 2002). As this study used 34 statements within the Q-sort, standard error was $1/\sqrt{34} = 0.1715$. In order to achieve 95% ($p < .05$) correlation coefficient utilization, the result is 1.96(SE) = 1.96(0.1715) = 0.3361. Previous Q-methodology research indicates that the correlation that are 2 to 2.5 times the SE was considered as statistically significant (Brown, 2004). According to the findings from PQ Method 2.33, there were 529 out of 900 correlation matrix between sorts which are significantly correlated or 58.78%. Only two of them indicated that there were no relationships at all between the sorts or 0.2%. This means there was a significant relationship between sorts.

Table 2 shows examples of summarizing statements of each factor which was generated by the PQ-method. Results of statements which reveal participants' viewpoints were ranked orderly from +4 to –4. Two of the factors were titled as follows:

Factor 1: Level 1 – Analysis (or Descriptive) of van Hiele Levels of Geometry Thinking

About 18 out of 30 participants or 60% of the students achieved the second level of van Hiele Levels of Geometry Thinking. Most importantly, the Analysis Level of van Hiele's Geometry Thinking includes the understanding of geometry concepts of shapes and spaces. It is important to these students that they can work step by step according to van Hiele levels of learning in geometry. They are very much aware that it requires an understanding of concepts which include recognition or visualization level of learning before they can be promoted to analytical level of learning.

In factor 1, participants were recognized at level 1or Analysis Level of van Hiele Levels of Geometry Thinking. Most of the students did not understand the statements of van Hiele level 2 (Informal Deduction). For the statements of least level of understanding, the students showed that they cannot link the geometry concept in order to combine the properties of the shapes and spaces. Students did not understand about parallelogram and what are the angles indicated in geometry figures. The statement of "A rectangular container without its cover is still known as a cuboid." showed that participants can visualize common figures according van Hiele theory but were less exposed to real life activities in learning geometry.

Factor 2: Level 0 – Recognition (or Visualization) of van Hiele Levels of Geometry Thinking

For this factor, about 12 out of 30 participants or 40% of the students were engaged at the first level of van Hiele levels of geometry thinking. The characteristic of this group was they were less likely to use the potential of imagination about the shapes and spaces and harness it as a learning skill, i.e., problem inflection and students' immediate feedback. There were also less enrichment for other perspectives and potential interference with work. This group reacted positively to the things which can be visualized and concrete. They were not concerned about imagining the shapes and spaces in practices. This group reacted at a basic level to the statements such as less of a sense of comparison between the shapes and spaces and tend to only use simple visualization in geometry thinking. Participants in factor 2 recognized few deficiencies in their application of concepts in delivering educational content; and their ability in learning shapes and spaces is limited at the visualisation level.

This group showed that they are at the least level of understanding and unsure about the statements of analysis level and informal deduction level according to van Hiele Levels of Geometry Thinking. This group showed the basic or lowest understanding in learning geometry. In results of z-scores, they tend to misunderstand and choose the statement of "A rectangle is a type of squares" and "Some rectangles have more than two lines of symmetry" in higher level of understanding. The choice of these invalid statements and ranking them at higher level showed that these students do not understand the higher level geometry concepts.

This group of participants also acknowledged that "A rectangular container without its cover is still known as a cuboid." at least understanding, the results consistent with group factor one which showed that they have less practice (exercises) in daily life problems. The same factor showed that participants shared similar opinions or values in their geometry understanding.

Table 3 shows the correlation between the primary factor scores and also demonstrates the significant differences between the factors. If the factors were correlated at a value greater than 0.5 (Job van Exel, 2005) then they are considered as significant. In this study factor 1 and factor 2 are correlated at a score of 0.7079 which shows they are at a high level of correlation. This is consistent with van Hiele theory of geometry thinking which postulates that the students must pass through the levels in a sequential order without skipping. The progression depends on the content and method of instruction at the level of the learner but not their age.

From the result of correlation analysis, it is clear that the understanding of geometry concept according to van Hiele theory is highly correlated and the learner must first achieve the visualization level before proceeding to analysis level in the order without skipping (van Hiele, 1986). Reliability between and within the factors were established by analysing each factor's distinguishing statements. Brown (2004) confirmed that reliability for an individual's operant subjective responses can be greater than or equal to 0.80. In this study, the composite reliability for factor 1 and factor 2 ranges between 0.980 and 0.986 as shows in table 4. Standard error of each factor score signifies the reliability for each of the identified Q-sorts.

DISCUSSION OR ISSUES FOR CONSIDERATION

The viewpoints obtained by Q-methodology indicate that students exhibited viewpoints which are consistent to worldwide findings (Ding & Jones, 2006; Noraini, 2007; Usiskin, 1982; Wu & Ma, 2005a) in terms of van Hiele levels of geometry thinking. This finding

Table 3. Correlations Between Factor Scores

	1	2
1	1.0000	0.7079
2	0.7079	1.0000

Table 4. Factor Characteristics

	Factor	
	1	2
Number of defining variables	18	12
Average relative coefficient	0.800	0.800
Composite reliability	0.986	0.980
S.E. of factor scores	0.117	0.143

Table 5. Higher Understanding Statements for Factor 1

No.	Statement	van Hiele Level	Rank
1	If four sides of a quadrilateral are equal in length, the figure is a parallelogram	1	+4
3	A rectangle is a quadrilateral which has 2 longer sides and 2 shorter sides.	1	+4
6	There are many ways to cut a square into two exact halves	1	+3
5	Rectangle has the shape of a long box.	0	+3
12	An encyclopedia book looks like a cuboid.	0	+3

Table 6. Least Understanding Statements for Factor 1

No.	Statement	van Hiele Level	Rank
13	An equilateral triangle has an angle that is larger than 60 degrees.	2	-3
15	If a quadrilateral has four equal sides, and three of its corners are 90 degrees each. Therefore, it must be a rectangle.	2	-3
29	A rectangular container without its cover is still known as a cuboid.	0	-3
9	A square is not a parallelogram because parallelograms are slanted.	2	-4
17	If a corner of an isosceles triangle is 60 degrees, then three sides of this triangle cannot be unequal.	2	-4

answered the first research question. The following discussion described how these viewpoints support findings of previous research.

Characteristic statements of Factor 1 which were ranked +3 and +4 indicated that participants are recognized at van Hiele Levels of Geometry Thinking of level 1 or Analysis Level. Most of the students did not understand the statements which corresponded to van Hiele level 2 (Informal Deduction). These higher understanding statements are shown in Table 5.

Understanding of these statements as well as the other positively loaded statements for Factor 1 indicated that students' van Hiele Level of Geometry Thinking loaded on this factor provided evidence that their achievement is at level 0 and level 1. Furthermore, students cannot link the properties of each figure according to different shapes and spaces. This factor also implied that their geometry thinking was enhanced by visualizing the concrete objects in the classroom. These elements support the findings of Usiskin (1982) and Schafer (2008). Usiskin (1982) found that the van Hiele's Levels of Geometry Thinking of 2361 high school students were at the pre-recognition level and Level 1. Similar result was obtained by Schafer (2008) who conducted a similar study on 144 students from Nigeria and South Africa. Studies conducted by several researchers have also revealed more or less the same phenomena (Ding & Jones, 2006; Noraini, 2007; Wu & Ma, 2009).

Table 6 shows that students have least understanding about the van Hiele Levels of Geometry Thinking at level 2 – informal deduction which requires the ability to logically order the "litany" of properties of figures previously identified, before they can begin to perceive the relationships between these properties and between different figures. Statements such as "A rectangular container without its cover is still known as a cuboid." is considered as a negative statement in level recognition but still students were unable to recognize it. The misunderstanding of this concept shows that students probably have less practice in activities related to daily life.

Characteristic statements of Factor 2 which were ranked +3 and +4 indicated that participants are recognized at van Hiele Levels of Geometry Thinking of level 0 or Recognition Level. Most of the students do not understand the statements of van Hiele level 1 (Analysis) and level 2 (Informal Deduction). These higher understanding statements appear in Table 7.

An understanding of these statements as well as the other positively loaded statements for Factor 2 indicated that students' van Hiele Levels of Geometry Thinking loaded on this factor provided evidence at level 0 only. Furthermore, students cannot understand and link the properties of each figure according to different shapes and spaces. This factor also shows that their geometry thinking is only enhanced by visualizing the concrete objects in the classroom. However, students recognized "A rectangle is a type of squares" and ranked it at higher understanding showed that these elements support the finding of Schafer (2003) who found, in his case study, students performed poorly in items characterized by the spatial visualization and orientation construct. This was largely due to limited capacity of spatial ability and world view of their spatial conceptualization. This is evidence again that most students only operate at the

Table 7. Higher Understanding Statements for Factor 2

No.	Statement	van Hiele Level	Rank
22	Roof of a house is normally in the shape of a triangle.	0	+4
12	An encyclopedia book looks like a cuboid.	0	+4
10	The shape of a square looks like a perfect box.	0	+3
6	There are many ways to cut a square into two exact halves	1	+3
20	A rectangle is a type of squares.	2	+3

Table 8. Least Understanding Statements for Factor 2

No.	Statement	van Hiele Level	Rank
15	If a quadrilateral has four equal sides, and three of its corners are 90 degrees each. Therefore, it must be a rectangle.	2	-3
8	A square is a subset of a cube.	2	-3
17	If a corner of an isosceles triangle is 60 degrees, then three sides of this triangle cannot be unequal.	2	-3
30	The sum of internal angles of a square is 360 degrees.	1	-4
18	Most rectangles are drawn horizontally or vertically and its length is two or three times the width.	2	-4

basic levels during the instruction of Shapes and Spaces in KBSR.

Additionally, Table 8 shows that students have least understanding about the van Hiele Levels of Geometry Thinking at level 1 (Analysis) and level 2 (Informal deduction) which support the findings of Noraini (1999, 2004 & 2007) who has proposed several factors to explain the learning difficulties in Geometry related to van Hiele's model which include geometry language, visualization abilities and ineffective instructions. More explicitly, the geometry language involves specific terminology; geometry requires visualizing abilities (Level 0); and misuse of geometry terminology (Level 1) can lead to misconceptions of geometry knowledge (Level 2). Many students cannot visualize and imagine three-dimensional objects on a plain surface. Students who lack concrete experiences will find it difficult to visualize cross sections of solids in geometry. Therefore, hands-on activities will be meaningful to the learners. By being able to "touch-see-and-do", gaining experiences through exploring and interacting with the objects themselves, students can become more imaginative and successful in learning geometry (Bishop, 1983).

From the determination of factor 1 and factor 2, this study found that students only perform at the second level according to van Hiele theory and there are nearly half of them still at first level or recognition which is a very basic stage in learning geometry especially in Shapes and Spaces at elementary level. This may answer the question why Mathematics results in UPSR 2008 and 2010 showed low performance in Shapes and Space and explained the statistics in TIMSS report whereby in the area of geometry, the Malaysian students' performance gradually progressively showed less competence in the international arena.

Factor 1 showed the van Hiele level 1- analysis which has 60% or 18/30 students, while factor 2 showed the van Hiele level 0 - recognition or visualization which consist of 40% or 12/30 students. Therefore, in the group factor 1, students at higher level of understanding include first and second level of van Hiele geometry thinking. At factor 2 students only perform at first level of van Hiele geometry thinking. This result supports van Hiele theory that learner must pass through the levels in a sequential order; the progression of level is dependent on the content and method but not on age of students. Instruction must happen at the same level of the students so that learning will occur, if instruction is delivered at a higher level, the student will have difficulty in the learning process (Crowley, 1987). Incidentally, this result answers the second research question which was "What is the relationship between the identified factors?"

Theory suggests that students pass through numerous levels of geometry thinking merely from recognizing geometrical shapes to constructing formal geometry proof (Abu et al., 2012). The van Hiele theory enables us to explain why many students encounter difficulties in their geometry courses. Students cannot progress into higher level of thinking if they have failed in the previous level. This is consistent with the finding that group factor 1 progress from visualization level to analytical level and it related closely to factor 2 upon first level to proceed to second level. It is clear that, to achieve effective geometry learning, the teacher has various roles to play, and these are more or less aligned with the view of current literature.

CONCLUSION

The results of this study support the van Hiele theory which influences and affects students' geometry thinking and achievement in learning geometry especially at elementary level. The links between the sequencing of level is important and interconnect in a manner that influences students' perceived geometry achievement.

Implementation of Q-methodology allowed for the extraction of shared perspectives of van Hiele Levels of Geometry Thinking in order to identify emerging factors under which participants aligned. The two factors, (a) Level 1 - Analysis (or Descriptive) of van Hiele Levels of Geometry Thinking; and (b) Level 0 – Recognition (or Visualization) of van Hiele Levels of Geometry Thinking provided insight into different perspectives leading to a common outcome - that the students' geometry understanding according to van Hiele Levels of Geometry Thinking is at the lower level. The majority of previous research focused on students' performance based on learning difficulties of geometry or what elements affected them. By contrast, the current study sought to provide students' geometry understanding according to van Hiele Levels of Geometry Thinking with an opportunity to explore why they perform lower or have low achievement and offer future researchers with a more comprehensive understanding about the positive influences on students' geometry thinking especially at primary level.

The two identified factors revealed that students who perceived their van Hiele level are due to their ability to reason about a geometrical shape in terms of its properties. The student now sees geometry shapes as collections of properties. Students can recognize and name properties of geometry figures, but they do not yet understand the relationships between these properties and between different figures. Although both groups relied on van Hiele level 0 – recognition level of learning geometry, all of the participating students recognized the importance of recognition or visualization and how it influences them in terms of the impact made by prior level of learning geometry, and that a learner must pass through the levels in a fixed sequence.

This study is not intended to investigate the van Hiele's geometry thinking in detail. However, the findings somehow have provided us with strong indications of the serious deficiencies pertaining to the nature of learning of geometry among students.

The results has shown that most of the students seem to be operating at the lower level of L0 (Recognition) and L1 (Analysis) during the learning of Shapes and Spaces in KBSR syllabus. These findings are more or less similar with those found by other researchers around the world as reviewed earlier. In other words, this finding coupled with those found by Noraini (2007) suggest that a substantially large proportion of Malaysian primary school children are operating at lower level of van Hiele Levels of Geometry Thinking as other school students around the world (United State, China, Taiwan and United Kingdom) do.

This phenomenon gives rise to several interesting points. First, the deficiency of van Hiele Levels of Geometry Thinking appears to be global in nature, crossing the boundaries of educational practices and curriculum. This deficiency might have been the major contributing factor to the learning difficulties encountered by learners around the globe. Secondly, the deficiency may hold the key that explains why Malaysian lower secondary school students had lower performance in TIMSS and PISA. It is to be noted that as far as van Hiele Levels of Geometry Thinking is concerned including higher thinking skills, decision making and mathematical concepts require the learners to operate at higher level in van Hiele theory. Thirdly, this finding may raise concern that many of the students did not seem to have acquired the targeted learning outcomes emphasised by Year 6 KBSR Maths as they are expected to reach at least L2 (Informal Deduction) at the end of the learning session. If this concern happens to be true, than we can expect this group of primary students will be encountering perhaps more serious learning difficulty in geometry at secondary school levels as the learning outcomes of KBSM Maths emphasises learning outcomes that reach higher levels of L3 (Deduction) and L4 (Rigor).

Dissemination of the results of this study may assist in providing future researchers with guidance for conducting additional exploration into the van Hiele Levels of Geometry Thinking. It will further serve to remind all teachers that effective geometry instruction or pedagogical approaches, applying appropriate learning theory such as van Hiele theory, couple with advances of computer technology with capabilities to include visualization-oriented activities is a need in re-evaluation especially at elementary level. Teachers need to select relevant materials and carry out hands-on explorations to develop geometry thinking, making conjectures and carrying out geometry projects and even guide classroom investigations and discussions to provide opportunities to deepen students' geometry understanding.

REFERENCES

Abu, M. S., Ali, M. B., & Tan, T. H. (2012). Assisting primary school children to progress through their van Hiele's levels of geometry thinking using Google SketchUp. *Procedia Social and Behavioral Sciences*, *64*, 75–84. doi:10.1016/j.sbspro.2012.11.010

Bishop, A. 1 (1983). *Space and Geometry.* In R. Lesh & M. Landau (Eds.) *Acquisition of Mathematics Concepts and Processes.* (pp. 175-203). Florida: Academic.

Brown, M. (2004). *Illuminating Patterns of Perception : An Overview of Q Methodology.* Federal Government Contract Number F19628-00-C-0003 with Carnegie Mellon University. CMU/SEI-2004-TN-026

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education.* doi:10.2307/749317

Clements, D. H., & Battista, M. T. (1992). *Geometry and Spatial Reasoning.* In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 420- 464). New York: Macmillan.

Crowley, M. L. (1987). The van Hiele model of the development of geometric thought. In M.M. Lindquist, Ed., *Learning and Teaching Geometry, K-12* (p. 1-16). Reston, VA: National Council of Teachers of Mathematics.

Ding, L., & Jones, K. (2006). Teaching geometry in lower secondary school in Shanghai, China. In *Proceedings of the British Society for Research into Learning Mathematics* (Vol. 26, pp. 41–46).

Effandi, Z., & Abdul Halim, A. (2012). The activities based on van Hiele's Phase-Based Learning : Experts ' and Preservice Teachers ' Views. *Journal of Mathematics and Statistics*, *8*(3), 385–395.

Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education.*, *3*(1988), 1–198.

Job van Exel, Gjalt de Graaf. (2005). *Q methodology : A sneak preview* (pp. 1–30). Version 05.05. www.jobvanexel.nl

Mason, M. (1998). The van Hiele levels of geometric understanding. In *The Professional Handbook for Teachers: Geometry* (Vol. 4, pp. 4–8).

Ministry of Education Malaysian. (2010). *Huraian Sukatan Pelajaran Matematik KBSM* (pp. 1-16). Kuala Lumpur: Ministry of Education.

Monaghan, F. (2001). What difference does it make? Children's views of the differences between some quadrilaterals. *Educational Studies in Mathematics*, *42*, 179–196.

National Center for Education Statistics. (2013). *Highlights From TIMSS 2011* (pp. 1–56). NCES, IES, U.S. Department of Education.

Noraini, I. (1999). *Linguistic Aspects of Mathematical Education: How Precise Do Teachers Need to Be?* In M. A. Clements & P. Y. Leong (Eds.), *Cultural and Language Aspects of Science. Mathematics and Technical Education.* Brunei: Universiti Brunei Darussalam.

Noraini, I., & Tay, B. L. (2004). Teaching and Learning of Geometry: Problems and Prospects. *Masalah Pendidikan*, 27. pp. 165-178. ISSN 0126-5024

Noraini, I. (2007). The effect of Geometers ' Sketchpad on the performance in geometry of Malaysian students ' achievement and van Hiele geometric thinking. *Malaysian Journal of Mathematical Sciences*, *1*(2), 169–180.

Organisation for Economic Co-operation and Development. (2012). *PISA 2012 Results in Focus.*(pp. 1- 32). Programme for International Student Assessment. OECD.

Özerem, A. (2012). Misconceptions in geometry and suggested solutions for seventh grade students. *Procedia Social and Behavioral Sciences*, *55*, 720–729. doi:10.1016/j.sbspro.2012.09.557

Schafer, M. (2003). *The impact of learners' spatial capacity and worldviews on their spatial conceptualisation: A case study.* Unpublished doctoral thesis, Curtin University of Technology, Perth.

Schafer, M., & Atebe, H. U. (2008). *"As Soon as the Four Sides Are All Equal, then the Angles Must be 90° Each". Children's Misconceptions in Geometry.* In M. C. Polaki, T. Mokulu & T. Nyabanyala (Eds.), *Proceedings of the 16th Annual Conference of the Southern Africa Association for Research in Mathematics, Science and Technology.* Maseru: SAARMSTE.

Schmolck, P. (2002). PQMethod (Version 2.33) [Computer Software]. Retrieved from www.lrz.de/~schmolck/qmethod

Senk, S. L. (1989). van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*, *20*(3), 309. doi:10.2307/749519

Usiskin, Z. (1982). *van Hiele levels and achievement in secondary school geometry: Cognitive development and achievement in secondary school geometry project.* Chicago: University of Chicago Press.

van Hiele, P. M. (1986). Structure and insight: A theory of Mathematics education by Pierre M. In *Education Study in Mathematics* (pp. 95–103).

van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. In *Developing Geometric Thinking Through Activities That Begin With Play* (Vol. 6, pp. 310–316).

Wu, D. B., & Ma, H. L. (2005a). A study of the geometric concepts of the elementary school students at the van Hiele level one. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings 29th Conference of the International Group for the Psychology of Mathematics Education,4*, 329-336. Melbourne, Australia: PME.

Wu, D. B., & Ma, H. L. (2009). An Application of GM (0.N) on Analyzing the First van Hiele Geometrical Thinking Level. *Journal of Grey System*, Vol. 12, No. 4, pp. 161-168. Dec. 2009. ISSN 1028-9488

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