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Validation of a questionnaire to evaluate mathematics teachers' beliefs about mathematics, teaching, and learning

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Abstract

The study of mathematics teachers' beliefs has become a significant research area in recent decades; however, more robust and reliable instruments are needed to assess these beliefs. This paper reports the design and validation of a multiple-choice questionnaire to assess mathematics teachers' beliefs about mathematics, teaching, and learning. It began with a systematic review of existing questionnaires, followed by an evaluation of the instrument's content validity using Aiken's V by seven expert judges. Finally, a total of 199 in-service mathematics teachers completed the instrument, demonstrating construct validity through confirmatory factor analysis and internal consistency. These results indicate that the instrument is reliable and valid for assessing mathematics teachers' beliefs and identifying three teaching profiles: instrumentalists, platonists, and problem-solving.

Keywords: beliefs about mathematics, beliefs about teaching mathematics, beliefs about learning mathematics, mathematics teachers, diagnosis of beliefs

INTRODUCTION

Studies on the beliefs of mathematics teachers continue to be a focal point in mathematics education research (Beswick, 2005; 2012; Šunderlík & Rybanský, 2015; Xenophons 2018). This is because the beliefs teachers develop throughout their professional journey wield a direct and substantial impact on the implementation of teaching practices (Dayal et al., 2019; Ernest, 1989; Nisbet & Warren, 2001; Pajares, 1992; Shoenfeld, 1998; Siswono et al., 2019; Wang & Cai, 2007; Xenophon, 2018). It has also been established that teachers' beliefs can significantly influence the academic performance of their students (Widjanjati, 2009). As a result, when teachers' beliefs align more closely with a constructivist perspective on mathematics, the potential for more successful learning outcomes among their students is notably heightened (Muhtarom, 2018).

Beliefs are permeated by the context to which the teacher belongs (Donoso et al., 2016; Vale et al., 2021; Xie & Cai, 2021), by their training and experience (Xie & Cai, 2021). Thus, these beliefs exert a palpable influence on the decisions teachers make in the classroom (Beswick, 2019; Zakaria & Maat, 2012). This aspect of teacher beliefs has garnered substantial interest among researchers, who seek to assess and comprehend these beliefs among educators. The aim of such inquiry is arguably to incorporate these insights into future teacher training endeavors, providing a means for educators to either strengthen or adapt their beliefs, which has the potential to enhance the overall quality of mathematics education (Dorimana, 2021; Koklu & Phan, 2020; Thurm & Barzel, 2020).

Given the challenges of assessing mathematics teachers' beliefs, it is observed that existing instruments are, in some cases, qualitative (Furinghetti & Morselli, 2011; Misfeldt et al., 2016; Vale et al., 2021), making it difficult to apply them to a large number of teachers. Additionally, there are issues when it comes to making statistical generalizations. Other questionnaires are quantitative (Ciftci & Karadag, 2019; Clark et al., 2014; Eichler & Erens, 2014; Goos et al., 2012; Kardanova et al., 2014; Lloyd et al., 2016; Misfeldt et al., 2016; Sunderlik & Rybansky, 2015; Wijaya et al., 2015). However, these instruments assess only one or two variables examined in this study, except for the questionnaires by Dorimana et al. (2021), Swan and Swain (2010), Xie and Cai (2021),

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Contribution to the literature

- This study identifies mathematics teachers' beliefs about the mathematics, teaching, and learning.
- It generates a starting point in the development of teacher training programs, which allows the improvement of teaching practice and teachers' beliefs.
- It provides a validated instrument to identify the mathematics teacher profile associated with instrumentalist, Platonist, and problem-solving beliefs.

Table 1. Beliefs about mathematics, teaching, & learning (Ernest, 1989)

Beliefs	Instrumentalist	Platonist	Problem-solving
Mathematics	Accumulation of rules, facts, & skills that are used to achieve a goal	A static, unified body of knowledge that has already been determined	A dynamic field of knowledge that is constantly evolving & man's invention
Teaching	Direct transfer of knowledge	Explanatory model, focused on understanding concepts and logic	Knowledge building through guidance & problem-solving
Learning	Memorization, repetition, & exercise of procedures	Reception of logical laws & mathematical procedures	Active knowledge building, debate, interaction, & problem-solving

and Zakaria and Maat (2012), which assess beliefs about mathematics, teaching, and learning using a Likert scale. Lischka and Garner (2016) and Safrudiannur and Rott (2021) with a scoring scale, leaving only Muhtaron et al. (2018) and Siswono et al. (2016, 2019), as the only works that evaluate the three variables specifically associated with problem-solving in mathematics and use multiple-choice responses with a single answer.

Therefore, this article presents the process of designing and validating a questionnaire, on the one hand, to evaluate the three dimensions (mathematics, teaching, and learning) and, on the other hand, to write items with three response options with which to identify a profile of the teachers about their beliefs, whether they are instrumentalist, Platonist, or problem-solving.

Theoretical Framework

Regarding teachers' beliefs about mathematics, the most widespread idea about its definition states that these are rules, conscious or unconscious concepts (Wang & Cai, 2007), and a hierarchical psychological system (Ernest, 1989) that guides and impacts the teaching practice (Beswick, 2012). The latter authors recognize three large groups of beliefs (**Table 1**): some that describe characteristics of mathematics, teaching, and learning that are less informed or less advanced in teachers (instrumentalist), others that could be described as informed or more advanced than the instrumentalists (Platonist), and those that favor student-centered teaching practices and, therefore, could be considered the most informed or advanced (problem-solving).

The ideal, at least for Ernest (1989), an author who has been supported by other researchers in more recent works (Calleja, 2021; Muhtarom et al., 2018), is that teachers in future professional development processes can be aware and self-reflective (Siswono et al., 2016) of their beliefs and, when being instrumentalist or Platonist in their teaching practice, they can move towards problem-solving beliefs those have shown to better motivate students to be more engaged in class and, ultimately, perform better in mathematics (Lerch, 2004; Ozturk & Guven, 2015).

METHODOLOGY

Initially, a systematic review of existing instruments was carried out to evaluate beliefs about mathematics, teaching, and learning, and to theoretically substantiate the structure of the questionnaire items; the need to build an instrument to evaluate teachers' beliefs in mathematics, teaching, and learning was also justified, therefore, three profiles can be characterized:

(a) instrumentalist,

(b) platonist, and

(c) problem-solving.

Then, the preliminary version of the questionnaire was constructed, Aiken's V was quantified using expert judgment criteria, and recommendations for improvement were addressed. The following is the statistical treatment from the administration of the instrument to a pilot group of 199 primary and secondary mathematics teachers to meet the criteria of content and construct validity and reliability.

Participants in Validation

To validate the content of the questionnaire, seven expert judges were used. The criteria for their selection were:

(a) to have a PhD in mathematics education or related to the study of beliefs about mathematics,

(b) to have participated in research on the study of beliefs about mathematics or related, and

(c) experience in training of secondary mathematics teachers.

Participants in Pilot Group

Once the content of the questionnaire was validated, an online test of the instrument was carried out with a sample of 199 in-service secondary school mathematics teachers, who participated voluntarily.

The sample size was based on the work of Gorsuch (1983), who suggested a ratio of at least five subjects per item and a size that was not below 100 people. The participants were 51.8% women and the remaining percentage men. The most representative age range was between 36 and 45 years old with 36.2%, while the lowest consisted of those under 25 years old with 4.5%.

In terms of teaching experience, the most frequent range corresponds to between five and 10 years with 23.1%; followed by between 16 and 20, 18.6%; 11 and 15 years old, 18.1%; 21 and 25 years old, 16.1%; more than 26 years with 10.1%. Concerning undergraduate education, 78.0% studied professional degrees in mathematics or basic education with an emphasis on mathematics, the remaining 22.0% have studies with a high mathematics component, such as engineering, administration, physics, and accounting, among others.

Regarding postgraduate training, 22.1% of the participants have not accessed it, 20.1% have a master's degree or specialization associated with mathematics, only 0.5% have PhD studies, and the remaining sample has done so in areas such as education, pedagogy, and technology.

Literature Review & Initial Questionnaire Development

A systematic review was carried out, using Scopus and Web of Science databases, to identify articles published in English between 2010 and 2021 that have focused on the beliefs of in-service mathematics teachers (those from the university level were not included). The search strategy used following criteria: "beliefs" AND "mathematics" AND "teachers" AND "secondary OR high school". The search was extended to Google Academic Search as a secondary source, finding 111 articles for a total of 3,055 papers. In the first review, 2,174 duplicate papers were eliminated thus 881 were considered for the next phase. In the second screening exercise, 422 studies were excluded as they were not directly related to teachers' beliefs about mathematics. Subsequently, in another exclusion exercise, only the studies that answered the question "What are teachers' beliefs about mathematics, teaching, and learning?" were chosen, thus reducing the articles included in this review to 29 studies of which 19 reported development of instruments to evaluate the beliefs of mathematics teachers and/or students.

According to **Table 2**, 10 studies assess beliefs about teaching and learning mathematics associated with pedagogical approaches of transmissionism and constructivism (Ciftci & Karadag, 2019; Clark et al., 2014; Goos et al., 2021; Guangbao & Timothy, 2021; Kardanova et al., 2014; Lloyd et al., 2016; Sunderlik & Rybansky, 2015; Wijesundera et al., 2021; Xie & Cai, 2021; Zakaria & Maat, 2012). Three studies are associated with epistemic beliefs about mathematics (mechanistic and realistic, absolutist, fallibilist) (Corkin et al., 2015; Lischka & Garner 2016; Wijaya et al., 2015), while the remaining seven studies are related to beliefs about mathematics, its teaching, and learning. These are distributed, as follows: two studies correspond to the worldviews theory (formalist, application, and process) (Eichler & Erens, 2014; Swan & Swain, 2010), and 5 studies are associated with Ernest's (1989) views (instrumentalism, Platonism, and problem-solving) (Dorimana et al., 2021; Lischka & Garner, 2016; Misdfeldt et al., 2016; Muhtaron et al., 2018; Siswono et al., 2019).

As can be seen in **Table 2**, nine studies reported Cronbach's alpha between the range of 0.60 and 0.87, therefore, their reliability can be considered acceptable (around 0.60) or optimal (values above 0.80). The reliability of these instruments may improve in further research but, despite having 20 papers with a wide range of items, most of them are based on Likert scales, which makes it difficult to characterize teachers by belief profiles (instrumentalist, Platonist, and problem-solving) primarily because they were not designed for such purposes. Only in the case of Safrudiannur and Rott (2021) and Siswono et al. (2016, 2019), given their multiple-choice and scoring scale-based formats, respectively, can the correct choice associated with a problem-solving belief define such profiles among teachers. Given this context, it could be stated that the instruments that can serve as a basis for improvement are those of these two authors.

Instrument Development

10 items from Safrudiannur and Rott's (2021) work were adapted and translated into Spanish. Subsequently, as a result of the questionnaire analysis and the qualitative statements from Siswono's (2016, 2019) studies, 23 additional items were constructed, considering the definitions presented by Siswono et al. (2019). These definitions correspond to the analysis the author performed on the teachers' cases, identifying them under Ernest's (1989) hierarchical belief profiles. For example, regarding teaching, Siswono et al. (2019) mentions that the teacher's role in helping solve problems can be that of a knowledge and skills transmitter, the teacher as an evaluator of students' work, or the teacher as a facilitator, which correspond to the instrumentalist, Platonist, and problem-solving views, respectively. This analysis refers to item 21 in the questionnaire.

After the recommendations of the experts and the validation process, the multiple-choice questionnaire on mathematics, teaching, and learning beliefs (see **Appendix**) consists of 33 items classified into three

Table 2. Instruments to assess teachers' beliefs

Reference	Scale*	n	Validity & reliability **	Beliefs evaluated	BA***
Swan and Swain (2010)	RS	28	Not reported	Mathematics, teaching, learning, & practice	TCD
Zakaria and Maat (2012)	LS	30	*a*=0.60	Mathematics, teaching, learning, & teaching practice	C
Clark et al. (2014)	LS	40	α=0.60 & EFA	Teaching, learning, learners' attitude, & declared awareness	T
Eichler and Erens (2014)	LS	NG	Not reported	Mathematical beliefs about teaching calculus	SAP
Kardanova et al. (2014)	LS	NG	EFA & CFA	School climate, general beliefs about teaching, conceptions of good teaching, & perceptions about one's own practices	CT
Corkin et al. (2015)	LS	28	*a*=0.70 in each dimension	Epistemic, self-efficacy & internal locus	AF
Sunderlik and Rybansky (2015)	LS	16	*a* is acceptable Cronbach's alpha & EFA	Effective teaching, general on teaching & learning, practices, & attitudes	SAP
Wijaya et al. (2015)	LS	16	Not reported	Teaching & learning mathematics, & context-cased tasks	MR
Lischka and Garner (2016)	RS	20	Probability curves	Mathematics, teaching, learning, & reform-oriented classroom discourse	AF
Misfeldt et al. (2016)	OQ	55	Qualitative triangulation study	From mathematics to technology	IPP
Lloyd et al. (2016)	LS	18	EFA	Regulations related to mathematics and teaching	C
Muhtaron et al. (2018)	MC	30	Validation by three expert peers	Consistency in students & teachers' beliefs about nature, teaching, & learning of mathematics (adapted from Siswono et al., 2016)	IPP
Ciftci and Karadag (2019)	LS	32	*a*=0.70	Teaching mathematics	CT
Siswono et al. (2019)	MC	18	*a*=0.60	Mathematics, knowledge in mathematical problem-solving, & teaching	IPP
Safrudiannur and Rott (2021)	RS	10	*a*=0.80	Mathematics, teaching & learning associated with problem-solving & relationship it has with students' mathematical skills	IPP
Goos et al. (2021)	LS	20	Not reported	Epistemological & pedagogical	CT
Dorimana et al. (2021)	LS	23	*a*=0.659	Mathematics, teaching & learning associated with problem-solving	IPP
Guangbao and Timothy (2021)	LS	16	*a* greater than 0.70 & CFA	Teaching & learning international survey, teachers' pedagogical beliefs & self-efficacy	C
Wijesundera et al. (2021)	LS	71	*a* values ranging between 0.749 & 0.966	Teaching mathematics in classroom, student-related constraints, & self-efficacy in teaching mathematics	CT
Xie and Cai (2021)	LS	26	*a*=0.87 EFA & CFA	Mathematics, teachers & students' teaching & learning	CT

Note. *LS: Likert scale; RS: Rating scale; OQ: Open-ended question; MC: Multiple choice; **CFA: Confirmatory factor analysis; EFA: Exploratory factor analysis; *a*: Cronbach's alpha; ***TCD: Transmissionist, connectionist, & discovery beliefs; CT: Constructivist & traditional beliefs; IPP: Instrumentalist, platonist, & problem-solving beliefs; SAP: System, application, & process beliefs; AF: Absolutist & fallibilist beliefs; MR: Mechanistic & realistic beliefs; C: Constructivist beliefs; T: Traditional beliefs; n: Number of items; & BA: Belief approaches

categories, with an equal number of questions for each variable (11). The instrument constructed is called COMEA by its Spanish acronym.

All the items have the same structure: a statement, where a situation is posed and three answer options (a, b, and c). The first option offers an instrumentalist-type response; the second, a Platonist-related answer; and the third, a problem-solving view (the desired or correct one). It should be clarified that in the items the term task is understood as an activity that includes several things, for example, performing exercises, constructing objects, solving problems, debating ideas, consulting, among others (Zakaria & Maat, 2012).

Content Validity

Content validity was carried out through a rubric that evaluated:

(a) coherence between the question and the indicator,

(b) clarity,

(c) agreement of responses with statements, and

(d) the correct formulation of answer options.

For each question, a level of fulfillment with each criterion was indicated on a four-point Likert scale, with four representing the highest level and one indicating non-compliance. Content validation by the seven judges was conducted by calculating the Aiken's V coefficient (V), which yields values between zero and one, where a value of one indicates the highest agreement among judges. To keep an item, a threshold of V≥0.71 was considered for the lower limit of the confidence interval and one for the upper limit (Escurra, 1988; Soto & Segovia, 2009; Torres-Malca et al., 2022).

Construct Validity

The data collected in the pilot test were analyzed using the Stata software to assess the construct validity. A correlational method, specifically confirmatory factor analysis (CFA), was employed, which is commonly used to report the internal structure of a measurement instrument (Morata-Ramírez et al., 2015). A CFA is a multivariate modeling technique used to estimate the measurement model and test hypotheses regarding the factor structure of a set of variables within an instrument (Martínez, 2021). In this technique, the researcher a priori defines the number of factors in the model and the relationships among its components based on theory (Herrero, 2010). Consequently, when examining the internal structure of the instrument, it ensures that it assesses what it was designed for and aligns with the supporting theoretical model (Jordan Muiños, 2021). To achieve this, CFA is employed to estimate the measurement model and establish its reliability and validity, considering that the theory was articulated with the creation of the instrument. Goodness-of-fit indices were evaluated to determine model's appropriateness.

Acceptable values for these indices include: ratio between chi-square and degrees of freedom of <3 (indicating adequate fit), between three and five (indicating acceptable fit), comparative fit index (CFI) values approaching 0.90 are considered acceptable (Bentler, 1992), standardized root mean square residual (SRMR) should be ≤0.08 (Hu & Bentler, 1999), root mean square error of approximation (RMSEA) should be ≤0.06 (Hu & Bentler, 1999), and coefficient of determination (CD) should range between zero and one, indicating a better fit as it approaches one.

Internal Consistency

The reliability of the instrument was quantified using Cronbach's alpha coefficient in which acceptable values range by consensus from 0.65 to 0.80 (Peña-Sarmiento et al., 2022). According to George and Mallery (2003), values below 0.5 are unacceptable, (α) greater than 0.5 poor, above 0.6 are moderately acceptable, >0.7 acceptable, >0.8 good, and >0.9 excellent. An indicator that to date guarantees the quality of the psychometric instrument (Taber, 2018). For this study, the coefficient was calculated to measure the reliability of the whole instrument, as well as its categories.

RESULTS

Content Validation

Table 3 presents the results of the content validation through the judgment of seven experts. The highest scores were obtained concerning the criterion of coherence (mean [M]=3.8; δ=0.234; V de Aiken=0.932), whereas the lowest score referred to the formulation of answer options (M=3.4; δ=0.341; V de Aiken=0.809).

Table 3. Aiken's V

Item	Coherence			Clarity			Agreement of answer			Formulation of answer option		
	M	δ	Aiken's V	M	δ	Aiken's V	M	δ	Aiken's V	M	δ	Aiken's V
M1	3.5	0.490	0.857	3.3	0.487	0.761	3.5	0.494	0.857	3.3	0.451	0.761
M2	3.3	1.030	0.761	3.3	1.030	0.761	3.4	1.049	0.809	3.3	1.030	0.761
M3	3.8	0.349	0.952	3.8	0.349	0.952	3.7	0.451	0.904	3.7	0.728	0.904
M4	3.8	0.349	0.952	3.7	0.451	0.904	3.7	0.451	0.904	3.3	0.699	0.761
M5	3.5	0.728	0.857	3.7	0.451	0.904	3.5	0.728	0.857	3.6	0.494	0.857
M6	3.4	0.494	0.809	3.1	0.638	0.714	2.7	1.277	0.571	3.0	1.069	0.666
M7	3.8	0.349	0.952	3.7	0.451	0.904	3.4	0.728	0.809	2.8	0.832	0.619
M8	4.0	0.745	1.000	3.4	0.728	0.809	3.0	0.759	0.666	3.0	0.755	0.666
M9	3.4	0.728	0.809	3.6	0.494	0.857	3.4	0.728	0.809	4.0	0.832	1.000
M10	4.0	0.000	1.000	4.0	0.000	1.000	3.8	0.349	0.952	3.3	0.880	0.761
M11	3.4	0.728	0.809	3.3	0.880	0.761	3.6	0.728	0.857	3.3	0.880	0.761
T12	3.4	1.049	0.809	3.4	1.049	0.809	3.1	1.069	0.714	3.3	1.030	0.761
T13	4.0	0.000	1.000	4.0	0.000	1.000	3.8	0.349	0.952	3.7	0.451	0.904
T14	3.7	0.699	0.904	3.7	0.699	0.904	3.7	0.699	0.904	3.1	0.832	0.714
T15	4.0	0.000	1.000	3.6	0.728	0.857	3.5	0.728	0.857	3.6	0.832	0.857
T16	4.0	0.000	1.000	3.4	0.451	0.809	3.8	0.349	0.952	3.4	0.728	0.809
T17	4.0	0.000	1.000	3.7	0.451	0.904	3.8	0.349	0.952	3.6	1.049	0.857
T18	4.0	0.000	1.000	4.0	0.000	1.000	4.0	0.000	1.000	3.7	0.699	0.904
T19	3.8	0.349	0.952	3.7	0.451	0.904	3.8	0.349	0.952	3.6	0.494	0.857

Table 3 (Continued). Aiken's V

Item	Coherence			Clarity			Agreement of answer			Formulation of answer option		
	M	δ	Aiken's V	M	δ	Aiken's V	M	δ	Aiken's V	M	δ	Aiken's V
T20	4.0	0.000	1.000	4.0	0.000	1.000	4.0	0.000	1.000	3.7	0.699	0.904
T21	3.8	0.349	0.952	4.0	0.000	1.000	4.0	0.000	1.000	3.7	0.699	0.904
T22	4.0	0.000	1.000	4.0	0.000	1.000	4.0	0.000	1.000	3.7	0.699	0.904
L23	3.7	0.699	0.904	3.7	0.699	0.904	3.7	0.487	0.904	3.6	0.494	0.857
L24	4.0	0.000	1.000	3.7	0.699	0.904	3.7	0.487	0.904	3.8	0.349	0.952
L25	3.8	0.349	0.952	3.6	0.728	0.857	3.2	0.755	0.761	2.8	0.989	0.619
L26	4.0	0.000	1.000	3.1	0.832	0.714	3.1	0.699	0.714	3.1	1.124	0.714
L27	3.4	1.049	0.809	2.8	0.989	0.619	2.8	0.000	0.619	3.1	0.989	0.714
L28	3.4	0.699	0.809	4.0	0.000	1.000	4.0	0.000	1.000	3.6	0.728	0.857
L29	3.8	0.349	0.952	3.7	0.451	0.904	3.7	0.487	0.904	3.6	0.728	0.857
L30	3.8	0.349	0.952	3.4	0.5	0.809	3.6	0.534	0.857	3.7	0.451	0.904
L31	4.0	0.000	1.000	3.6	0.728	0.857	3.4	0.786	0.809	3.8	0.349	0.952
L32	3.8	0.349	0.952	3.8	0.349	0.952	3.4	0.786	0.809	3.3	0.699	0.761
L33	4.0	0.000	1.000	3.7	0.451	0.904	3.8	0.377	0.952	3.8	0.349	0.952
Total	3.8	0.234	0.932	3.6	0.304	0.871	3.6	0.353	0.859	3.4	0.341	0.809

Table 4. An example of a modified item

Initial item: Doing mathematics a. requires logical-formal derivation and abstraction and formalization ability. b. requires a lot of practice to follow and apply routines and calculation schemes. c. means understanding facts, noticing relationships, and having ideas.			
Quantitative assessment			
Coherence: M=4.0, **δ**=.745, & Aiken V=1.0	Clarity: M=3.4, δ=.728, & Aiken V=.809	Agreement with responses: M=3.0, δ=.759, & Aiken V=.666	Formulation of response options: M=3.0, δ=.755, & Aiken V=.666
Qualitative evaluation J1: b) may be "takes a lot of practice to apply". J2: Agree on what is understood or what it means to do math. J3: a) & b) are interchanged. In c) word "means" reduces mathematical work to a single thing, it is not correct. I suggest changing means for consider. J4: Items a) & b) are swapped. Answers are not of same nature, item a) refers to cognitive requirements, b) to aptitudes, & c) to what it means.			
Modifications: Initial statement was modified, items a) & b) were interchanged, & answer options were improved.			
Final wording of item: To understand mathematics, what is most required is a. lots of practice to apply routines and calculation schemes. b. logical-formal derivation and the capacity for abstraction and formalization. c. considering incorporating facts, noticing relationships, and having ideas.			

Items from 1 to 11 refer to mathematics beliefs (M), items from 12 to 22 have to do with teaching beliefs (T), and items from 23 to 33 relate to learning beliefs (L).

The items were reviewed individually based on the scores given by the experts in the four evaluated criteria and their qualitative evaluation. Items with Aiken's V<0.71 were modified. Therefore, modifications were made to items M6, M7, M8, L3, and L5. As an example, the modification of item M8, which presents difficulties in the criteria of agreement and correct formulation of the answer options, can be seen in **Table 4**. According to the quantitative evaluation obtained from the expert judges, no item was eliminated since none of them obtained an Aiken's V<0.71 in three or more criteria.

Construct Validation

The model fit was analyzed based on three latent exogenous variables: beliefs about mathematics, beliefs about the teaching of mathematics, and beliefs about the learning of mathematics, which are interconnected. Each exogenous variable comprises 11 endogenous variables (11 items). Maximum likelihood estimation was used for the latent variables because its objective is to find parameter values that make the data most probable (Gómez-Mejía, 2020).

Chi-square statistic is calculated (chi2=740.994 with 248.994 degrees of freedom, p<.001), indicating the absolute fit of the model. The obtained chi-square value implies a discrepancy in the model; however, it is important to note that this value is highly sensitive to sample size (Bentler, 1980, 1990; Pilatti et al., 2012), which, in this case, is less than 200, as well as to multivariate normality (Kaplan, 1990) and the estimation method used (Lévy et al., 2006). For this reason, the focus of interpretation is on the chi-square-to-degrees-of-freedom ratio.

Table 5. CFA results

Goodness-of-fit indices for COMEA				
Index	Perfect fit criterion	Acceptable fit criterion	Finding	Result
	0-3	3-5	2.975	Perfect fit
RMSEA	0.00≤RMSEA≤ 0.05	0.05≤RMSEA≤0.10	0.051	Acceptable Fit
CFI	0.95≤CFI≤1.00	Trending to 0.90	0.715	Good fit
SRMR	0.00≤ SRMR≤0.05	0.05≤SRMR≤0.08	0.075	Acceptable Fit
CD	Tend to one		0.959	Perfect fit

Table 6. Cronbach's alpha value per dimension

Reliability statistics		
Category	Numbers of items	Cronbach's *a*
Mathematics	11	.590
Teaching mathematics	11	.628
Learning mathematics	11	.703

Table 5 presents the goodness-of-fit index values obtained for the entire model.

The coefficient obtained between the chi-square and the degrees of freedom indicates a perfect fit of the model. RMSEA attains a value of 0.051, which, according to Martínez Ávila (2021), suggests an acceptable model fit for values ≤0.08. In relation to CFI, a value of 0.715 was obtained, indicating that the model's items are related at a 71.5% level, not being so far from the recommended threshold of 0.90 that is proposed for adequate model fit. It is important to note, however, that this value is dependent on sample size (Lai, 2020; Mulaik et al., 1989), and this limitation is present in this case. Despite this, all the values conform to the model, even if CFI value is considered poor.

Internal Consistency

Regarding the overall internal consistency of the instrument, Cronbach's alpha value obtained is 0.803. Therefore, the instrument is considered to have good internal consistency and reliability.

As for the consistency within each dimension, the values presented in **Table 6** are observed. It is noted that both beliefs about teaching and beliefs about learning mathematics yielded acceptable values, while beliefs about mathematics exhibited a poor value due to being less than 0.6 but higher than 0.5. The fact that the variable "mathematics" had a poor value might be attributed to the sensitivity of Cronbach's alpha when dealing with small samples (Peña-Sarmiento et al., 2022). Furthermore, excluding any of the items from this variable does not enhance the reliability; on the contrary, it diminishes the internal consistency of the variable and the entire instrument.

DISCUSSION & CONCLUSIONS

This article presents the process of design and validation of a single-select multiple-choice questionnaire to determine teachers' beliefs about mathematics, teaching, and learning. According to the answers, the respondents could be classified into three teacher profiles:

(a) instrumentalist,

(b) platonist, and

(c) problem-solving.

A non-Likert scale was chosen because, as suggested by Philipp (2007), this type of measurement from one to five (from least to greatest agreement) provides little or no context of teachers' beliefs and increases the tendency for respondents to respond according to how society would expect them to do (Safrudiannur & Rott, 2021). Second, single-select multiple-choice questionnaires could facilitate the classification of teachers' specific profiles based on the answer options chosen, in this case, the identification of instrumentalist, Platonist, and problem-solving beliefs.

Regarding content validity, the seven experts evaluated the instrument based on four criteria, and on average, the values obtained were above the lower limit of the confidence interval (V≥0.71). Specifically, the scores were, as follows: coherence 0.932, clarity 0.871, agreement of responses 0.859, and formulation of answer options 0.809. Consequently, the instrument, including the adjustments requested for items M6, M7, M8, L3, and L5, met the specific domain of what it aimed to assess: beliefs about mathematics, teaching, and learning.

Concerning CFA conducted using STATA, the structure of the model is confirmed, with acceptable values obtained for the calculated goodness-of-fit indices. This is a significant finding because factor analyses are typically performed with continuous variables (Cupani, 2012). However, in the case of this instrument, which involves a nominal variable (neither dichotomous nor ordinal), CFA was applied for validation, as the model is theoretically used to confirm the theory (Herrero, 2010).

A coefficient of determination very close to one was obtained, indicating an excellent fit, as well as a coefficient between chi2 and degrees of freedom below three, suggesting that the instrument fits well despite having a sample size of less than 200. According to Batista-Foguet et al. (2004), this can affect chi-square results, as can the nominal nature of the variable. Nevertheless, the most crucial aspect is quantifying the degree of model fit.

Considering the goodness-of-fit values, RMSEA of 0.051 and CFI of 0.715 from CFA, these values are consistent with the model. However, there are authors who prefer RMSEA to be higher than 0.06 and CFI to approach 0.9. Nevertheless, authors such as Méndez-Giménez (2014) accept these values. To enhance these fit indices in future research, increasing the minimum sample size to 300 subjects and ensuring statistically normal distributions could be considered, which is not always achievable with multiple-choice instruments

It is worth concluding that the literature does not frequently report factor analyses for instruments with discrete variables (single-select multiple-choice) due to the previously described reasons. Nevertheless, and grounded on the justification that the items were constructed based on theory, it can be concluded that CFA is acceptable.

The coefficients of internal consistency were calculated using Cronbach's alpha, yielding a value of 0.803, which is considered optimal for the overall set of items. Additionally, the reliability coefficients for each factor obtained values of 0.590, 0.628, and 0.703, indicating that the measurement obtained with the instrument is reliable (George & Mallery, 2003). It is important to note that Cronbach's alpha is highly sensitive to small sample sizes due to fluctuations in alpha errors (Peña-Sarmiento et al., 2022).

Therefore, it is concluded that the instrument designed and validated (COMEA) is consistent and reliable due to its adequate coherence, syntax, and content that evidences the relationship between the variables and their items. This allows the questionnaire to finally assess teachers' beliefs about mathematics, teaching, and learning and in turn to characterize teacher profiles.

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APPENDIX: BELIEFS IN MATHEMATICS, TEACHING & LEARNING (COMEA)

COMEA Questionnaire

Section I. General information

Name or pseudonym ____

Please, mark with an X

Gender: Male ______ Female _______ Other ________

Age: Less than 25 _________ 25 to 35 ________ 36 to 45 ______ 46 to 55 __________ More than 55 ___________

Years of experience: Less than five____________ Between five & 10 __________
Between 11 & 15 _________ Between 21 & 25 ___________
More than 26 __________

Academic degree: Normal school_________ Bachelor's degree_________
Professional _________ Specialization _________
Master's degree _______ Doctoral degree __________

Write your undergraduate education: ____

Section II. About beliefs

Dear teacher, the following instrument is intended to identify secondary school teachers' beliefs associated with mathematics nature, teaching, and learning. The question corresponds to the multiple-choice model with a single answer. All the items have the same structure: a statement where a situation is posed and three response options (a, b, & c). Please mark with an X option that aligns most with your way of thinking. There are no right or wrong answers.

Beliefs about nature of mathematics

1. Concepts in mathematics are understood as
   a. an accumulation of facts, rules, and skills that are useful in everyday life.
   b. logically sequenced and interconnected within an organized structure.
   c. constructed through a dynamic process carried out by humans.
2. Regarding the truth of mathematical concepts, it can be stated that they are
   a. absolute truths, without conflicting interpretations.
   b. objective truths, revealed by humans, but not determined by them.
   c. truths established by humans and constructed by them.
3. Mathematical knowledge is
   a. a static body of knowledge that exists on its own and is expressed through a set of rules and procedures that can solve any situation.
   b. a science that establishes patterns expressed in symbols, does not evolve, and is revealed by humans.
   c. a set of mathematical objects created by the human mind that responds to the needs of society.
4. For culture, mathematics should be conceived as
   a. the most important science because it contributes to the development of others, such as physics, chemistry and/or human social sciences.
   b. a science characterized by the use of ideas or forms found in the world, represented through symbols.
   c. a science that emphasizes the development of reasoning skills and both critical and creative thinking.
5. The most important aspect that mathematical skills should enable is
   a. to calculate and perform established mathematical procedures.
   b. to understand the branches of mathematics, both pure mathematics and those used in other professions.
   c. to establish relationships between mathematics and everyday life situations.
6. The most appropriate medium-term relationship between mathematics and real life is
   a. utilitarian, enabling one to perform calculations, choose the appropriate formula, and then solve real-world problems.

  b. disjunctive, in which it is identified which knowledge solves only the problems of mathematics or which solves those of real life.
  c. dialogical, in which real life serves as input to create mathematics, and mathematics allows for understanding real-life problems.

7. Mathematics can be understood as
  a. a set of procedures and rules that precisely determine how a task is solved, an activity that involves thinking about problems and acquiring knowledge.
  b. a logically non-contradictory structure with clear, precisely defined terms and unequivocally demonstrable claims.
  c. an activity that involves thinking about problems and acquiring knowledge.
8. To understand mathematics, what is most required is
  a. a lot of practice to apply routines and calculation schemes.
  b. logical-formal derivation and the capacity for abstraction and formalization.
  c. considering incorporating facts, noticing relationships, and having ideas.
9. The most important thing when doing mathematics is
  a. memorization and the application of definitions and formulas, facts, and mathematical procedures.
  b. the formal rigor of mathematical argumentation.
  c. intuition, as well as thinking and reasoning, both related to content.
10. The main component considered in mathematics is
  a. mathematical formulas and procedures.
  b. exact and precise mathematical terminology.
  c. ideas, terms, and connections.
11. The central aspects of mathematics are
  a. definitions, rules, and formulas.
  b. impeccable formalism and formal logic.
  c. content, ideas, and cognitive processes.

Beliefs about teaching mathematics

12. What should be prioritized when teaching a formula is
  a. how to use it through examples.
  b. explaining where the concepts originate.
  c. having students discover it through situations.
13. The most important action the teacher should take when assigning a task in class is
  a. providing students with hints about which formula to use and how to use it.
  b. giving students hints to solve it and ensuring they understand the procedures.
  c. motivating students to create strategies to solve it and guiding them if they request assistance.
14. The most relevant moment to introduce problem-solving during teaching is
  a. before or after explaining the topic, with a focus on having students apply mathematical procedures.
  b. after students have learned the appropriate concepts and algorithms.
  c. at the beginning of the topic so that students can use their prior ideas.
15. The primary role that the teacher should assume to help students solve tasks is
  a. explaining the specific topic related to the proposed activities so that they can reach the answer.
  b. evaluating the process at each step to reduce the errors they may make.
  c. guiding only when assistance is requested, so that they can develop different strategies that lead them to the solution.
16. The source that the teacher prioritizes for planning tasks and suggesting work to their students comes from
  a. textbooks because the activities are already established and correspond to a single topic and way of being resolved.
  b. textbooks or the internet because they restructure information or questions according to the lesson's objectives.

c. those proposed by their students during the class because it encourages debate, creativity, and collaborative work.

17. To clarify the difficulties that students have during the instruction of a topic, the teacher should prioritize:
    a. explanation, focused on students' doubts, emphasizing the execution of procedures.
    b. providing a detailed explanation guided by the questions the teacher asks the students.
    c. creating spaces for interaction among students so that they can share their doubts and solution strategies.
18. When a student cannot solve a task, the teacher should say:
    a. "Watch how I solve it, and then you do it."
    b. "Tell me where you are having difficulty, and I will tell you how to do it in that part."
    c. "How have you thought it can be done."
19. The best key to teaching mathematics is
    a. strictly following the objectives in the curriculum and established procedures.
    b. establishing strategies for understanding mathematical concepts and procedures.
    c. promoting mastery and the discovery of procedures for completing tasks.
20. The actions that the teacher should encourage in the teaching of mathematics are
    a. explanation, generalization, and imitation.
    b. explanation and understanding of concepts and procedures.
    c. participation, experimentation, and interaction.
21. The teacher should preferably be seen by his students as
    a. the transmitter of knowledge and skills.
    b. a mathematical authority and an evaluator.
    c. the one who guides students to construct their own knowledge.
22. In mathematics teaching, error is
    a. penalized and immediately corrected.
    b. used to explain a procedure again.
    c. an opportunity to recognize knowledge and learn.

Beliefs about learning mathematics

23. What students should learn from mathematics is
    a. the correct way to use and memorize them.
    b. understanding the variables they relate.
    c. the relationship they have with other contexts.
24. To learn how to solve problems, the student's priority must be
    a. remembering and applying the procedure to solve them.
    b. solving them correctly and explaining how they did it.
    c. creating their own strategies to solve them.
25. The most important activity for the teacher to carry out in order for students to learn to solve tasks is
    a. proposing many similar tasks to reinforce concepts and skills.
    b. applying the procedures explained by the teacher as it is the best way to solve different types of tasks.
    c. assigning different situations for them to create solution strategies.
26. The best strategy for students to learn to solve tasks is
    a. only the ones explained and worked on by the teacher in class.
    b. both the ones explained by the teacher and those agreed upon with the teacher.
    c. the ones proposed by them during task completion.
27. The main purpose of learning formulas in mathematics should primarily enable students to
    a. solve tasks and save time.
    b. solve tasks and understand the process of formula derivation.
    c. recognize their existence but also explore new alternatives.

28. In the learning of mathematics, the most important aspect of students using technology (calculator and applications) is that
    a. it should only be used to verify the obtained answer, not to learn mathematics.
    b. it can be used after they have learned how to perform mathematical procedures.
    c. it facilitates the creation of their own strategies in solving situations.
29. The most essential thing in learning mathematics is
    a. accepting and repeating the knowledge received.
    b. establishing relationships between concepts and procedures.
    c. creating guesses from knowledge.
30. Students learn mathematics if they
    a. remember the mathematical language, formulas, and definitions.
    b. interpret the definitions and underlying procedures.
    c. solve challenging mathematical tasks.
31. The best way for students to learn mathematics is
    a. by observing, listening, and imitating, individually.
    b. by exploring, reflecting, and prioritizing individual work.
    c. by debating and creating through interaction.
32. The learning of mathematics mainly takes place
    a. individually, prioritizing content, assigning routine tasks, and practicing procedures.
    b. generally individually, following the given examples, learning the formulas and concepts addressed.
    c. through both group and individual work, generating discussion and the creation of strategies to solve various tasks.
33. The best way for students to learn mathematics is
    a. memorizing definitions, formulas, keywords, and repeating solutions provided by the teacher in different tasks.
    b. relating definitions to formulas, solving exercises, and addressing the tasks proposed by the teacher.
    c. solving different types of tasks through strategies and conjectures proposed by themselves.