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Fostering the Mathematics Learning of Language Learners

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Fostering the Mathematics Learning of Language Learners: Introduction to Trends and Issues in Research and Professional Development

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ABSTRACT

The introduction of the EURASIA Special Issue argues why fostering the mathematics learning of (monolingual and multilingual) language learners is crucial with respect to equitable access to mathematics. It provides a structured list of parallel questions for research and design on the classroom level as well as on the professional development level. The overview on the articles of the special issue shows how widely the field must be spanned in order to grasp the complexities of the learning content (language demands specific to mathematical topics and genres), the learners' and the teachers' processes.

Keywords: language demands, language learners, mathematics learning, professional development

RELEVANCE FOR CONSIDERING LANGUAGE LEARNERS IN MATHEMATICS EDUCATION RESEARCH

The role of language for mathematics learning has been in the focus of research in mathematics education since three decades (e.g. Pimm, 1987; Ellerton & Clarkson, 1996). Language has been identified as *learning medium* and as *learning goal* (Lampert & Cobb, 2003). The increasing research focus on equity and access for all learners (Secada, 1992; DIME, 2007) has added a third function, language as *unequally distributed learning prerequisite*, since limited language proficiency in the language of instruction can constrain the mathematical learning opportunities in mathematics classrooms (Snow & Uccelli, 2009). This does not only apply to students whose family language differs from the language of instruction (students with minority languages or immigrant status, Haag, Heppt, Stanat, Kuhl, & Pant, 2013; OECD,

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2007; Barwell et al., 2016) but also for monolingual students with under-privileged socio-economic status (Heinze, Reiss, Rudolph-Albert, Herwartz-Emden & Braun, 2009; Prediger, Wilhelm, Büchter, Benholz, & Gürsoy, 2015). That is why monolingual and multilingual students with low language proficiency in the language of instruction are subsumed under the unifying construct "language learners" in this volume (cf. Moschkovich, 2010a).

Raising Important Questions on the Level of Classrooms and Professional Development

The gap in the mathematics achievement of students with high and low language proficiency has often been shown in large scale studies (e.g. OECD, 2007; Haag et al., 2013; Prediger et al., 2015), but these studies alone cannot provide an empirical foundation for fostering the mathematics learning of language learners. Instead, many further questions must be answered, in parallel for the level of classrooms and professional development (PD):

Questions that need to be addressed in research and design on the classroom level:

(Q1) What language demands are most relevant in mathematics classrooms?
(Q2) How can instructional approaches be designed to support language learners' access to mathematics and the required language? How can these approaches, in the case of multilingual learners, connect to the students' language resources?
(Q3) Which effects and challenges do different instructional approaches have for supporting language learners in mathematics classrooms?

Questions leading research and design on the level of professional development:

(Q4) What do mathematics teachers need to learn for being able to support language learners in mathematics classrooms?
(Q5) How can PD be designed to enable teachers to support language learners?
(Q6) Which effects and challenges do different PD approaches have for enabling teachers to support language learners in mathematics classrooms?

Contributions to Specifying Language Demand for Language Learners in Mathematics Classrooms (Q1)

Although all articles in the special issue focus on language learning in mathematics, only two articles explicitly address the WHAT-question of what language demands are crucial for mathematics learning (Q1).

- Rezat & Rezat (2017) investigate language demands connected to the mathematics-specific genre of geometric construction texts. They argue why the text level must be considered in research, as genre-specific aspects must be taken into account and should be articulated with the students explicitly. This study gives an interesting example for the communicative function of language in the mathematics classroom.
- Prediger & Zindel (2017) suggest a research program how topic-specific language demands can be specified empirically in a design research framework (Prediger, Gravemeijer, & Confrey, 2015). They show for the mathematical topic of functional

relationships that video-based learning process studies are required to extrapolate language demands in learning processes, e.g. of developing conceptual understanding. The discourse practice of explaining is tightly connected to lexical demands on the word level, but also to syntactical demands on the sentence level. The article focuses on the epistemic function of language, i.e. the tight connection between mathematical thinking and language.

Furthermore, implicit contributions to the research program of specifying language demands are provided by two further articles:

- Moschkovich's (2017) deconstruction of early research on language specifics and number names make clear that number names alone are not the most crucial language demand in mathematics classrooms, not even in early arithmetic. Instead, wider discourse practices must be taken into consideration. Her article shows that when multiple languages are involved (e.g. for multilingual students), then the languages do not determine what is thought in each language frame because learners activate their multilingual repertoire as a whole, not separately.
- Hagena, Leiss, and Schwippert (2017) show that general reading proficiency may not be the main language demand in the mathematics classroom, since an intervention for fostering general reading abilities does not increase the ability to solve word problems in mathematics.

All these studies call for addressing and investigating language demands not in a generic way, in terms of some form of general 'academic' language, but in a subject-specific or even topic-specific way. The unit of investigation can for example be a specific genre such as geometric construction texts or a specific mathematical topic such as functional relationship or fractions (as claimed by Moschkovich, 2010b). This research agenda will have to continue in further studies.

Contributions to Developing and Investigating Instructional Approaches On Classroom Level (Q2-Q3)

Although the HOW-question is logically subordinated to the WHAT-question (van den Heuvel-Panhuizen, 2005), most studies combine both, specify what to learn and investigate how students can learn them. The design of instructional approaches is often combined with a qualitative or quantitative investigation of the initiated teaching and learning processes or learning outcomes. As Planas (2014) has called for, these studies aim at better understanding mechanisms and effects of teaching interventions on students' topic specific mathematics learning:

- Hagena et al. (2017) show in a randomized controlled trial that fostering students' general reading proficiency does not increase their ability to crack word problems. Whereas controlled trials without significant effects are mostly not published, the editors of the special issue found this negative results specifically important as it

contributes to empirically founding the knowledge that language and mathematics learning cannot be fostered separately.

- Prediger and Zindel (2017) present a design how to foster the conceptual understanding of language learners by design principles of relating registers and systematic variation of texts. In the qualitative investigation of the initiated learning processes, they show how conceptual compaction of mathematical concepts is aligned with language condensation; these empirical insights contribute to elaborating the theory of the epistemic function of language.
- Schüler-Meyer's (2017) research is embedded in a project that builds upon multilingual students' resources (Barwell, 2009), here in their home language Turkish. He investigates the functioning of a bilingual German-Turkish intervention for fostering the students' conceptual understanding of fractions. As this instructional approach has led to very different learning gains for different students, the article presents an in-depth analysis with respect to students' identities as multilingual learners. It shows how the fruitfulness of the students' learning processes is shaped by the interactive co-construction of students' identities. Hence, a design principle is not per se productive or not, but heavily depends on the conditions of enactment in the classroom.
- In a similar sense, Rezat and Rezat's (2017) brief empirical insight into one teachers' ways of teaching the mathematics-specific genre of geometric construction texts provide starting points for problematizing challenges while fostering a mathematics-specific genre.
- Finally, Short (2017) presents an instructional approach in the SIOP-model which has been developed over decades and proven to be effective for robust language learning gains under different conditions of implementation. The model is based on the idea of systematically combining mathematical and language learning goals in each lesson and provides the teachers with concrete planning and realization tools.

These articles show that integrating mathematics and language learning can be beneficial for fostering students' learning, with respect to mathematical as well as language learning goals. However, the implementation in classrooms is shown to be a complex challenge for most teachers, that is why teacher professional development must also be taken into account.

Contributions to Specifying What Teachers Should Know and How They Can Be Promoted to Learn to Support Language Learners (Q4-Q6)

The described studies on the classroom level already provide interesting answers to the questions of what teachers need to learn:

- Language demands comprise much more than general reading proficiency (Hagena et al., 2017) or technical words like number names (Moschkovich, 2017). The language demands in mathematics classrooms have to be specified more holistically by starting from the text level with subject-specific genres (Rezat & Rezat, 2017) or from the discourse level by starting with topic-specific discourse practices (like explaining

meanings of the mathematical topic 'functional relationships', Prediger & Zindel, 2017). This is consequently done in the SIOP model described by Short (2017).

- Language learning for increasing mathematics achievement cannot be separated from mathematics learning (Hagena et al., 2017), instead, language and content integrated approaches are necessary (Short, 2017; Prediger & Zindel, 2017; Schüler-Meyer, 2017; Rezat & Rezat, 2017).
- Instructional approaches should take into considerations multilingual language resources, if existent, and how they are enacted in the classroom (Moschkovich, 2017; Schüler-Meyer, 2017).

In line with these research results on the classroom level, three articles explicitly treat the level of professional development. These articles contribute not only to the What-question, but also to the how-question on the PD level:

- The SIOP model (Short, 2017) which has been developed for the classroom level has been disseminated in various implementation projects. Accordingly, the author can draw on a lot of evidence and experience to address questions of what teachers need to learn for enabling them to work with the language and content integrated instructional approach of SIOP successfully. In her article, she summarizes results on effects and conditions of several implementation studies.
- In a similar manner, Hajer and Norén (2017) based their specification of what teachers need to learn starting from research and design on the classroom level. In their article, they present the content of an online-PD-module for professional development which is disseminated in Sweden. The module shows nicely what it means to consequently integrate language and mathematics.
- The third article by Lange and Meaney (2017) on the PD level investigates a teachers' individual professionalization process when trying to foster primary students' writing in mathematics classrooms. Although being intensively accompanied by facilitators, the process shows the institutional and individual complexities which promote or constrain teachers' development.

Different Research Approaches

In sum, the eight articles of the special issue provide a wide picture of the current trends and issues on research on the classroom and professional development level. All articles share the basic assumption that language should be investigated as learning medium, learning goal and unequally distributed learning prerequisite, and all articles contribute to showing why this must be done subject-specifically.

Above that, the special issue shows the need for diverse research approaches. The broad range of research foci for investigating questions of fostering the mathematics learning of language learners, under a classroom learning perspective (Q1-Q3), and under a professional development perspective (Q4-Q6), goes hand in hand with a broad range of approaches:

- Rezat and Rezat (2017) and Moschkovich (2017) mainly present theoretical analyses which are strengthened by references to empirical (descriptive) findings.
- In contrast, the other articles all start from designing approaches for students or teachers (e.g., Hajer & Norén, 2017) and five of them then empirically investigate their functioning:
    - Quantitative methods are applied by Hagena et al. (2017) and Short (2017) for providing quantitative evidence for the (non-)effectiveness of approaches,
    - The others investigate the initiated teaching learning processes qualitatively (Schüler-Meyer, 2017; Prediger & Zindel, 2017; Lange & Meaney, 2017), showing the complexities
        - of the connection between language and mathematics (Prediger & Zindel, 2017),
        - of student learning in interaction (Schüler-Meyer, 2017)
        - and of teacher learning (Lange & Meaney, 2017).

All the different approaches rest upon a common foundation: All of these five studies could not have been conducted without first designing learning opportunities, and this is an important progress in the research on language and mathematics. By collecting these different questions, research approaches and highly interesting findings in one special issue, the editors hope to initiate a further vivid research discourse on how to foster the mathematics learning of language learners. This would be an important step for enhancing equity.

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Using Reading Strategy Training to Foster Students' Mathematical Modelling Competencies: Results of a Quasi-Experimental Control Trial

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ABSTRACT
Ever since the national standards for teaching and learning mathematics in Germany were published, investigation of ways to support students' acquisition of mathematical competencies has increased. Results of these studies have been of special interest in empirical educational research. In this context, several recent studies have focused on the enhancement of students' reading comprehension skills as a means of supporting students' development of subject-specific competencies. Taking into account previous research, the empirical research project FaSaF investigated to what extent students' mathematical modelling competencies can be fostered using a 15-week training in reading strategy. Treatment effects have been investigated in three conditions: EC A, integrated reading strategy training; EC B, separate reading strategy training; and EC C, no reading strategy training. Data from German secondary school students (N = 380) who were about 13 years old were analyzed. The results indicate that students who have participated in reading strategy training experience an increase in mathematical modelling competencies but that the same increase can also be observed in students who have not participated in reading strategy training. Thus, the issue of fostering the acquisition of modelling competencies using reading strategy training is still open for debate.
Keywords: mathematical modelling, reading comprehension, intervention study, fostering mathematical modelling competencies, reading strategy training

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State of the literature

- Mathematics achievements, amongst others, are influenced by individual reading comprehension.
- Reading comprehension seems to be important for working on mathematical modelling problems. Although students' reading comprehension abilities play a prominent role in working on modelling problems, previous intervention studies have not focused on working on improving reading comprehension to support students in building up mathematical modelling competencies.
- Reading comprehension can be fostered using reading strategy training.

Contribution of this paper to the literature

- This study aimed at fostering selected mathematical modelling competencies with the help of reading strategy training. In this context, the impact of differences in the organization of reading strategy training was investigated.
- Our results indicate that students who have participated in the reading strategy training in our intervention experienced an increase (although low) in the selected mathematical modelling sub-competencies but that the same increase can also be observed in students who have not participated in our intervention.
- Our study points out that more research on mathematics-specific reading strategies focusing on the interplay between reading and finding mathematical relations is required.

INTRODUCTION

The issue of how to design learning-conducive, competency-oriented mathematics instruction is a key challenge in the research discourse and in educational policy. The *Fach-an-Sprache-an-Fach* (FaSaF[1]) study has been addressing this research gap, taking as an example competency in mathematical modelling, which is a central component of the German Education Standards. In particular, it explores means of supporting students in developing this competency using reading strategy training, acknowledging that reading is an important facet of school learning. This article includes (1) a basic description of the interplay between language and mathematics, (2) key ideas in mathematical modelling, and (3) a discussion of fostering reading comprehension using reading strategies in general. Finally, we present (4) an intervention study in which we (5) investigate to what extent the acquisition of selected sub-competencies of mathematical modelling can be facilitated through targeted promotion of reading strategies. On the basis of the results, we (6) reflect on the intervention and the measurement instruments used in the study.

THE INTERPLAY BETWEEN LANGUAGE AND MATHEMATICS

For several years, "it [has been] widely acknowledged within the field of mathematics education that language plays an important (or even essential) role in the learning, teaching, and doing of mathematics" (Morgan, 2013, p. 50). In this context, various studies have demonstrated that mathematics achievements are influenced by individual language proficiency (Abedi & Lord, 2001; Baumert & Schümer, 2001; Heinze, Rudolph-Albert, Reiss, Herwartz-Emden, & Braun, 2009). Furthermore, language proficiency not only influences

multilingual students' mathematics achievement (Heinze et al., 2009) but also influences monolingual students' mathematics achievement, particularly those with low socioeconomic status (Prediger, Renk, Büchter, Gürsoy, & Benholz, 2013).

One aspect of language proficiency is individual reading comprehension: the active (re)construction of a text's meaning, a complex ability made up of various sub-processes (Lenhard, 2013). Empirical replication studies have determined that reading comprehension is an influential predictor for the successful completion of mathematics problems (Fuchs, Fuchs, & Prentice, 2004; Grimm, 2008). Since students who possess insufficient reading comprehension skills show deficits in dealing with mathematical test items (Leutner, Leopold, & Elzen-Rump, 2007), the process of extracting meaning from texts has been regarded as the precondition for understanding mathematical phenomena encountered in everyday life (Kaiser & Schwarz, 2003). However, despite an extensive body of research, an open question still remains: How can knowledge about the interplay between reading comprehension and mathematics achievement be used to develop adequate intervention programs in mathematics education? In order to explore answers to this question, we designed the present study to investigate empirically the possibility of fostering modelling competencies (as part of mathematics achievement) by fostering reading comprehension using reading strategy training.

REALISTIC PROBLEMS IN MATHEMATICS INSTRUCTION

As part of a changing problem-solving culture in mathematics instruction motivated by the German students' disappointing results in solving realistic problems in international school comparison studies, there have been increased efforts in the past years to integrate mathematical modelling problems into daily teaching practice (see, among others, Kultusministerkonferenz, 2003; National Council of Teachers of Mathematics, 2000). In contrast to the algorithmic mathematics problems long dominant in German mathematics instruction, mathematical modelling problems are realistic word problems involving the application of mathematics to situations outside of mathematics (Blum, 2011; Pollak, 2007). The goals in integrating mathematical modelling problems into daily teaching practice are to teach students the significance of mathematics for everyday life and to enable them to apply mathematics in a thoughtful way to present and future real-life problems (Niss, Blum, & Galbraith, 2007). It is thus a means for fostering "mathematical literacy": "an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts" (OECD, 2013, p. 17).

Mathematical Modelling Process

The completion of mathematical modelling problems involves complex translation processes between reality and mathematics that may be illustrated by so-called modelling cycles (for an overview, see Borromeo Ferri, 2006). An example of an idealized modelling cycle describing the modelling process in seven cognitive steps is presented in Figure 1. The cognitive steps involved in solving the modelling problem "Annual Movie Theater Pass" (see Figure 2) are explained in Table 1.

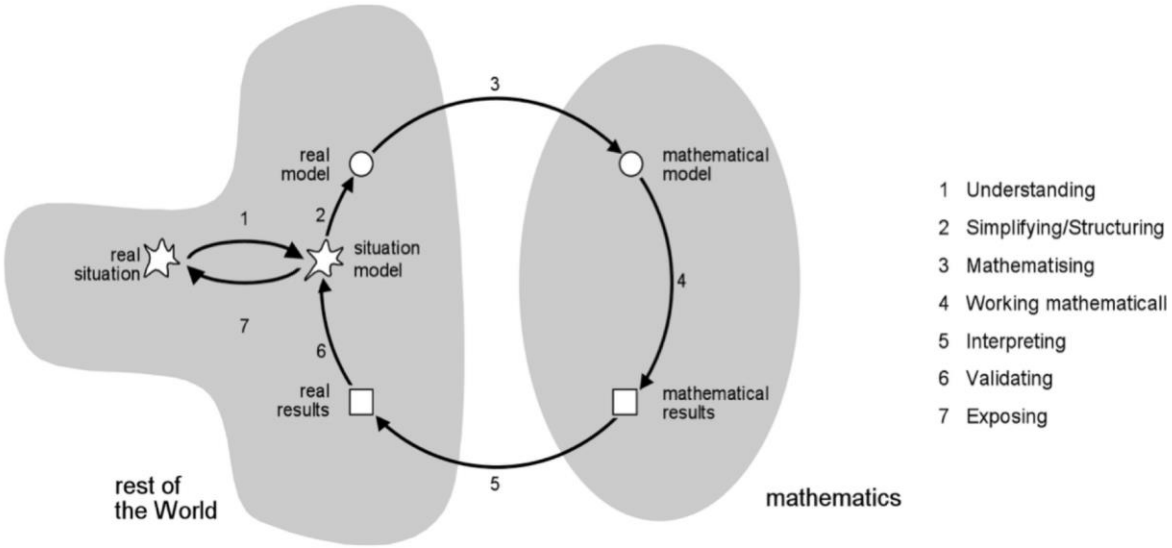

Figure 1. Modelling cycle according to Blum and Leiss (2007)

Annual Movie Theater Pass

Mr. Morgan comes across an interesting offer in the newspaper. The movie theater chain Kinomaxx is selling annual movie theater passes for 399 €. The pass allows one to go to the movies as often as one wants for an entire year. Mr. Morgan, a big movie fan and a regular moviegoer, is considering whether to buy an annual pass.

Decide whether it's worth it to buy an annual movie theater pass. Provide reasons for your decision.

Figure 2. Sample modelling problem "Annual Movie Theater Pass"

The performance of the steps presented in Table 1 describes the process of mathematical modelling in its entirety. In the context of this process, the ability and the willingness to perform a modelling process are understood as mathematical modelling competencies: "In short: modelling competency in our sense denotes the ability to perform the processes that are involved in the construction and investigation of mathematical models" (Niss et al., 2007, p. 12). The individual sub-competencies necessary for performing a modelling process in detail are defined with reference to the cognitive steps of the modelling cycle (see Table 1).[2] Since single cognitive steps of the modelling process can hardly be distinguished empirically (Borromeo Ferri, 2006), and since competencies in mathematizing are largely dependent on

Table 1. The seven modelling steps involved in solving the modelling problem "Annual Movie Theater Pass"

1. Understanding	The problem-solving process begins with reading the text and examining the accompanying picture. The reader must understand the circumstances in order to make the problem accessible (situation model): *"Is it less expensive to buy an annual pass for 399 € or to pay for admission each time one goes to the movie theater?"*
2. Simplifying/ Structuring	In order to formulate the problem in mathematical terms, the problem solver needs to make independent assumptions about the estimated costs of going to the movie theater and/or the estimated amount of times Mr. Morgan will go to the movie theater, information that is not provided in the formulation of the problem: *"Admission to the movie theater costs around 8 €."*
3. Mathematizing	The problem solver needs to use mathematical concepts to treat these assumptions within the framework of a mathematical model: *399 € : 8 € = x*
4. Working mathematically	In the next step, the problem solver needs to perform the necessary mathematical operations in order to arrive at a mathematical result: $x = 49{,}875$
5. Interpreting	The mathematical result must then be translated into reality and rounded off in a meaningful way: *"Mr. Morgan would have to go to the movie theater at least 50 times in a year for the annual pass to be worth the cost."*
6. Validating	Finally, the problem solver needs to reflect on the individual steps of the problem-solving process and the result: *"Does the movie theater even show that many films that Mr. Morgan really wants to see? Would he go to the movies every week even in the summer?"*
7. Exposing	The problem solver then formulates the final result in written or oral form: *"If Mr. Morgan went to the movies once a week, it would be worth it for him to buy the annual movie theater pass, but in that case he would really have to be a big movie fan."*

the steps of understanding the task and simplifying the problem (Biccard & Wessels, 2011), different experimental studies concerning ways of fostering modelling competencies have used an adapted version of the modelling cycle. The adapted cycle has been reduced to three cognitive steps: (1) Understanding/mathematizing the task, which comprises the first three phases of the modelling cycle; (2) working mathematically; and (3) explaining the results, which includes interpreting and validating the result (see Djepaxhija, Vos, & Fuglestad, 2015; Schaap, Vos & Goedhart, 2011; Zöttl, 2010).

In the following we concentrate on the ability to simultaneously conduct the two cognitive steps (which we will refer to as sub-competencies) mentioned above—understanding and simplifying/structuring—because they cannot distinguished empirically (Borromeo Ferri, 2006, see above).

Fostering Mathematical Modelling Competencies

Because performing each individual step in the modelling process can cause problems for students (Galbraith & Stillman, 2006), a variety of studies have shown that modelling problems are difficult for students (Blum, 2011). More precisely, even the comprehension processes at the beginning of the modelling process, namely the translation from the real situation given in a written task to a mathematical model, can pose cognitive obstacles in the modelling process (Borromeo Ferri, 2006; Galbraith & Stillman, 2006; Reusser, 1994). However, it has to be stressed that "the translation of one's understanding of a problem situation into a mathematical model constitutes a key step in the process of mathematical modelling" (Van Dooren, De Bock, & Verschaffel, 2013, p. 385). Based on this idea, it is not enough to merely extract the numbers included in the modelling problem and enter them into whatever mathematical algorithm seems to suggest itself—yet this is a strategy followed by many students (De Corte, Verschaffel & Op't Eynde, 2000). According to these findings, the comprehension processes at the beginning of the modelling process influence mathematical modelling performances (Maaß, 2007; Voyer, 2010). Since students' comprehension processes show themselves in the so-called situation model, which is "a representation of the content of a text, independent of how the text was formulated and integrated with other relevant experiences" (Kintsch & Greeno, 1985, p. 110), students' understanding is related to problems with language (Maaß, 2007). Although Maaß did not describe in detail the meaning of "problems with language," the main finding was that understanding the content of a text in a modelling task is crucial for starting to work on the problem. In other words (as pointed out by Biccard & Wessels, 2011; Leiss, Schukajlow, Blum, Messner, & Pekrun, 2010), students' abilities in reading and reading comprehension play a prominent role for the construction of the situation model. At present, there is not enough empirical knowledge about fostering students' mathematical modelling performances by promoting reading comprehension in mathematics education.

In addition, investigating ways of supporting students to successfully build up mathematical competencies has been promoted in the context of the development of national education standards for mathematics in several countries (Kultusministerkonferenz, 2003; National Council of Teachers of Mathematics, 2000). Empirical studies have also shown that students have an insufficient level of modelling competency by the end of lower secondary school (Blum, 2011; OECD, 2013). Therefore, researchers have conducted intervention studies on students at various grade levels to foster mathematical modelling competencies (for further discussions see, e.g., Blum, 2011). Some of these studies have demonstrated that support of students' strategy use improves students' mathematical modelling competencies (Schukajlow, Krug, & Rakoczy, 2015; Schukajlow, Kolter, & Blum, 2015; Stillman & Galbraith 1998; Stillman, 2011; Zöttl, Ufer, & Reiss, 2010). For this reason, Blum emphasized that an effective way of fostering students' modelling competencies is "to teach learning strategies, cognitive strategies as well as metacognitive strategies such as planning, controlling, or

regulating" (Blum, 2015, p. 88). Unfortunately, only a few interventions have aimed to foster the acquisition of said strategies in mathematics classrooms (Leiss, 2007).

As argued above, reading comprehension plays a prominent role in working on modelling problems. Since an effective way of fostering students' modelling competencies is to support students' strategy use, we thus discuss the use of reading strategies for fostering general reading comprehension.

READING COMPREHENSION IN THE FOCUS OF SOCIAL AND SCIENTIFIC INTEREST

The starting point for a variety of studies in the past few decades that have focused on fostering students' reading comprehension has been the unsatisfactory reading competencies of German middle school students, as tested by the international comparative school achievement study, PISA 2000 (Artelt, Schiefele, Schneider, & Stanat, 2002; Kirsch et al., 2003). Since students' reading comprehension has proven to be a promising target dimension for interventions (Artelt et al., 2002), progress in students' reading comprehension has recently been made (Naumann, Artelt, Schneider, & Stanat, 2010). However, students still need further support to improve their reading comprehension. Since adequate reading comprehension is the complex result of an active examination of a text, it is influenced by a number of different factors that are related to, on the one hand, the text (the type of text, the complexity of the micro- and macrostructure of the text, and the amount of new information it includes) and, on the other hand, the individual (decoding skills, prior knowledge, lexicon, and affective factors such as motivation and self-perception) (De Corte, Verschaffel, & Van de Ven, 2001; following Hiebert & Raphael, 1996; Cromley & Azevedeo, 2007).

Whereas changing factors inherent to a text leads only to a short-term improvement in reading comprehension (namely only with regard to that particular text), promoting factors related to the individual can bring about long-term improvements. However, not all individual factors are equally suited for use as target dimensions of interventions designed to foster general reading comprehension. While it is difficult to support dimensions such as the capacity of working memory or basic cognitive skills, the carefully considered use of strategies has been a promising target dimension for supporting students' reading comprehension (Edmonds et al., 2009; Gersten, Fuchs, Williams, & Baker, 2001; Nordin, Rasihd, Zubir, & Sadjirin, 2013). Furthermore, findings of empirical research show that good and poor readers often differ with regard to their use of appropriate cognitive strategies as well as their metacognitive monitoring of comprehension (Mokhtari & Reichard, 2002; Paris, Lipson, & Wixson, 1983).

The Influence of Reading Strategies on the Development of Reading Comprehension

Reading strategies are related to learning strategies that support students in acquiring knowledge and in influencing and controlling their motivation (Friedrich & Mandl, 2006): "Strategic readers actively construct meaning as they read and interact with the text" (Nordin et al., 2013, p. 470). The term reading strategies is defined as any processes that

readers are conscious of executing in order to facilitate understanding from written texts (Artelt et al., 2002; Nordin et al., 2013).

In the PISA study, knowledge of reading strategies contributes substantially to the explanation of individual reading competence and is additionally even the second strongest predictor for general reading competence when controlling for basic cognitive abilities, verbal self-concept, and general decoding skills (Artelt et al., 2002). These findings reinforce the claim that reading strategies, which are still given only scant attention in school learning, should be integrated into daily teaching practice (Pressley, 1998; Lenhard, 2013). Therefore, "it is important to teach the strategies by naming the strategy and how it should be used" (Kükçükoğlu, 2013, p. 710). Furthermore, teachers should give students opportunities to practice the strategies, either in pairs, small groups, or individually, and offer structured feedback to students (Kükçükoğlu, 2013).

There are three general categories of reading strategies: cognitive strategies, which involve processes of extracting and processing information; metacognitive strategies, which focus on planning, controlling, and monitoring the learning process; and resource-based strategies, which are used to ensure a suitable learning environment (De Corte et al., 2001; Lenhard, 2013). Each of these three categories contains a substantial number of individual strategies (De Corte et al., 2001). Since readers use strategies to understand what they read before, during, and after reading (i.e., pre-reading, while reading, and post-reading) (Nordin et al., 2013), in the following, we present examples of several cognitive and metacognitive reading strategies of these stages that have already been useful (Gersten et al., 2001; Pressley, 1998):

Pre-reading: Research indicates that readers use strategies before they begin to read. In doing so, students are likely to make the texts more accessible during reading. While "pre-reading activities assist readers to activate what they know about a topic and foresee what they will read" (Nordin et al., 2013, p. 470), one major strategy before reading is *activating prior knowledge*. By using the title, table of contents, or pictures, readers are instructed to formulate their own prior knowledge before reading the text to be processed. After reading, the readers must see if their predictions are validated by the text. Research has shown that readers improve their individual understanding comprehension by making predictions (Duke & Pearson, 2002; Kintsch, 1994).

While reading: There are a variety of strategies effective readers use to build their understanding of the text and to become engaged in the reading process during reading. Most of these strategies are monitoring strategies to make sure that readers understand what they are reading. Since the relationship between reading comprehension and vocabulary knowledge is widely acknowledged, one monitoring strategy is *dealing with unclear text passages* by identifying and interpreting comprehension obstacles with the help of context or external aids (Gersten, 2001).

Post-reading: Since constructing meaning from text does not end with the termination of reading, readers have to identify and summarize major information of a text (Nordin et al., 2013). In order to do this, *dividing the text into thematic sections* and *highlighting keywords* can be very helpful. While dividing the text into thematic sections and giving each section a heading, the reader becomes sensitive to the structure of a text. By highlighting key words,

the reader identifies a text's necessary information. Afterwards, readers can make meaningful connections between pieces of information (Kükçükoğlu, 2013). These types of meaningful connections can be done by creating a *concept map*. A concept map is "a type of graphic organizer that is distinguished by the use of labeled nodes denoting concepts and links denoting relationships among concepts" (Nesbit & Adesope, 2006, p. 415). Concept maps were developed as organizational tools to represent knowledge and are useful learning tools (Novak & Cañas, 2007).

(Reading) Comprehension Strategy Training in Mathematics

Even though in recent years the effectiveness of comprehension strategies for working on mathematical word problems has come under scrutiny (Capraro, Capraro, & Rupley, 2012; Kintsch & Greeno, 1985; Verschaffel, Greer, & De Corte, 2000), the literature on successful reading strategies—one kind of comprehension strategy—for working on mathematical word problem is limited. Because students' difficulties with mathematical word problems are often related to poor reading comprehension and because teachers normally tend to take students' reading competences for granted and focus only on teaching subject-specific skills, there have been calls for a more language-sensitive teaching in recent years (Thürmann, Vollmer, & Pieper, 2010). In this context, initial studies have been conducted to identify the interplay between reading and finding mathematics relations. One of these studies was conducted to investigate how to foster students' comprehension strategies (including reading comprehension strategies) for multi-step algebraic word problems. The findings of this study suggest that an interplay of six different strategies supports students' comprehension processes. Some of these six strategies focus on supporting students in finding relevant information and in making meaningful connections between pieces of information (Prediger & Krägerloh, 2015). However, further research is required to investigate the possibility of transferring these findings to other mathematical contexts, especially to mathematical modelling.

The underlying research gap

Since we do not know much about the influence of reading strategy training on students' modelling competencies, we created an intervention study based on the following ideas: (1) Students' comprehension processes play a prominent role in students' modelling performances. (2) The construction of the situation model is related to students' abilities in reading comprehension and difficulties can often be traced back to deficits in students' comprehension strategies. (3) An effective way of fostering students' modelling competencies is to support students' strategy use. In our intervention study, we investigate the effectiveness of reading strategy training on the comprehension processes at the beginning of the modelling process, namely the modelling sub-competencies associated with understanding and simplifying/structuring. Therefore, students received support in making use of the selected reading strategies presented here (activating prior knowledge, dealing with unclear text passages, dividing the word problem into thematic sections, highlighting key words, and creating a concept map). While the first three strategies are applied to ensure students' understanding of the text, the last two strategies focus on supporting students in finding information and in making meaningful connections between information.

Because a mathematical word problem is generally structured differently than a narrative or an expository text (Thürmann et al., 2010), we also analyzed whether it was more effective to foster reading strategies directly while working on modelling problems (integrated strategy training) or separately as an interdisciplinary aid (separate reading strategy training).

INTERVENTION STUDY IN THE FRAMEWORK OF THE RESEARCH PROJECT *FACH-AN-SPRACHE-AN-FACH*

The interdisciplinary research project FaSaF has been investigating the effectiveness of reading strategy training on the mathematical modelling competencies of seventh-grade students (about 13 years old) within the framework of a 15-week intervention study. The primary concern of the study described here was to foster mathematical modelling competencies by focusing on the comprehension-oriented sub-competencies of understanding and simplifying/structuring in the modelling process. In detail, the study pursued two research questions:

- To what extent is it possible to foster seventh-grade students' selected mathematical modelling sub-competencies with the help of reading strategy training?
- Are there differences in the efficacy of two different teaching approaches (integrated vs. separate reading strategy training) on the development of selected mathematical modelling sub-competencies?

Design of the Study

In the academic year 2014-2015, we conducted an intervention study (starting in November 2014 and ending in April 2015) undertaken in the interdisciplinary research project FaSaF. The study compared the effects of three different experimental conditions: Experimental Condition A (EC A), integrated reading strategy training; Experimental Condition B (EC B), separate reading strategy training; and Experimental Condition C (EC C), wait-list control group (see Figure 3).

In the course of ECs A and B, seventh-grade students participated in an optional reading strategy training for solving mathematical modelling problems. At seven different schools, we established an EC A and an EC B with a maximum of 16 students parallelized in accordance with basic mathematical skills and general language skills, including reading comprehension. The students received 90 minutes of reading strategy training from trained teachers one afternoon each week in addition to their regular lessons. Hence, the intervention covered a period of 4.5 months, during which the students received a maximum of 15 afternoons of weekly extra lessons, excluding vacations. On average, the students participated in extra lessons on 10.53 afternoons (standard deviation 3.84). In addition to ECs A and B, we established an EC C that did not receive any reading strategy training (wait-list control group). Therefore, in two additional schools our research instruments were administered to all seventh-grade students.

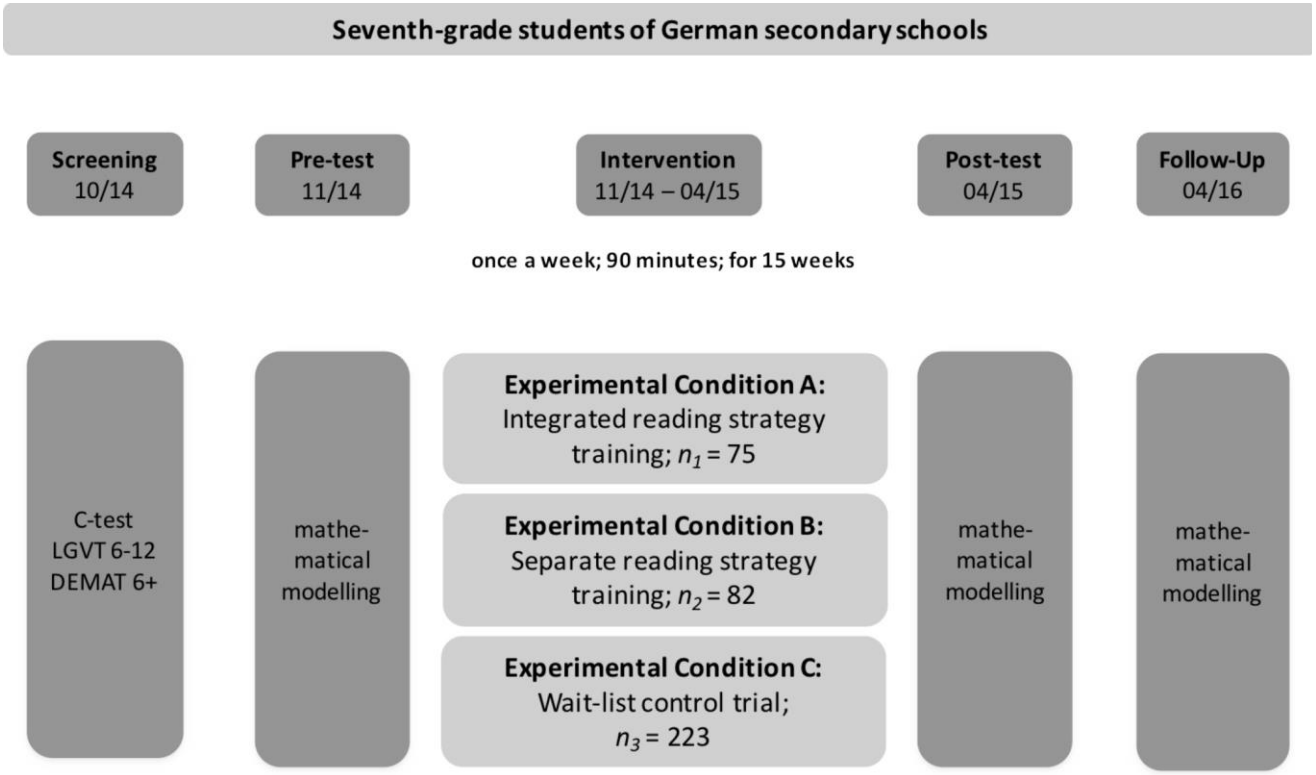

Figure 3. Design of the intervention study

Research Instruments

To investigate the results with regard to the research questions discussed above, we employed a variety of research instruments completed by the students. Before the beginning of the intervention study, the students underwent a 90-minute screening to determine their initial level of basic mathematical abilities, their general language skills, and their reading comprehension. As the instruments (C-Test, LGVT 6–12, and DEMAT 6+) were standardized research instruments, we evaluated them on the basis of the available evaluation forms and standardization tables. Since we took these results as a basis for establishing parallelized ECs A and B, students in EC C did not participate in the screening.

C-TEST

We used a C-test to measure the general language skills of the participating students. The C-test is a written test that is regarded as particularly valid for measuring general language proficiency (see Grotjahn, 2013).

LGVT 6–12

We used the LGVT 6–12 to measure the reading comprehension and the reading speed of the students participating in the study. It is a proven standardized reading speed and comprehension test developed for sixth- to twelfth-grade students that involves reading a

continuous narrative text. A validity test has confirmed that the LGVT 6–12 can serve as a valid measure for reading comprehension (see Schneider, Schlagmüller, & Ennemoser, 2007).

DEMAT 6+

We used part of DEMAT 6+ to measure the students' basic mathematical skills at the beginning of the intervention study (see Götz, Lingel, & Schneider, 2013).

To determine the students' subject-specific initial learning level with regard to mathematical modelling competencies, we assigned a pre-test to all three experimental conditions assessing mathematical modelling competencies before the start of the intervention (November 2014). Finally, upon completion of the intervention in April 2015, we again tested the students' mathematical modelling competencies in a post-test in order to measure possible increases in performance.

Mathematical Modelling

We used a research instrument designed specifically for the intervention study to measure the understanding and simplifying/structuring sub-competencies. Based on other empirical studies (see the Mathematical modelling process section), we did not try to distinguish these two cognitive steps of mathematical modelling empirically. Thus a total of 30 items were available for measuring these two sub-competencies. Each of these items was characterized by a moderately long informational text (10–16 lines and a picture) that included the relevant information for completing the item as well as information that was unnecessary for completion of the item (for a sample item, see Figure 4). Successfully completing the

Apple Juice

Apple juice is the classic fruit juice: Germans drink around 12 liters of apple juice per year. However, drinking apple juice is no innocent pleasure. After all, a glass of apple juice has more calories than a glass of cola: Apple juice contains almost 190 calories per 250 ml glass, while cola has 140 calories per 250 ml glass. But what happens to an apple on its way from the tree to the bottle? In 2014, 600,000 tons of apples were used to make 400 million liters of apple juice in Germany. After harvesting, rotten spots were first removed from these apples, and then they were carefully cleaned and chopped up into small pieces. The apple pieces were then put through the press. Finally, the pressed juice was heated to 80 °C to ensure that it will keep for at least two years. In the end, the apple juice was bottled. Thus, each 1 liter bottle of apple juice now available for sale contains approximately 1.5 kg of cleaned, chopped, pressed, and pasteurized apples.

Henry wants to calculate how many calories a German takes in on average each year through the consumption of apple juice.

Underline all of the numbers Henry really needs to calculate how many calories a German takes in on average each year through the consumption of apple juice.

Figure 4. Sample item "Apple Juice"

individual items involves, among other things, deciding which information provided in the text is relevant for performing the mathematical operations necessary for answering a given question. By selecting the necessary information to complete these items, the students provided the data for measuring the mathematical modelling understanding and simplifying/structuring sub-competencies (see the Mathematical modelling process section).

Twelve of the 30 items were used specifically at each of the two measurement time points. Another six items served as anchoring items (see Figure 5). This allowed us to use the probabilistic Rasch test model (OPL) to illustrate the students' competencies for the two measurement time points on a scale. Fifteen out of a total of 30 items were provided in multiple-choice format and coded dichotomously, while the other 15 items were provided in partial credit format (with 0, 1, and 2 points). For scaling purposes, the students' data was arranged to consider as pseudo-observations those students who had participated repeatedly in the testing due to the longitudinal design. Hence, a total of 883 observations were available for scaling, including 760 students who had participated in the study at both measurement time points. With the exception of one item (whose discrimination was somewhat too high [MNSQ = 0.79]), the scaling (performed with the program ConQuest) resulted in good item fits ($0.8 \leq MNSQ \leq 1.2$). Nevertheless, we left this item in the test instrument as it only exhibited a very minor deviation and all of the other characteristics of this item were rated very highly. The reliability of the scale for measuring the selected sub-competencies of mathematical modelling may also be described as good (EAP-Rel. = 0.810). In the end, we wrote out the person parameters as a WLE and then standardized them across all cases to a mean of 100 and a standard deviation of 20. For the following analyses, we adopted the students' WLEs as their new performance values.

Intervention

In the framework of ECs A and B (see Table 2), the students were given training in the five selected cognitive reading strategies, which have already been shown to be important aspects of fostering reading comprehension (see the Influence of reading strategies on the development of reading comprehension section). Furthermore, the participating students in ECs A and B completed five complex modelling problems developed especially for the project. The level of mathematical proficiency required to solve the modelling problems was controlled by only including mathematical concepts (various size ranges, functional relations, and direct and inverse proportionality) the students had dealt with previously in regular lessons in order to avoid making the modelling problems even more challenging for the students than they already were.

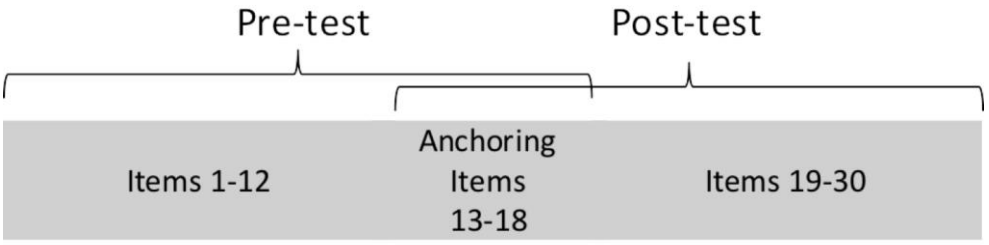

Figure 5. Distribution of test items across the measurement time points

Table 2. Differences in the treatments

EC A: Integrated reading strategy training	
	In interpreting the content of selected modelling problems, the students in EC A received explicit support in applying reading strategies designed to help them perform the modelling process independently and in a thoughtful way.
Reading strategy training	Within the integrated reading strategy training the students practiced the selected reading strategies while working on different modelling problems.
Completion of mathematical modelling problems	Although the intervention focused on supporting the students in performing the comprehensibility-enhancing processes of the modelling process, the students also completed the mathematical operations involved in solving each of the selected modelling problems. Leaving the mathematical operations out could have possibly had a negative effect on the students' motivation. After completing the modelling problems, the students discussed and reflected on their results in a verification phase.
Advantage	The integrated reading strategy training made it possible to take into account the specific structure of mathematical modelling problems.
Disadvantage	At the same time, however, the students were also regularly interrupted in the modelling process, as the focus of the support changed constantly between the reading strategies to be learned and the modelling problems to be completed.
EC B: Separate reading strategy training	
	We separated language support from subject-specific support by providing the students separate reading strategy training in the first 10 weeks of the intervention; the students then worked on selected mathematical modelling problems in the subsequent five weeks.
Reading strategy training	In the separate reading strategy training, the students were familiarized with selected reading strategies as interdisciplinary aids. While reading various factual texts the students practiced the selected reading strategies.
Completion of the mathematical modelling problems	To enable a comparison of how the students in the two experimental conditions dealt with mathematical modelling problems, we also gave the students in EC A the selected modelling problems that had formed the basis for the integrated reading strategy training in EC B. The students presented and discussed their results in a verification phase.
Advantage	The decision in favor of separate strategy training made it possible to focus first exclusively on the reading strategies and then on the completion of mathematical modelling problems. Thus, the students did not have to deal with a glut of new information.
EC C: Wait-list control group	

Table 2. contiined

Disadvantage	At the same time, the decision to separate language and subject learning came with the disadvantage that the mathematical modelling problems were completed without the explicit use of the previously trained reading strategies. Accordingly, the students were not explicitly instructed to use the reading strategies they had acquired on the mathematical modelling problems.
EC C: Wait-list control group	
	EC C did not include reading strategy training. The students in EC C thus received no additional language or modelling-specific support between the pre-test and the post-test. Other than regular lessons, there was no additional intervention.

While the mathematical modelling problems did not differ between ECs A and B, there were design-related differences in the material used to help the students acquire the reading strategies. The students in EC A acquired the various reading strategies with the help of mathematical modelling problems, while in EC B the strategies were introduced and applied with the help of selected factual texts.

Sample

The sample consisted of seventh-grade students (about 13 years old; N = 380) from nine secondary schools who answered both the tests on mathematical modelling competencies. Two groups of students (EC A = 75; EC B = 82) participated in the intervention, while another group of students (EC C = 223) only filled in the research instruments before and after the intervention.

RESULTS

We measured the selected mathematical modelling sub-competencies (understanding and simplifying/structuring) in pre- and post-tests. This performance data is presented in the following sections with reference to our two research questions. In the first section, we present descriptive data and correlations of all manifest/latent variables being used for answering the research questions. In sections two and three we present more in-depth analyses in the form of group comparisons (between the experimental conditions) and analyses of variance.

Descriptive Data and Correlations

Three hundred and eighty students answered both the tests at measurement points one and two. Descriptive data for these students is given in Table 3 (both separated for the experimental conditions as well as summed up for all participating students). Having a closer look at the correlations of this data, it becomes obvious (see also Table 4) that all variables correlate significantly; correlations range from r = .30 to r = .73.

Although the students in EC C did not take part in the screening, we assumed that their performances in these tests (DEMAT 6+, LGVT/C-test) would be comparable to those of the students in ECs A and B because of our random sample (see Bortz & Döring, 2009).

Table 3. Mean scores of general mathematical knowledge, language skills including reading comprehension, and mathematical modelling competencies

	Screening	Pre-test	Post-test
DEMAT 6+			
Whole sample (n = 133)	8.75 (3.45)	---	---
EC A (n = 64)	8.47 (3.32)	---	---
EC B (n = 69)	9.04 (3.57)	---	---
EC C	---	---	---
LGVT/C-test			
Whole sample (n = 133)	6.03 (2.41)	---	---
EC A (n = 64)	5.98 (2.37)	---	---
EC B (n = 69)	5.97 (2.52)	---	---
EC C	---	---	---
Mathematical modelling			
Whole sample (n = 380)	---	98.20 (18.88)	103.24 (19.96)
EC A (n = 75)	---	93.64 (17.65)	97.34 (19.31)
EC B (n = 82)	---	96.07 (19.81)	102.06 (21.98)
EC C (n = 223)	---	100.51 (18.63)	105.65 (19.02)

Table 4. Correlations of all research instruments (DEMAT 6+, LGVT/C-test, and pre- and post-test mathematical modelling) used in the study

	DEMAT 6+	General language skills and reading comprehension (LGVT/C-test)	Pre-test mathematical modelling	Post-test mathematical modelling
DEMAT 6+	1			
General language skills and reading comprehension (LGVT/C-test)	.298**	1		
Pre-test mathematical modelling	.559**	.422**	1	
Post-test mathematical modelling	.657**	.488**	.726**	1

*: Significant with $p < .05$ (two-tailed).
**: Significant with $p < .01$ (two-tailed)

Research Question 1: Is it possible to foster selected mathematical modelling sub-competencies (understanding and simplifying/structuring) of seventh-grade students with the help of reading strategy training?

The results of selected mathematical modelling sub-competencies from the pre- and post-test serve as a basis for answering this research question. As pointed out in Table 5, the students who participated in reading strategy training (EC A + EC B) in the context of this intervention study (n = 157) scored a mean of 94.91 points on the pre-test for measuring the selected mathematical modelling sub-competencies and a mean of 99.81 points on the post-test. A t-test for paired samples confirmed that this increase was significant ($p < 0.001$).

However, the difference between the two measurement time points was minor, as shown by the effect size ($d = 0.25$; see Table 5).

Research Question 2: Are there differences in the influence of two different teaching approaches (integrated vs. separate) on the development of selected mathematical modelling sub-competencies (especially understanding and simplifying/structuring)?

Taking this (the combined results of EC A and EC B) as a basis, we discuss in the following what specific effect the two different teaching approaches (EC A and EC B) had on the students' performance in the area of the selected mathematical modelling sub-competencies.

As shown in Table 5, the students in EC A (n = 75) scored a mean of 93.64 points on the pre-test for measuring the selected mathematical modelling sub-competencies and a mean of 97.34 points on the post-test. The students from EC B (n = 82) scored a mean of 96.07 points on the pre-test for measuring selected mathematical modelling sub-competencies and a mean of 102.06 points on the post-test (see Table 5). Furthermore, the minimum mean score increased from 25.58 to 50.52. The descriptive results indicate that both groups achieved increases in the selected mathematical modelling sub-competencies. Although these increases were significant in both experimental conditions (EC A, p = 0.018; EG B, p = 0.001), they must be rated as small on the basis of the effect sizes calculated for the two groups (EC A, d = 0.20; EC B, d = 0.29). A direct comparison does not show any significant differences between these two experimental conditions before (p = 0.420) or after the intervention (p = 0.156). However, at a descriptive level, there was a slightly larger effect for the students from EC B (d = 0.29; see Table 5). However, neither of the two reading strategy trainings proved to be more suitable for fostering the selected mathematical modelling sub-competencies. Both the students in EC A (integrated reading strategy training) and the students in EC B (separate reading strategy training) experienced a slight increase in their levels of the selected mathematical modelling sub-competencies.

Table 5. Differences in the mean scores between the measurement points (pre-test and post-test mathematical modelling)

	n	min.	max.	mean	sd		
Reading strategy training (EC A + EC B)							
Pre-test mathematical modelling	157	25.58	140.55	94.91	18.79	t(156) = -4.29	d = 0.25
Post-test mathematical modelling		50.52	161.56	99.81	20.81	p < 0.001	
Integrated reading strategy training (EC A)							
Pre-test mathematical modelling	75	48.40	140.55	93.64	17.65	t(74) = -2.42	d = 0.20
Post-test mathematical modelling		50.52	148.34	97.34	19.31	p = 0.018	
Separate reading strategy training (EC B)							
Pre-test mathematical modelling	82	25.58	140.55	96.07	19.81	t(81) = -3.56	d = 0.29
Post-test mathematical modelling		50.52	161.56	102.06	21.98	p < 0.001	
Wait-list control group (EC C)							
Pre-test mathematical modelling	223	25.58	147.65	100.51	18.63	t(222) = -5.29	d = 0.27
Post-test mathematical modelling		39.58	161.56	105.65	19.02	p < 0.001	

In order to ensure that the observable increases can be attributed to participation in the intervention, we examine the performance data of the students in EC C (no reading strategy training) in the following. The students in EC C (n = 223), who did not receive reading strategy training, scored a mean of 100.51 points on the pre-test for measuring the selected mathematical modelling sub-competencies and a mean of 105.65 points on the post-test (see Table 5). In a t-test for paired samples (see Table 5), the increases observed between the two measurement points in the students in EC C also turned out to be significant ($p < 0.001$). The difference between the two measurement time points is nominally lower here ($d = 0.27$) than in the students who participated in a separate reading strategy training (EC B). A single-factor analysis of variance with repeated measures did not reveal any significant differences in the increase between the three experimental conditions. This indicates that there are no differential courses of development in the various experimental conditions across all three conditions. Furthermore, a test of the levels of the selected mathematical modelling sub-competencies, conducted on the basis of a linear regression analysis controlling for the initial level of the selected mathematical modelling sub-competencies and the support conditions, did not reveal any differential effects either.

Consequently, the increases observed in the students in EC C do not differ from the increases confirmed in the students in EC A or EC B. With regard to the research questions under discussion in this study, we may thus conclude that while the students who participated in one of the two reading strategy trainings did experience minor increases in the selected mathematical modelling sub-competencies, the same increases can also be observed in students who did not participate in reading strategy training.

DISCUSSION

The students who participated in reading strategy training during the intervention (ECs A and B) achieved significant increases (with low effects; $d = 0.25$) in the selected mathematical modelling sub-competencies understanding and simplifying/structuring over time (see first research question). These increases may be observed for those students in the integrated reading strategy training (EC A, $d = 0.20$) as well as for those students in the separate reading strategy training (EC B, $d = 0.29$) (see second research question). Similar low effects can also be observed in other studies: Biccard and Wessel (2011) reported on an intervention study involving 12 seventh-grade students solving a variety of modelling problems over a period of 12 weeks. Their descriptive analyses showed that modelling competencies developed slowly and gradually. However, as the students in this study in the wait-list control trial (EC C, $d = 0.27$) also achieved significant increases, and as the courses of development did not differ across the three experimental conditions, the success of the intervention must be called into question. The low increases therefore cannot be attributed to a general slow development of modelling competencies, but must be explained differently.

In the following, we discuss which issues could be responsible for the lack of intervention-related increases in the context of our analyses. We therefore distinguish between issues associated with the choice of our specific method and issues related to the content of our intervention.

Discussion of the Method

Validity of the test instrument

In interpreting the results, it is necessary to consider the possibility that the test instrument we used in the intervention was not sensitive enough for the content. Although the instrument was shown to be suitable from the perspective of test theory (see the Research instruments section), this says nothing about its intervention-specific validity. The first question that needs to be addressed in this context is whether the linguistic complexity of the selected items requires the application of reading strategies to the extent provided by the test instruments, as the use of reading strategies has only proven to be effective for problems with a subjectively average level of difficulty (Hasselhorn, 2010). Secondly, it will be necessary to investigate whether the pre- and post-tests were set up to enable the students to apply elaborate reading strategies: The task of completing an extensive achievement test in a limited amount of time might have hindered the students from applying the reading strategies, which are already demanding per se (Lenhard, 2013): "Students who learned in ISL [informed strategies for learning] to think about the title, to skim before and after reading, to monitor comprehension, not to skip unknown words, and to reread text would be unable to use these strategies in the time-constrained testing procedure" (Paris & Oka, 1986, p. 52). Perhaps qualitative settings are needed to investigate whether students' comprehension processes at the beginning of the modelling process can be fostered by supporting students in using reading strategies.

Duration of the intervention

It also must be taken into consideration that the intervention lasted for only 15 lessons and that these 15 lessons were not only given separately from the regular school lessons but were also spread over a period of about 4.5 months: Students participating in the study took part in only 1 lesson per week. Therefore, many other variables may have influenced the effect of the intervention: "Studies of shorter duration were found to be more effective than long interventions" (Jacobse & Harskamp, 2011, p. 6; for some discussion about the influence of the duration of an intervention on the interventions' effect size, see, e.g., Hattie, Biggs, & Purdie, 1996).

Treatment control

It is also necessary to consider that the implementation of the reading strategy training involved various teachers due to the substantial number of groups receiving training. The training was taught by 14 different teachers, who received weekly instructions and were requested to design the lessons in accordance with a detailed handbook; however, we were not able to monitor each individual training session. As a means of providing treatment control, future interventions should involve another researcher attending at least a selection of the actual training sessions to take notes or film them.

As we did not measure the students' reading comprehension again in the post-test due to a lack of time, it is also unclear to what extent the intervention actually succeeded in fostering the students' reading comprehension skills and their knowledge of reading strategies.

Discussion of the Content of the Intervention

The significance of reading comprehension for performing the modelling process

Whereas the previous points addressed the intervention as such and the test instruments used in the intervention, we now turn to the significance of reading comprehension for the mathematical modelling process. The results of previous studies have demonstrated that reading comprehension skills are essential for the modelling process. Ultimately, however, it must be taken into consideration (also on the basis of the first point of discussion) that reading comprehension skills are a necessary yet by no means sufficient condition for the ability to adequately understand and simplify or structure mathematical modelling problems. Accordingly, it is conceivable that even these initial steps in the modelling process (understanding and simplifying/structuring) require an understanding of the basic principles of mathematics. Other studies (see Ludwig & Reit, 2013) have shown that "students simplified problems based on the mathematics they wanted to use on the problem" (Biccard & Wessels, 2011, p. 380). This impression is also borne out by the highly significant correlations between the individual test instruments used in the study (Table 4). It turns out that the mathematical modelling competency test instruments (especially the understanding and simplifying/structuring competencies) show highly significant correlations not only with the test instruments for measuring general language proficiency and reading comprehension skills (C-TEST and LGVT) but also with the test instrument for measuring basic mathematical skills (DEMAT 6^+).

The key potential obstacles to understanding and simplifying/structuring a modelling problem might lie not only in students' poor reading comprehension but also in conceptual obstacles – semantic problem structures which are connected to students' access to different basic models (Prediger & Krägeloh, 2015). These so-called conceptual obstacles are often linked to comprehension obstacles. The comprehension of the underlying mathematics relations is necessary for understanding the given task (Prediger, Wilhelm, Büchter, Gürsoy, & Benholz, 2015).

Selection of reading strategies

We have already discussed students' reading comprehension skills being a necessary yet by no means sufficient condition for the ability to adequately understand and simplify or structure mathematical modelling problems. However, we have not discussed whether our selected reading strategies were suitable for working on mathematical modelling problems successfully.

The study presented here focuses on a general reading strategy training. Although the students of EC A were practicing the selected reading strategies while working on different modelling problems, the selected reading strategies were formulated in a very general manner. According to Prediger and Krägeloh (2015), instructional approaches for overcoming mathematics word-problem obstacles have to focus on mathematics-specific reading strategies. Students are in need of strategies that help them to focus on the interplay between reading and finding mathematical relations. For this purpose, in the intervention students received support in creating a concept map. However, students could not use this

strategy for working on the 18 post-test items due to a lack of time. Perhaps rather than supporting general comprehension strategies, more support on other mathematics-specific comprehension strategies focusing on relations connecting information would have been more useful.

Discussion of the Reasons for the Increases in Competencies across All Three Experimental Conditions

After discussing various reasons for the lack of intervention-related increases, we turn in the following to possible reasons for the significant and comparable increases in competencies across all three experimental conditions. How is it possible that students experience an (although low) increase in the selected mathematical modelling sub-competencies if the intervention was not successful?

The completion of test instruments as an opportunity for learning

According to Lipowsky (2015), the completion of test instruments can serve as an educational aid for fostering learning processes. In line with this description, empirical studies have confirmed that the completion of test instruments initiates learning-conducive effects (Roediger & Karpicke, 2006). It is thus possible that all the students profited from memory effects in completing the post-test due to having previously completed the pre-test.

Development of competencies

In addition, the students' increases in competencies can also be explained by their participation in regular lessons (Hofe, Pekrun, Kleine, & Götz, 2002). After all, the students attended their regular lessons for five months between the pre-test and the post-test. Unfortunately, we do not know anything about the content students were being taught in their regular lessons between the pre-test and the post-test. Thus, we cannot exclude the possibility that the students were practicing mathematical modelling in their regular mathematics lessons.

Our analyses did not succeed in confirming that reading strategy training has an influence on selected mathematical modelling sub-competencies (especially understanding and simplifying/structuring), that is, on the ability to understand and appropriately simplify or structure modelling problems. The aim of the remaining analyses will be to study whether there are any group-specific differences with regard to open modelling items the students had to work on in addition to the modelling tests presented here. Moreover, the extensive corpus process data analyzed during the intervention will also need to be analyzed at a qualitative level. These analyses will perhaps allow us to evaluate the success of the intervention in more specific terms. If the number of students per school allows for it, we also plan to conduct further quantitative analyses for specific types of schools. Finally, much research on the interplay of linguistic complexity and successful modelling processes is needed to be able to interpret the results being pointed out here: To what extent are difficulties in understanding and simplifying/structuring a modelling problem caused by difficulties in understanding singular words, difficulties in understanding singular sentences, or difficulties depending on the connections between the underlying

mathematical ideas and comprehension obstacles (e.g., the working group *Fach-und-Sprache* has been working on these basic problems in teaching and learning mathematics with regard to linguistic difficulties; see Leiss, Domenech, Ehmke, & Schwippert, submitted)?

What we can learn from our findings: As empirical research has demonstrated relationships between mathematics achievements and individual language proficiency, we also see in our results strong correlations ($r \geq .422$) between the mathematical modelling sub-competencies of understanding and simplifying/structuring and students' general language skills, including their reading comprehension. However, our findings did not succeed in confirming that our general reading strategy training had an influence on the selected mathematical modelling sub-competencies. This means that even if there is a strong connection between students' mathematical modelling sub-competencies of understanding and simplifying/structuring and students' general language skills, including their reading comprehension, it is not enough to merely foster students reading comprehension using general reading strategy training. Because obstacles to understanding and simplifying/structuring a modelling problem might lie not only in students' poor reading comprehension but also in conceptual obstacles, students are in need of mathematics-specific reading strategies. These strategies should help students to focus on the interplay between reading and finding mathematical relations. Further studies are needed to investigate this relationship.

SUMMARY

This article begins by examining possibilities for fostering mathematical modelling competencies described in the context of current empirical and educational policy discussions on designing competency-oriented mathematics instruction (see the Realistic problems in mathematics instruction section). In this context, we called attention to a research gap: Is it possible to foster the development of mathematical modelling competencies by providing students with reading strategies? As differences in reading comprehension have been explained by knowledge of reading strategies, among other factors (see the Reading comprehension in the focus of social and scientific interest section), we conducted a study to investigate whether it is possible to foster the mathematical modelling sub-competencies of understanding and simplifying/structuring in seventh-grade students by means of reading strategy training. Taking into account requests for language-sensitive teaching, we therefore developed two different teaching approaches for fostering the acquisition of reading strategies (integrated reading strategy training and separate reading strategy training) and tested them in the context of a 15-week intervention (see the Intervention study in the framework of the research project *Fach-an-Sprache-an-Fach* section). Our analysis of the intervention shows that while students who had participated in reading strategy trainings (ECs A and B) experienced a (low) increase in the selected mathematical modelling sub-competencies, the same increase could also be observed in students who had not participated in reading strategy training (EC C) (see the Results section). Based on these sobering results, we reflected on the intervention and the measurement instruments used in the study (see the Discussion section). We assume that solid reading comprehension skills are a necessary but probably not a sufficient condition for performing the first steps of the

modelling process (understanding and simplifying/structuring). Hence, the key potential obstacles to understanding and simplifying/structuring a modelling problem might lie not only in a lack of reading comprehension skills but also in the connection between the underlying mathematical ideas and comprehension obstacles. In order to support students' mathematical modelling competencies, more mathematics-specific reading comprehension strategies focusing on the interplay between reading and finding mathematical relations are needed. On the basis of the available process data as well as the open modelling items completed by the students, we aim at conducting further, more specific analyses.

NOTES

1 Fach-an-Sprache-an-Fach ("Subject to Language to Subject") is funded by the Mercator Institut für Sprachförderung und Deutsch als Zweitsprache [Mercator Institute for Language Acquisition and German as a Second Language].

2 Furthermore, researchers suggest that other competencies should be considered, e.g., metacognitive competencies (Stillman, 2011). The result is a complex combination of sub-competencies that serves as the necessary basis for mathematical modelling competencies in general (Niss et al., 2007).

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Teachers' Knowledge about Language in Mathematics Professional Development Courses: From an Intended Curriculum to a Curriculum in Action

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ABSTRACT
Explicit language objectives are included in the Swedish national curriculum for mathematics. The curriculum states that students should be given opportunities to develop the ability to formulate problems, use and analyse mathematical concepts and relationships between concepts, show and follow mathematical reasoning, and use mathematical expressions in discussions. Teachers' competence forms a crucial link to bring an intended curriculum to a curriculum in action. This article investigates a professional development program, 'Language in Mathematics', within a national program for mathematics teachers in Sweden that aims at implementing the national curriculum into practice. Two specific aspects are examined: the selection of theoretical notions on language and mathematics and the choice of activities to relate selected theory to practice. From this examination, research on teacher learning in connection to professional development is proposed, which can contribute to a better understanding of teachers' interpretation of integrated approaches to language and mathematics across national contexts.

Keywords: content and language integrated learning, curriculum implementation, knowledge about language, mathematics teachers, pedagogical content knowledge, professional development

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State of the literature

- The importance of engaging pupils in oral practice for meaning-making is underlined in many studies on learning mathematics. A growing linguistic heterogeneity in classrooms has brought awareness of the need for literacy development as part of subject learning.
- To bring ambitions from an intended curriculum to a curriculum in action, subject teachers need specific knowledge about the language of the subject and practical skills.
- The design of professional development programs for content and language-integrated learning requires a focus on a specific language register, such as for mathematics. Previous research has identified required teacher skills, such as scaffolding students' language use.

Contribution of this paper to the literature

- Professional development programs can function as starting points for examining teachers' role in the development of students' mathematics-language register in mathematics classrooms.
- The planning of teachers' professional development needs to be examined in terms of existing knowledge about teachers' competencies, particularly with a curriculum that aims to integrate students' language and mathematics learning.
- The design of professional development programs with a focus on the language in mathematics can be described as: a) the selection of relevant content, and b) teacher learning activities.
- Systematic description is meant to contribute to enabling comparisons of such programs despite their different national contexts.

INTRODUCTION

The importance of academic language learning within content areas has been underlined in many studies and policy documents focusing on the education of migrant and second-language learners (OECD, 2010). Successful realization of ambitions in this direction, formulated at the national curriculum level, is a complex process that requires a multidisciplinary approach. Educational research can contribute to an understanding of teachers' role in curriculum implementation processes, while applied linguists and researchers in the field of mathematics education can contribute by selecting core content for professional development (PD) to build understanding of the processes of language and mathematics learning (van Eerde & Hajer, 2008).

Different intervention programs also address teachers' Professional Development (PD) for content and language-integrated approaches in multilingual classes (Vogt, Echevarria, & Short, 2010; Short & Echevarria, 2016). Despite the availability of subject-independent PD programs, mainstream content teachers often fail to identify with the role of providing language and literacy support to second-language learners in their classrooms (Davison, 2016; Hajer, 2006; Little, 2007; Norén, 2015).

Several factors have been proposed to explain these difficulties. First, the program may lack a subject-specific focus. Second, the relation between theoretical understanding and practice may be unbalanced, focusing too much on either of them (Hattie &

Timperley, 2007), or perhaps not making the connections between them sufficiently explicit. Third, crucial aspects of content and language-integrated teaching can be missed in such programs, as suggested in a Dutch study (Hajer, 2006), which identified the provision of feedback as important for both the language and content aspects of students' utterances. Finally, the duration of PD programs and their components—explicit instruction, experimenting with new instructional tools, the provision of tutoring, etc.—would affect program outcomes (Short, 2013).

Teachers' role in realizing a curriculum change is often taken for granted and remains a black box in large-scale efficacy studies. In his classical curriculum study, Goodlad (1979) distinguishes between different curriculum levels: the intended, the implemented, and the attained curriculum (Van den Akker, 2003, 2010, see Table 1).

In order to gain a better understanding of PD in the process of implementing language and mathematics integrated teaching, it is necessary to look closer at Goodlad's dimensions at the stage in which teachers explore curriculum objectives in relation to their role in bringing the curriculum into the classroom. Even though this model may suggest a top-down perspective, teachers can be seen as autonomous professionals working within a given curriculum frame.

The aim of this project was to investigate: how PD programs can be designed to enable teachers to develop competencies for integrating language and mathematics learning. In addressing this question, we investigated a PD program that specifically aims to link an intended curriculum to an implemented curriculum, through enhancing mathematics teachers' knowledge about the role of language in mathematics teaching and learning, as well as their skills to change mathematics classroom practices. A final aim for the PD, not investigated in this article, was the attained curriculum itself—the learning perceived by the learners and the outcomes of the changed practices.

Table 1. Typology of curriculum representations (Van den Akker, 2003, following Goodlad, 1979)

Intended	Ideal	Vision (rationale or basic philosophy underlying a curriculum)
	Formal, written	Intentions as specified in curriculum documents and/or materials
Implemented	Perceived	Curriculum as interpreted by its users (especially teachers)
	Operational	Actual process of teaching and learning (also, curriculum-in-action)
Attained	Experiential	Learning experiences as perceived by learners
	Learned	Resulting learning outcomes of learners

This article does not have the character of a typical research report. Empirically and theoretically we examined a specific PD program in Sweden that was developed to support teachers in realizing the official, intended curriculum that envisions a mathematic and language integrated curriculum (Skolverket [National Agency for Education], 2011). In its design, the program aims to bridge the space between teachers' theoretical understanding—in Goodlad's terms the 'perceived' curriculum, as interpreted by the teachers—and teachers' actual classroom practice—the 'operational' curriculum, in effect through teaching and learning. The issues of selecting relevant course content (Section 2) and handling the theory-practice dimension of the PD program (Section 3) will be discussed. The main part of this article (Section 4), examines the Swedish PD program. To set the scene, the background is described: a national, web-based program for primary and lower secondary teachers in Sweden, structured around collaborative team learning, of which the language and mathematics integrated module is a part. We will examine several parts of the module in more detail to find out how teacher learning in the PD program could be analysed in depth. In the concluding Section 5, we discuss how Goodlad's (1979) distinction between curriculum levels can help explain teachers' role in bringing curriculum changes to the classroom. Here we relate the selection of relevant knowledge about language with curriculum design principles for teacher learning activities (Bakkenes, Vermunt, & Wubbels, 2010).

Based on general considerations around the relationship between language and mathematics learning, and teachers' roles, the focus will be on the design of PD programs. In formulating a PD program focusing on language development in multilingual mathematics classes, the questions can be formulated as follows:

- How can specific knowledge, including know-how and concrete skills, about language in mathematics learning be included in a PD program that relates theory to practice?
- How can a PD program, explicitly designed to link theory and practice on language in mathematics, initiate and change teaching practices in mathematics?

In the concluding section, we will discuss how the specified outcomes of a teachers' PD program on language and mathematics can be studied systematically in the future.

SELECTION OF CONTENT IN PD

Choosing relevant course content in any PD program for mathematics teachers is a multifaceted endeavour. It requires the translation of findings on student learning and the language of schooling into teachers' practices. Characteristics of the language of schooling have been identified by scholars such as Schleppegrell (2004; Schleppegrell & O'Hallaron, 2011) and have been specified for mathematics where students have to mediate at least three linguistic registers: everyday language, the language of schooling, and the technical language of mathematics (Prediger, Clarkson, & Bose, 2016). For students with a mother tongue other than the language of instruction (even more obviously for newly arrived students), a disadvantage exists with respect to

listening skills, reading comprehension of written texts, and expressing themselves in written text and mathematical talk. The development of second-language skills at a high proficiency level can take several years after students' arrival (Short & Echevarria, 2004). During the last decade, aspects of subject-specific literacies have been included in national curricula, as in Australia and Sweden (Australian Curriculum and Reporting Authority (ACARA), 2015; Skolverket, 2011). Curriculum implementation nevertheless is highly dependent on teachers' understanding and interpretation of the language dimension as well as their skills in planning their lessons from a language- and content-integrated perspective. For classroom teachers who are teaching all subject areas in primary schools, this is not always obvious. Even more, content teachers at the upper primary and secondary level can be unaware of their potential role in language development for mathematics learning.

Worldwide, language diversity in classrooms has brought a growing awareness of the importance of PD for in-service teachers, including knowledge of the characteristics of subject-specific literacy and the role of language proficiency in content learning, and skills in including language development in their subject teaching. This has been referred to as Knowledge About Language (KAL) (Love & Humphrey, 2012), which can be considered a specific part of teachers' broader Pedagogical Content Knowledge (PCK) (Shulman, 1986; Hill, Ball, & Schilling, 2008). If teachers are well prepared, they can include academic language skills throughout the school year as a natural part of subject teaching (Lee, 2004). Thus, they can play a role in fostering students' language proficiency in mathematics.

Characteristics of KAL for mathematics teachers can be derived from linguistic analyses of mathematics textbooks, assignments, and classroom practices. Turkan, de Oliveira, Lee, & Phelps (2014) argue that knowledge about literacy aspects of different disciplines should be addressed in teacher preparation. Key factors in helping to produce a mathematically literate citizen are that reading and writing support students as they analyse, interpret, and communicate mathematical ideas, and as they interpret the validity of information and evaluate sources of information. Different scholars have taken steps towards creating such a knowledge base (Kilpatrick, Swafford, & Findell, 2001), drawing on different conceptualizations of mathematical literacy and particular social practices in the math classroom. Engaging pupils in oral practice for meaning-making is underlined in many studies on learning mathematics (Adler, 1998, 2001; Moschkovich, 2002, 2007, 2013). However, classroom observations in, for example, Quebec and Zimbabwe (Cleghorn, Mtetwa, Dube, & Munetsi, 1998) have shown limited active participation of students in mathematics classrooms. This has also been shown in Spain and the Netherlands, specifically, in multicultural classrooms with a greater linguistic heterogeneity (Civil & Planas, 2004; Deen, Hajer, & Koole (Eds.), 2008). These studies show that teachers play different roles in their support of students' development of content-specific literacy skills in mathematics (Österholm & Bergqvist, 2013).

Though the importance of pupils' inclusion in oral classroom communication is often highlighted, pupil participation in school mathematics is not only oral. Schleppegrell's

(2007) research review characterises meaning-making systems in mathematics as multiply semiotic: mathematics uses symbolic notations, oral language, and written language, as well as graphs and visual displays. Examining the grammatical patterns, she shows the characteristics of technical vocabulary, dense noun phrases, specific verbs, conjunctions with technical meanings, and implicit logical relationships (Schleppegrell, 2007, p. 142). Theoretical knowledge about language and language development, including metalinguistic terminology, is required for teachers in order to understand the rationale behind content- and language-integrated pedagogy; 'KAL is understood in a broad sense, encompassing any implicit or explicit reference to language, communication, and learning' (Arnó-Macià, 2009).

Apart from improving knowledge about specific register characteristics, training programs include practical skills in preparing subject- and language-integrated lessons. In general, PD for teachers of second-language learners includes three core issues in lesson planning (Vogt, Echevarria, & Short, 2010; Hajer & Meestringa, 2014; den Brok, van Eerde, & Hajer, 2010): being able to make new math concepts comprehensible and relate these to contexts that are familiar to students ('contextualization'); promoting active involvement in classroom interaction ('interaction'); and offering feedback and scaffolding with a specific focus on language use ('scaffolding' or 'feedback'). The definition and development of specific teaching skills for language- and math-integrated teaching has not been studied extensively. However, the importance of active participation in classroom interaction has been found in several studies of multilingual classrooms in different national contexts. These reports discussed the limitations of individual seatwork, for instance, and the importance of fostering interaction with learners during short moments of support from the teachers. The latter would require teachers' awareness of language, walking through the classroom, and using these short moments for individual, tailored scaffolding (Elbers, Hajer, Koole, & Prenger, 2008). Second-language learners, particularly, are dependent on active participation in classroom interaction and on planned content and language-integrated learning.

Specifications of teachers' various practices can be derived from classroom observations (such as Deen et al., 2008), which examine different teachers guiding pupils' learning (Den Brok et al., 2010). Around the world, from the 1990s onwards, many PD programs on content- and language-integrated learning with theoretical backgrounds in second- and foreign-language learning have been delivered, though mainly in secondary education and upper primary schools (Eurydice, 2006; Wisemann, 2008). Marsh, Mehisto, Wolff, and Frigols (2009) formulated teacher competencies in content- and language-integrated learning (CLIL). One widespread program is the SIOP approach to integration of academic language in content areas especially for adolescent second-language learners (Short, this volume). It structures PD around teacher steps in lesson planning, from formulating content and language objectives to building background, providing comprehensible input, supporting strategies, focusing on interaction, organising practice and application, lesson delivery, and planning for assessment. The approach has its roots in second-language pedagogy.

Before being able to measure the effects of PD programs on student learning, the learning outcomes of teachers as actors in curriculum implementation have to be studied. PD for teachers in content- and language-integrated teaching has been examined at a general level, not taking into account subject-specific pedagogies and pedagogical content knowledge. Hajer (2006) mentions the lack of subject specificity as a factor when discussing the success of PD programs in the Netherlands. For example, she mentions that the focus on reading strategies for longer texts and vocabulary did not fit mathematics teachers' needs as much as they met the needs of biology or history teachers, where other text types are part of the subjects' pedagogy. The same conclusion of a too-general approach to meet mathematics teachers' needs can be drawn for PD programs and materials available within Content and Language Integrated Learning programs (CLIL) in Wales and Kiribati (Coyle, 2009; Marsh, 2002).

Given the specific language requirements of mathematics, PD programs for mathematics teachers can be expected to include specified Knowledge About Language and well-selected skills for lesson preparation related to the identified register characteristics of the subject. Specific PD materials have been developed for teachers of a range of subjects, including a specific manual for teachers of mathematics (Vogt, Echevarria & Short 2010; see also Short, in this volume). Researching PD specified for different content areas can enable more insights to be found in the role of teachers in bringing language- and content-integrated curricula into the classrooms.

THE THEORY: PRACTICE DIMENSION OF PD DESIGN

Apart from their own selection of course content, teachers will be most influenced about the intended curriculum and the possibilities for putting it into action by the form and outline of PD programs.

Few studies have focused on teachers' learning about language in mathematics. Smit (2013) reports an educational design research study on one primary teacher learning to include math language in her lessons. She focusses on teachers' language scaffolding skills around one domain, interpreting graphs. In this domain, Smit shows the teacher's growing understanding of the mathematics register, putting this into explicit activities that foster pupils' metalinguistic awareness of differences between daily wordings and mathematical language by offering them well-prepared linguistic scaffolding. In this Smit follows the ideas of Schleppegrell (2007) and Gibbons (2009) about the importance of explicitly connecting and moving on the continuum between daily and academic registers in the functional context of doing mathematics in classroom practice. This type of qualitative research clarifies teachers' learning from selected knowledge about math language and math learning in connection to PD program characteristics, including supervision and reflection on classroom experiments through the use of videotaping.

Relating theory to practice is a general concern for PD; (Timperley, Wilson, Barrar, & Fung, 2008), in that teachers should find connections between course content and their own classroom routines. Hattie (2012) argues that continuous and systematic learning can occur where teachers and principals are encouraged to purposefully develop ways

of teaching grounded in research and proven experience in local schools. School contexts should include structured peer meetings/collegial interactions where teachers feel safe enough to reflect on strengths and weaknesses in their teaching and students' learning (Timperley & Phillips, 2003). According to Timperley and Phillips, teachers may not want to change and modify their teaching unless they believe that change will result in learning improvements. They propose an iterative changing process, wherein teachers' beliefs, actions, or teaching outcomes are built on each other. Established domain knowledge has to be challenged (Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow, 2005), and new domain knowledge has to be built.

Pedagogical Content Knowledge (PCK) (Shulman, 1986) is subject-specific in nature. It includes not only teachers' subject knowledge (mathematics) and generic pedagogical knowledge, but also topic-specific insights into what students think about, or how they can best be supported in their development of particular subject matter and skills (Hill et al., 2008). Joubert, Back, De Geest, Hirst, and Sutherland (2010) note that within school-based initiatives in Guyana focusing on general improvement of PCK in mathematics, reflective activities like group discussions or writing in a diary were important to the teachers. A similar result is found in Bakkenes et al. (2010), in a study on an innovation program for secondary teachers, who reported that learning occurred 'mostly through experimentation and reflection on their own teaching practices' (p. 544). Bakkenes et al. (2010) list possible learning outcomes of teachers participating in formal and informal learning environments as changes in knowledge and beliefs, intentions for practice, changes in practice, and changes in emotion. In their research, Bakkenes et al. relate learning outcomes to the types of learning that teachers have been exposed to. These can be formal, organised in classes and following a program, as well as informal. Four types of learning activities in PD programs are discerned:

- learning by experimenting (e.g., trying out instructional materials or scaffolding strategies),
- learning in interaction with others (other teachers, researchers),
- learning using external resources (e.g., publications), and
- learning by consciously reflecting on one's own teaching practice).

Further examination of these activities would enable a closer examination of their relationship to learning outcomes in PD programs.
In summary, PD programs that would enable teachers to integrate language development in their mathematics classes should

- include a subject-specific body of Knowledge About Language,
- offer different learning activities that relate theory to practice, and
- be organised in the setting of collaborative learning in school teams.

BACKGROUND OF THE PD IN THE MATHEMATICS BOOST PROGRAM

The nationwide curriculum reform of 2011 in Sweden offers an interesting setting to construct the characteristics of a PD program against the literature review presented

above. First a general background to the PD program is given, and then the PD module on language in mathematics will be examined.

A (language) Perspective on Swedish Mathematics Education and Teachers' Intended Learning

The PD program, called 'Mathematics Boost' (Matematiklyftet), should clearly be seen from the perspective of the National Curriculum for the Compulsory School, introduced in 2011 (Skolverket, 2012). Mathematical communication is highlighted in this new syllabus, which aims to educate students in the exchange of mathematical ideas and thoughts with others. Long-term goals state that students should be given opportunities to develop the ability to formulate problems, use and analyse mathematical concepts and relationships between concepts, lead and follow mathematical reasoning, and use mathematical expressions to discuss, argue, and explain the issues, calculations, and conclusions (Kilpatrick et al., 2001). Explicit work on language proficiency is therefore essential for students to achieve the curriculum's long-term goals in mathematics (Adler, 1998).

In order to implement this curriculum in the classroom, teachers should understand it and be willing and able to translate it into practice (Goodlad, 1986). The observation that students' individual seatwork is more usual in Swedish mathematics classrooms (i.e., substantial time is spent in silence) was a strong incentive for the Swedish government to implement a mathematics teachers' PD program. Research had shown the need for a more communicative, interactive mathematics education (Kilpatrick et al., 2001). Already in 2004, the Swedish National Agency for Education noted that the total time students spent working independently in mathematics textbooks had increased (Skolverket, 2004). The amount of time that teachers instruct an entire group has declined in the last 20–25 years. In 2004, approximately 6 per cent of the time in Swedish mathematics classrooms at all levels was devoted to 'inquiry based' mathematics and laboratory practices where more conceptual than procedural learning could be applied (Skolverket, 2004). Liljestrand and Runesson (2006) explored how classroom organisation, tasks, and content shape the interaction as well as learning potential, and showed that classes typically began with an introductory plenary session that was followed by individual seatwork from a textbook. These studies of mathematics education uncovered the minor role that teachers played in actual classroom interaction, while students increasingly worked on their own with mathematics books. Several other researchers have noted this relationship (Kling Sackerud, 2009; Sjöberg, 2006; Österholm & Bergqvist, 2013).

The National Agency for Education plays a steering role, requiring schools to arrange PD, using the kit of PD materials as a condition for getting funding for PD, and providing training for supervisors and tutors leading the PD. At the same time, an active role of teachers in their own learning is expected, drawing on Hattie and Timperley's recommendations for collaborative learning in teams of teachers (Schnellert, Butler, & Higginson, 2008). The content of the PD program is structured around students' collaborative and interactive learning, and teachers are expected to

learn within the context of communities of practice with colleagues in their own school. An assumption is that teachers' interactive and collaborative learning should start with the teachers bringing a substantial body of knowledge into the collaboration.

The mathematics Teachers' Professional Development Program

The Swedish government decided to spend a total of 649 million crowns (roughly 76 million USD) starting in the school year 2012/13 and continuing for three additional academic years so that all teachers who teach mathematics within the school system would be able to participate in Mathematics Boost. The funds were used for program development and support for schools, for example in the form of compensated hours for teachers to participate. In addition, tutors—specialised mathematics teachers—were educated at different universities to lead and support the teams of teachers using Mathematics Boost in their schools.

The PD material is published for mathematics teachers as a national, web-based program with didactical support material (www.matematikportalen.se). The National Agency for Education consulted with universities' and colleges' mathematics education staff members, who were assigned to create the web platform content. The construction of web-based materials can be seen as a wider pathway in contrast to PD that involves off-site activities, where physical attendance can become an impediment. The main materials on the web platform are training packages, called modules, which teachers are supposed to work through collaboratively in planned sessions.

In addition to providing teachers with professional development, the overarching aim of Mathematics Boost is to increase students' achievement in mathematics through the strengthening of mathematics teaching (Skolverket [National Agency for Education], 2012). In other words, the purpose of the program is to influence two processes, namely the mathematics classroom practices (the teaching, of which working with language development is one aspect), and the professional development culture, to engage teachers in processes of collective learning where they relate new knowledge to their classroom experience. The construction of the PD program leans heavily on an assumption that mathematics teachers are considered to be in need of Continuing Professional Development (CPD) (Joubert & Sutherland, 2009) in order to implement the intended curriculum (Goodlad, 1979, 1986). Even though Swedish teachers may have participated in collaborative learning before, Mathematics Boost strongly articulates this as a way to develop teachers' teaching. State funding of teachers' participation requires schools to follow the framework and learning activities. Thus, the PD program supports collegial learning in communities of practice (Wenger, 1998), in which colleagues' structured collaboration aims to integrate new knowledge into day-to-day practices (Smit, & van Eerde, 2011). In the program, participating teachers work with the various modules consisting of didactic materials to use for discussing, planning, and evaluating mathematics teaching.

Modules typically consist of eight parts, which are meant to be the focus for one school term (20 weeks), during which all teachers spend two hours a week for a total of 40

hours. The fixed format for each part is meant to structure the collaborative work of teachers. Each part consists of four sections called A, B, C, and D.
Section A is an individual preparation for each teacher, who reads an introductory article and/or watches a video clip relating to the parts' theme. This would take about 45–60 minutes. Section A represents the intended curriculum, including the theory (Goodlad, 1979, 1986). Section B is related to Goodlad's perceived curriculum: in a meeting, teachers discuss the literature and video, aided by a number of focus questions and led by a tutor or supervising teacher. From these discussions, practical applications of didactical ideas in the teacher's own classroom is prepared (90–120 minutes). Section C is then the actual classroom activity that is part of the ordinary classroom work of each teacher. Thus, Section C forms the curriculum in action (Goodlad's operational curriculum. Section D consists of another group meeting where the teachers reflect on their experiences with class activities and draw conclusions about the part-theme (45–60 minutes), thus relating the implemented curriculum to perceived outcomes, the attained curriculum (as illustrated in Table 1).

Within a module, learning activities for the teachers are always repeated in these four-cycle sections: a) individual studies/work: read an article and/or watch a film sequence; b) group discussion on the articles and films, and plan lesson collaboratively; c) conduct lesson in one's own class/group, observe other teachers' teaching; and d) group discussion, follow-up, and collaborative evaluation of the conducted lesson.

By the end of 2015, about 14,000 teachers across the country had gone through a year of the Mathematics Boost program (Jahnke, 2015; Ramböll, 2015). In the summer of 2016, 76 per cent of all mathematics teachers in Sweden had participated in the program to various extents (Skolverket, 2016b). This translates into 35,580 teachers.

THE MODULE ON LANGUAGE DEVELOPMENT IN MATHEMATICS: DESIGN AND CONTENT

General Description of Course Content and Learning Activities

In May 2016, eight PD modules for compulsory school and seven for upper secondary school had been developed, many of them focussing on specific content areas like 'graphs', 'arithmetic', or 'geometrical forms'. The module we focus on in this article has a more general focus, 'Language in mathematics' for compulsory school. The module targets mathematics teachers working with pupils in the age range of 7–16. Different disciplines were represented when constructing the module: mathematics pedagogy, educational linguistics, and second-language learning. All materials were peer-reviewed in two cycles by the National Agency for Education, researchers, and teachers before being published on the open-access website.

Through this module, teachers should realize that students' oral and written communication is essential for learning mathematics. To enhance classroom communication for students' mathematical learning, a major part of the module was to establish teacher practices that take into account reading mathematical texts, and writing such texts (Österholm & Bergqvist, 2013). Thus, this module for teachers of

mathematics was developed to prepare for designing and delivering lessons under the intended curriculum, in which an explicit focus is on students' language development in mathematics.

The following is an overview of this module that presents the content and provides references to didactical models. We then describe in more detail three parts (3, 4, and 6) that have been mentioned by teachers in practice to be the most fruitful for their work (Norén, Ramsfeldt, & Österling, 2016). We also account for the attained curriculum in an activity in school year 5, conducted by a teacher who participated in the PD program, using videotaped data. The video has been shortened and published on the web for other teachers to view when working with Part 6. We go on to describe and analyse the selected knowledge about language included, the link to mathematics content, and the learning activities that aim to link theory to teachers' classroom practice, formulating the potential learning trajectories. Table 2 presents an overview of the modules' eight parts, its content focus, didactical models, and learning activities.

In Part 3, 'Communication from a formative perspective', the practice of two teachers in their actual classroom interaction is compared using original classroom transcripts derived from Deen et al. (2008) to show how teachers' daily practice can be more or less supportive for language development. In Section A, the learning activities for the teachers start with an introductory text directly linking the knowledge about language to a specific content area (Schleppegrell, 2004): reading and understanding graphs (van Eerde & Hajer, 2008). The importance of hearing students' thoughts in order to adapt teaching to their prior knowledge and existing language skills is a key aspect of Part 3. In addition, working with concept maps is introduced as a practical activity (also in Section A), starting from a list of relevant key terms from the mathematics syllabus. Concept maps visualize and make explicit the relationships between words and the required connecting words. In a concept map, a focus question is formulated that organises conceptual knowledge (Novak, 1990). Planning lessons in which concept maps are used to visualize prior knowledge, or to elaborate on new course content, are suggested in Section B. Section C (delivering the planned lesson) includes the gathering of students' maps, which stimulates teachers to promote students' active use of language and thus grasp students' prior knowledge at the beginning of a new mathematical unit. In this way, the crucial step of formulating language objectives in math lessons is presented. Teachers explicitly have to emphasise concepts like charts, graphs, line, curve, rise, fall, and line charts. In addition to the individual terms, the relations between the terms and descriptions of those relationships need to be considered. For the development of language, an overall plan for progression is suggested: a) The relationship between simple graphs and daily phenomena can be discovered in conversations, for example in discussing the temperature or distance. b) In small groups, students discuss the different graphs, identify features, and make comparisons. The teacher listens, supports by paraphrases, and shares formal words. c) Students draw and interpret graphs, and relate them to functions. They present results and are expected to express themselves with mathematical language. In Section D, the teachers' individual lessons are evaluated and then discussed with colleagues.

Table 2. Overview of the module, including the content and reference to didactical models

Parts	**Content Focus**	**Key reference**	**Learning activities**
	Section A	**to didactical models**	Section C
1. Language- and content-knowledge approach	Characteristics of language in mathematics teaching and learning	Three principles for sheltered instruction (Vogt et al., 2010) and CLIL	Plan, carry out, and evaluate lesson, with relevance for principles 1 and 2 of the 3 principles for CLIL
2. The mathematical language	The mathematics language's segments and their relation to other genres	Representations, a 'thinking board' or matrix (McIntosh, 2006), in which students draw and write four different mathematical representations: picture, material artefacts (hands-on material, manipulatives), symbols, words	Plan, carry out, and evaluate lesson: What do students know about informal and formal words and symbols in the mathematics register?
3. Communication with formative purposes	Connecting to students' prior mathematical knowledge. Content focus on graphs and their representation	Concept maps, (Novak, 1990)	Plan, carry out, and evaluate lesson: Construct conceptual map on current teaching of mathematics content
4. Scaffolding the mathematics language	Scaffolding language. Content focus on dynamic geometrical program, sequencing, visualization, reformulation, contrast	Micro (interactional) scaffolding, macro scaffolding (Hammond & Gibbons, 2005)	Plan, carry out, and evaluate lesson: Macro scaffolding in relation to the current teaching of mathematics content

Table 2. contiuned.

Parts	**Content Focus**	**Key reference**	**Learning activities**
	Section A	**to didactical models**	Section C
5. Interaction in the mathematics classroom	How to put questions to students, various questions, open questions in classroom interaction	IC model (Inquiry & Cooperation) (Alrø & Skovsmose, 2004)	Plan, carry out, and evaluate lesson: Student interaction around the current teaching of mathematics content
6. The teaching learning cycle: Text tasks in mathematics	Analyses of mathematical text problems	The teaching learning cycle, (Gibbons, 2002; Derewianka, 2003)	Plan, carry out, and evaluate lesson: Text problems in the current teaching of mathematics content, analyses together with students
7. To produce texts in mathematics	Writing and production of mathematical texts	The teaching learning cycle (Derewianka, 2003)	Plan, carry out, and evaluate lesson: Construct text problems together with students
8. Reflecting and looking forward	How one's own teaching in mathematics has developed, and can be more developed, regarding language development in mathematics	Metacognition, reflection	Discuss and evaluate with colleagues

Part 4 focuses on 'Scaffolding language in mathematics'. Offering students various opportunities to communicate mathematics is at the forefront. To communicate in the mathematics classroom means to exchange information with others about mathematical ideas and thoughts, orally and in writing, using different forms of expression (Love & Humphrey, 2012). In teaching, students have the opportunity to develop a more precise mathematical language to independently adapt their talks and presentations to various recipients or purposes. As key knowledge about language, the concept of 'scaffolding' (Gibbons, 2002; Hammond & Gibbons, 2005) is foregrounded.

After watching a film (Section A) from a classroom in which the mathematics teacher macro- and micro-scaffolds a whole class or students in pairs at different occasions, teachers are encouraged to give examples of how they already scaffold students in their own teaching. In Section B, the teachers discuss their individual experiences collectively and plan a lesson collaboratively to be delivered in each teacher's class (Section C). Teachers are asked to relate their lesson to sequencing, reformulation, visualizing, and contrasting (Hammond & Gibbons, 2005). In Section D, individual evaluations of the lessons are discussed.

Part 6 introduces the teaching-learning cycle for elaborating on texts in mathematics. Throughout Sweden, the generic Australian pedagogy had reached many classrooms. In this approach (Rose & Martin, 2012; Gibbons, 2002), an explicit focus on written texts is introduced in mathematical content areas. Even within the profession of mathematics, its texts have specific challenges for readers and writers. Therefore, it is argued, teachers should explicitly talk with students about the characteristics of a 'good' mathematics text and practice writing such texts themselves (Rezat & Rezat, 2017).

Part 6, Work on Mathematics Texts, through Sections A–D: An Illustration

The individual preparation for each teacher in Section A includes reading an introductory article, 'The teaching and learning cycle: Texts in mathematics', that discusses how teachers can practically work with language development in mathematics instruction using the teaching and learning cycle. It can be considered a clarification of the intended curriculum (Goodlad, 1979). A reflecting question for the teachers to individually consider is: What are your experiences of paying attention to your students on the language, structure, and context of different texts in mathematics? Teachers are told to prepare for Section B by selecting a few mathematical texts from the mathematics topic they are currently working on in their own classes. They have to justify their choices based on experiences with reading the article. The selection will serve as a basis for analysis of mathematical texts' characteristics in terms of language, structure, and context. Some texts will also be chosen as typical examples, and the teachers will discuss what makes the chosen tasks interesting to analyse in terms of language, structure, and context.

Teachers write down their reasoning about the issues above and bring examples of texts to the collegial work in Section B. Section B is related to Goodlad's (1979) perceived curriculum: the teachers meet and discuss the article and their chosen mathematical texts. They also watch a three-minute video clip of a Swedish teacher talking about the teaching and learning cycle. Aided by the focus question from Section A, and led by a supervising teacher, the discussions serve as preparation for practical applications of didactical ideas in the teachers' own classrooms. The teachers are instructed to analyse one or two mathematical texts, with respect to language, structure, and context.

Section C is the actual classroom activity. Thus, Section C forms the curriculum in action, or the implemented curriculum, relating to Goodlad. The video (Skolverket,

2016a) we discuss below is part of a recorded lesson, and shows an example of implementing the analysis of mathematical texts in the classroom.

Finally, Section D consists of a second meeting (45–60 minutes), where teachers reflect on their experiences with classroom activities and draw conclusions about the theme, thus relating to the attained curriculum. Focus questions are:

Evaluate

Did the lesson fulfil its purpose? What helped to make it fulfil the purpose? What obstacles did you experience?

- What aspects of the organisation of the lessons worked well?
- What worked less well? Why?
- In what ways did you adapt scaffolding during the lesson?
- How do your experiences of classroom activities differ?

Reflect

- In what way do you think the teaching and learning cycle supported the lesson planning and implementation?
- Which ways promoted in the lesson helped students develop mathematical language?
- How would you be able to jointly develop student skills in analysing the language, structure, and context of other types of texts in mathematics that they will encounter and produce in mathematics?
- How could you work with other kinds of texts in mathematics using the support of the teaching and learning cycle?

Part 6, Video: One Lesson in School Year 5 (students 11–12 years of age)

The aim of the lesson is to make students aware of mathematical and everyday vocabulary in a written mathematical text, but also to extend their vocabulary generally. In the introduction to the lesson, the teacher tells her students that she has moved and her way to school is now a longer distance from home than before. The students are invited to talk about their own way to school. The students animatedly describe how they walk, bike, or are driven to school by their parents. They talk about hilly and flat parts on their way, and how talking to other students along their way might make them walk at a slower pace. After about eight minutes, the teacher introduces a mathematical text that she and the students will analyse together, in line with the teaching learning cycle (Gibbons, 2002):

Early one Tuesday morning, Pelle rides his bike to school. He maintains a high average speed until half the distance to school is covered. There at the big oak, he stops and waits for Fia. Suddenly he realizes that he should have fetched Fia at her house. He rides back two-thirds of the stretch of road he had already cycled, at the same speed as he had before. After a short waiting time when Fia unlocks her bike, they ride together

to school. They talk so their speed is only half of what Pelle's speed was before. When they are halfway to the big oak from Fia's house, Pelle looks at the clock and sees that now they need to hurry. They increase their speed so that they ride twice as fast as Pelle cycled, from the beginning, the rest of the way to school. They arrive on time.

The teacher starts by telling the students to underline the words in the text that they find a bit unusual and difficult to understand, and she goes on reading the text aloud.

Figure 1. A student is underlining unusual words

The first word underlined comes from Jacob, who says tillryggalagd. The translation to English is distance travelled, but what it really means is distance you put behind your back. On the whiteboard, the teacher has written headings for two columns: mathematical words and everyday words. The students are invited to talk about the word. Students make suggestions like: he has already done it; he has it behind himself. Other suggestions for explaining the word include put behind the back, completed, ready. The students agree that the word is an everyday word, arguing that it is not mathematical. More words are discussed, elaborated on, and defined: average speed [medelhastighet], a mathematical word; two-thirds (två tredjedelar, two out of three), mathematical words; half [hälften], a mathematical word; the distance [sträckan], a word that can be both mathematical and used in everyday language. At the end of the 60-minute lesson, the whiteboard looks like in Figure 2:

The next lesson covers drawing a graph. On the y-axis it says: home, Fia, oak, school. The x-axis is the time.

The 'intended' curriculum in the national curriculum states, Students have to develop their ability to conduct mathematical reasoning, and as stated in the PD program in a more operationalized way, Texts in mathematics present challenges of different kinds for the students. Elaboration on various types of mathematical texts anchors and deepens students' knowledge, both in terms of the mathematical content and mathematical language. The 'attained' curriculum, what students experience and learn, is exposed in the lesson, a lesson that is the teacher's 'implemented' curriculum.

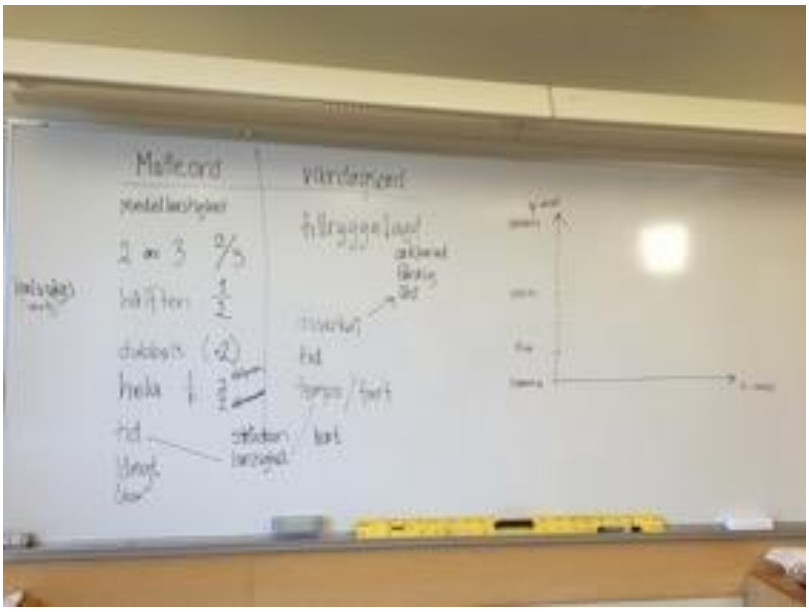

Figure 2. The writings on the whiteboard at the end of the lesson

DISCUSSION

In the discussion, we expand from the research questions: on specific knowledge, in a PD program, about language in mathematics, and how the knowledge can include know-how and concrete skills; on how the design of a PD program can link theory to practice by initiating and changing teaching practices in mathematics; and on the specified outcomes of PD programs and how they can be studied in the future.

Specific Knowledge on Language in Mathematics, Teachers' Learning, and PD Programs

The language module in the larger Swedish Mathematics Boost PD program offers an interesting example that meets the requirements of subject-specific Knowledge About Language. The PD program for mathematics teachers does not impose a language perspective onto their teaching role, but enlightens the language dimension as a natural part of mathematics subject content and pedagogy. The scale on which the program is spread throughout Sweden, and the similar conditions of the structured ABCD sections, offer possibilities for a closer examination of teacher learning, putting the intended curriculum into practice. In staff meetings, for instance, teachers could discuss the relevance of theoretical concepts (Parts A and B), illustrate how they use and develop skills in the classroom (Part C), and reflect upon their experiences (Part D). Up to now, evaluation studies of the program have been large-scale and focussed on appreciation of the PD structure, its setting in collaborative meetings, and time factors. One study showed 6,000 teachers' appreciation of the courses and examined conditions for its web-based nature of in-service training in combination with

collaborative learning contexts in school teams (Ramböll, 2015). The average outcomes show a positive rating of the material's relevance for teaching mathematics. One of the main factors in teachers' judgments is the content and structure of modules. The report judges the conditions for realization offered by the National Agency for Education as 'good'. One recommendation is that more flexibility should be enabled in using parts of modules and adapting the in-service portion to specific needs in the school team. The report did not examine teachers' learning within PD around specific modules. Further examination could discern the role of diversity within school contexts, teachers' individual development, and students' needs at different stages of learning.

The Design of the PD Program and Teachers' Change of Practice

Concerning the learning activities chosen, we can see that each part in a module consists of four sections: a) individual studies/work: read an article and/or watch a film sequence; b) group discussion on the articles and films, and plan lessons collaboratively; c) conduct lesson in one's own class/group, reflect on own teaching; and d) group discussion, follow-up, and collaborative evaluation of the conducted lesson. The design of the PD matches Loucks-Horsley, Stiles, Mundry, Love, and Hewson's (2009) professional development design framework, building on reflection about and revision of teaching. Considering the four teacher/learning activities listed by Bakkenes et al. (2010), we find each of them in the different sections of the module. Each Section C focuses on learning by experimenting (trying out instructional materials). Each Section B and D contains learning in interaction with others (recurring collegial discussions with other teachers). In the preparation, teachers are asked to read and watch external resources (film and article). Within Sections A, B, C, and D, consciously reflecting on one's own teaching practice is promoted, and it is proposed to do this collaboratively.

Apart from the recurrent, theory-practice linking of activities, a strength of the PD program is that it fits closely with the intended national curriculum on mathematics and pedagogical context and traditions in Swedish schools, likely due to the active involvement of the National Agency for Education. The materials reach schools not as part of a language pedagogy PD program, but as part of the national Mathematics Boost program addressing mathematics teachers' concerns, and through relating explicitly to the mathematics curriculum and syllabus guidelines. The content of the PD program helps teachers to implement the curriculum. The fact that authors of the materials worked in a multidisciplinary team has contributed to the program's mathematics-specific nature. What is more, through the process of designing the module, in which experts and practitioners were actively involved, the present materials can be seen as a state-of-the-art example of the Swedish idea regarding relevant knowledge about mathematics language.

Even though the module is not specifically constructed for schools with second-language learners only, it has its roots in research on second-language learners and content-based instruction. It is aimed at teachers' awareness of learning, teaching, and ways of using language/s in relation to a specific school subject (Marsh, 2002). We can

discern, in the module's core knowledge, a strong focus on language development in interaction, characteristics of mathematics language (in vocabulary as well as text types), and a strong focus on teacher scaffolding and feedback in interaction. Given the high number of newcomer students in the Swedish schools, one remaining question is whether the module provides sufficient understanding of the specific second-language pedagogy aspects required in newcomer classes.

In its actual use, the PD program may look different because the teachers and their tutors choose mathematics content from their daily teaching or textbooks. Here, we can expect major differences in teacher practices. The PD program is being used throughout the country at large, reaching hundreds of schools and thousands of teachers in different contexts. Group tutors leading the team work within the PD could certainly adapt the program to current concerns of teachers, be it addressing the needs of newly arrived pupils or including pupils in group work, just to mention two examples. The realization of the PD will therefore look different in different contexts, as will the outcomes.

One can raise the question of how the pedagogical tools offered in the module can be used as part of a comprehensive approach of teaching within a thematic unit or a certain area of mathematics. If we compare the program to the SIOP approach (Short & Echevarria, 2004), no comprehensive planning tool is offered, from introduction of new concepts and terminology to assessing student learning at the end of a unit (Hajer, 2006). It would be interesting to see how teachers take up and include the suggestions in their daily routines and planning.

Studying the PD for Language in Mathematics

In examining the design and content of PD for language in mathematics, desired outcomes have to be described in national settings (the attained curriculum). In Sweden, language learning has been integrated as an aspect of effective mathematics education that offers opportunities for all learners in multilingual classrooms. The design of the PD content and delivery were organised as a transparent, nationwide effort, made public through web-based materials. However, there is a tension between the need for uniformity and large-scale PD and more tailor-made programs adapted to specific school contexts, generating a requirement for comparative studies on strengths and weaknesses of PD programs. We argue that there is an absolute need for better understanding of teachers' work in constructing the syllabus, which should foster a communicative mathematics education in multilingual classrooms. Using Goodlad's (1986) distinction between the curriculum aimed for, interpreted, and realized, teachers are of crucial importance in understanding, being willing, and being able to plan and deliver their mathematics teaching in line with the national curriculum guidelines. In order to understand how they do this and develop new routines, the delivery of the Mathematics Boost PD offers an opportunity to gather data and compare teachers' learning in collaborative groups, taking pedagogical activities to their classrooms and reflecting on the outcomes.

There is a need for a better understanding of teachers' learning and the quality of their development of know-how and skills about language in mathematics. Davison (2016) states that many mainstream teachers fail to identify with the role of providing language and literacy support to second-language learners in their classrooms. To address this issue, Hammond (2014, p. 503) calls for 'more wide-ranging, theoretically robust accounts of teacher learning' to specifically support these learners. Our way of describing PD course content in general terms, subject specificity and the learning activities are meant to contribute to enabling comparisons of research on PD in various national contexts. If PD programs could be described in a similar way, the next step in creating a rich knowledge base would be to synchronize data gathering. In order to compare the PD programs in a systematic way, a better description of content and types of learning activities and synchronized assessment of learning outcomes would be required. Selected Knowledge About Language, chosen learning activities as well as achieved changes in knowledge and beliefs, changes in intentions for practice, and changes in actual practice are all relevant. If these categories could be described in similar ways, different PD programs and contexts could be compared within an international perspective, thus deepening our understanding of their effectiveness. In future research on PD we propose to examine how Simon's construct of Hypothetical Learning Trajectories (Simon & Tzur, 2004; Simon, 2014) could be of help. Explicating Hypothetical Learning Trajectories for teachers' learning and bringing theory to practice in selected parts of PD programs could enable a closer examination of teachers' learning. Although the authors of Mathematic Boost materials did not explicitly formulate such hypotheses, researchers as part of their evaluation studies could formulate them.

We argue that formulating underlying assumptions about fostering teachers' roles in students' language development in mathematics is a prerequisite for further studies on teacher PD as the crucial link in implementing language- and mathematics-integrated curricula.

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APPENDIX

Translation of self-reflection inventory instrument from the *Language in Mathematics* module

Min matematikundervisning [*My mathematics teaching*]

Exempel på hur jag gör det [*Examples of how I do*]

1 Jag gör det matematiska innehållet begripligt genom att utgå ifrån elevernas erfarenheter och förkunskaper. [*I make the mathematical content comprehensible by departing from students' experiences and prior knowledge*]

2 Jag främjar aktiv språkanvändning genom att skapa tillfällen för eleverna att omväxlande tala, läsa, skriva och lyssna, under en lektion och under en serie lektioner. [*I promote the active use of language by creating opportunities for students to alternately speak, read, write, and listen, during a lesson and for a series of lessons*]

3 Jag planerar för att eleverna ska få syn på samband, likheter och skillnader, mellan matematikspråk och vardagsspråk. [I plan for students to get sight of the connections, the similarities, and differences between the mathematical language and everyday language]

4 Jag planerar för att ge eleverna många tillfällen att använda de olika delarna av matematikspråket. [*I plan to give students many opportunities to use different aspects of mathematics language*]

5 Jag ger exempel på framgångsrika strategier för att tolka matematiska problemtexter. *[I give examples of successful strategies to interpret mathematical problem texts*]

6 Jag uppmärksammar språkliga aspekter när jag formulerar syftet med mina matematiklektioner. [*I pay attention to linguistic aspects when I formulate the purpose of my math lessons*]

7 Jag organiserar aktiviteter för att få syn på elevernas förkunskaper inom ett område. [*I organise activities to get hold of students' prior knowledge in a mathematics area*]

8 Jag planerar aktivt en varierad språklig stöttning. (Makrostöttning) [*I actively plan a varied linguistic scaffolding. (Macro scaffolding*)]

9 Jag anpassar den språkliga stöttningen medan undervisningen pågår. (Mikrostöttning) [*I adapt linguistic scaffolding while teaching is in progress (Micro scaffolding)*]

10 Jag planerar aktiviteter så att alla elever ges möjlighet till muntlig interaktion. [*I plan activities so that all students are given the opportunity for verbal interaction]*

11 Jag planerar undervisningen så att eleverna utvecklar den matematiska kvalitén I sina samtal, under en4114ectionn och under en serie lektioner. [*I plan teaching so that students develop the mathematical quality of their talk during a lesson and for a series of lessons*]

12 Jag uppmärksammar språkliga drag i olika matematiktexter, såsom faktarutor, typexempel, problemlösningstexter och redovisningar. [*I pay attention to linguistic features in different mathematics texts such as facts, typical examples, problem solving texts, and presentations]*

13 Jag ger eleverna möjlighet att producera olika sorters matematiska texter, tillsammans med mig och enskilt. *[I give students the opportunity to produce different kinds of mathematical texts, along with me and individually]*

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Teacher Narratives about Supporting Children to Read and Write in Mathematics: The Case of Kay

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ABSTRACT
The impact on one teacher of a short professional development project run in a school in a low socio-economic area in a small city in rural Australia is investigated in this case study. The project aimed to support teachers to improve students' writing in mathematics. The teacher's reflections about her work with a small group of Year 3-4-5 students are discussed in relationship to what supported or hindered her to change her practices. Over the two months of the project, the teacher supported the children to comprehend and produce their own word problems. However, the process of deciding how to change what she did to meet the needs of the students was messy because different combinations of factors affected her willingness to try alternative practices. Her narratives, from watching the videos on her lessons and in joint meetings with the other teachers and researchers, indicated that reflecting on what she was doing contributed to her taking more risks in her teaching. This resulted in the students having more opportunities to use their mathematical literacy skills to comprehend and respond to word problems.

Keywords: professional development, students' language, mathematics word problems, low socio-economic area, teacher reflection

PROFESSIONAL DEVELOPMENT AND INCREASING STUDENT ACHIEVEMENT

The scaling up of professional development (PD) is often based on models which expect teacher learning, acquired during the PD, to increase student achievement in a linear fashion (see for example, Carpenter et al., 2004; Higgins & Bonne, 2011). However, as Joubert, Back, De Geest, Hirst, and Sutherland (2010) indicate, the process of teacher learning is messy, due to a combination of factors, that involve interactions between the teacher, the students and the context, including the mathematics being learnt. Generally models of PD do not consider how contextual features affect teacher learning. In this paper, we use a case study to describe how one teacher, Kay, viewed a PD project and its impact on improving students'

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State of the literature

- The relationship between professional development, including on language learning, and improved student learning outcomes is complicated and messy.
- Teachers have an important role in scaffolding students' acquisition of different aspects of the mathematics register.
- For teachers to be able to undertake this scaffolding they need input about mathematics register content and the processes of scaffolding.

Contribution of this paper to the literature

- The teacher's context of being a non-permanent member of the school staff affected her public comments about her teaching, which then affected her possibilities for receiving suggestions for alternative practices.
- Learning about writing and interpreting word problems requires teachers to recognise that students need to attend to a large number of different aspects simultaneously.
- Teachers need awareness of scaffolding strategies connected to developing students' acquiring aspects of the mathematics register in order to provide activities appropriate for students' needs.

interpretation and production of standard Australian English in the writing and solving of mathematical word problems. Although the students seemed to increase their mathematical understandings through developing their language skills, the teacher's participation in the professional development did not always seem to have a positive effect on changing her practice. Therefore, our focus is not on the students' learning outcomes, but on the teacher's learning and how it was connected to her reflections about what she did. These reflections seemed to provide Kay with deeper understandings of her options and the researchers, who were also the professional development facilitators, with a better understanding of how contexts affected the impact of the PD.

As PD facilitators researching our own practices (Lange & Meaney, 2013), it was important to understand the messiness of the relationship between professional development and teacher change. Although this relationship has been characterised in a range of different ways, evidence for a link to student outcomes remains unclear (Joubert & Sutherland, 2009). Early models, such as Guskey's (2002), see Figure 1, indicate that sustainable change in teachers' beliefs and attitudes occurs after teacher practices have changed, which leads to improvement in student learning.

Other models, such as Clarke and Hollingsworth's (2002), include similar components but allow for different ordering, depending on the teacher. They found that sometimes input from the PD changes teachers' beliefs and attitudes before their classroom practices, something that Guskey (2002) had argued as not being likely. They also commented on the effect of the school environment on teacher learning:

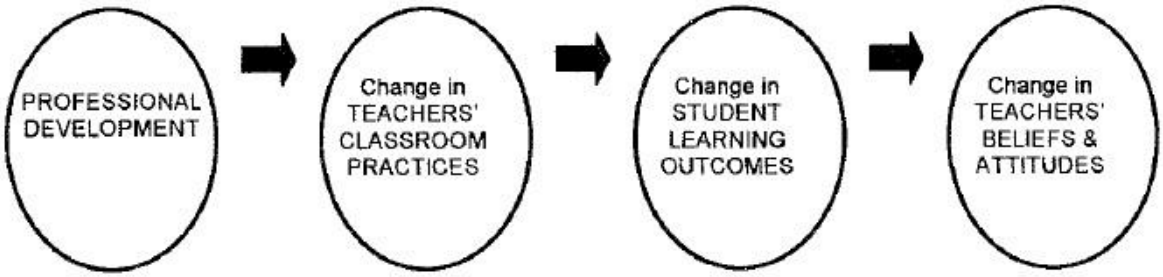

Figure 1. Guskey's model of teacher change (Guskey, 2002, p. 383)

> The school context can impinge on a teacher's professional growth at every stage of the professional development process: access to opportunities for professional development; restriction or support for particular types of participation; encouragement or discouragement to experiment with new teaching techniques; and, administrative restrictions or support in the long-term application of new ideas. (p. 962)

When PD projects are scaled up, concerns about the impact of different factors on outcomes have been raised. For example, Coburn (2003) called for a reconceptualization of scaling up that:

> … emphasizes the spread of norms, beliefs, and pedagogical principles both between and within classrooms, schools, and districts. And it includes an additional outcome—the shift in ownership—that may prove key to schools' and districts' abilities to sustain and spread the reform over time. (p. 8)

She saw it as essential that ownership did not reside with PD facilitators or other external bodies but with districts, schools and teachers.

In trying to capture some of the contextual factors that affect the outcomes of PD, Joubert et al. (2010) produced a complex model based on socio-cultural understandings about learning, in which they identified a range of factors that could affect the outcomes for both students and teachers (see Figure 2). In this model, they indicated that the planned PD is based on the motivations, beliefs, knowledge and experiences of the designers, but would also take into consideration contextual features that could affect its implementation. The designers would also identify the specific aims of the PD, related to the intended changes in practices and improved students learning, which were to be the outcomes of the PD. Teachers would then identify, from their motives, beliefs, knowledge and experiences, opportunities within the PD that they would want to adopt. The actual PD would arise from the interactions and contribute to changes in practices, which would affect students' learning, depending on previous and ongoing interaction between the students and others. Although already complicated, Joubert et al. (2010) stated:

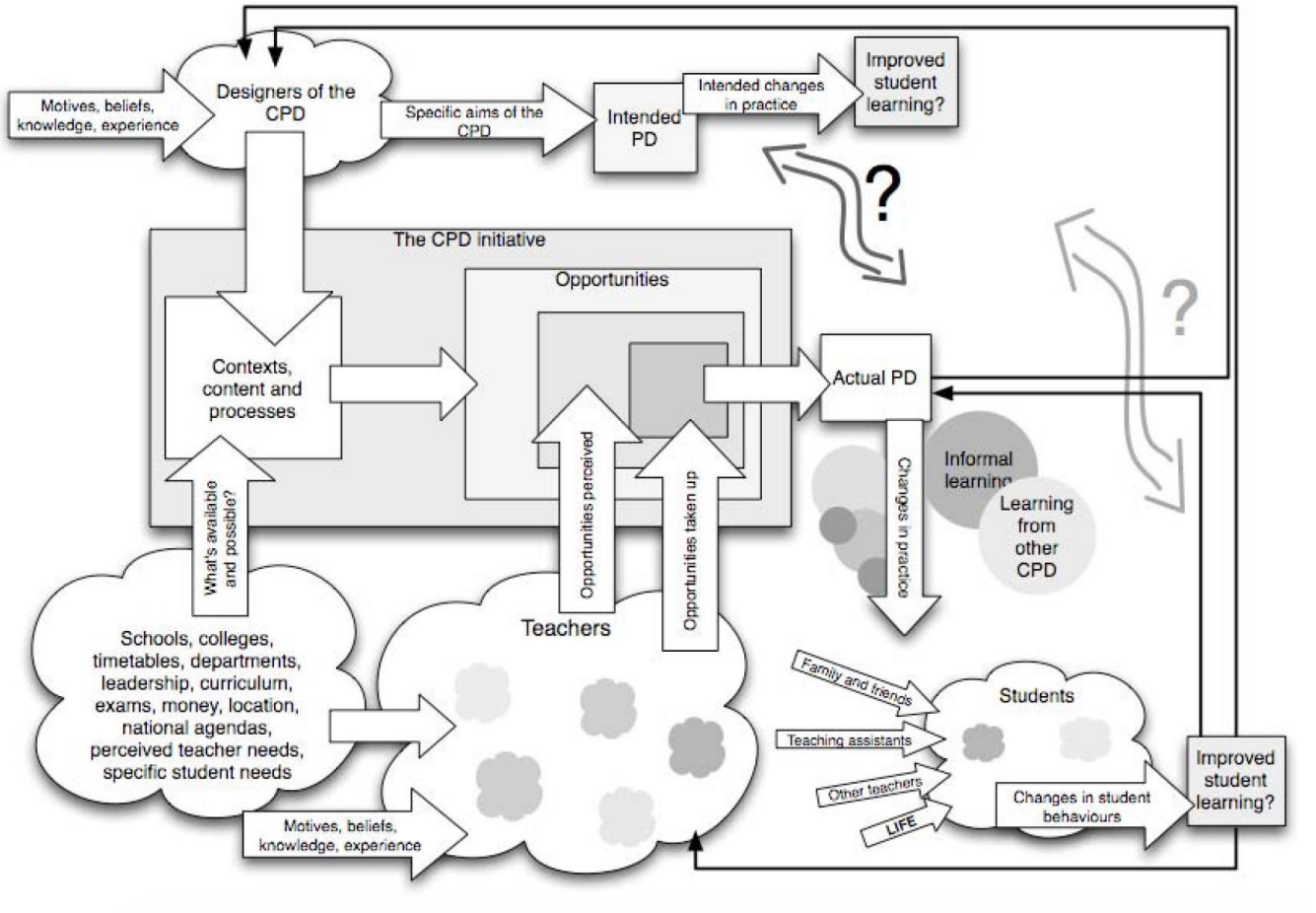

Figure 2. Joubert et al. (2010, p. 1163) model showing the relationship between Continuing Professional Development (CPD) and improved student learning

> As with many analytical frameworks, this representation could be seen as 'too neat', yet the data is messy and complex. Further, it is a static diagram which cannot represent the ways in which the nature of the CPD may be dynamic and changing in response to feedback from teachers and their changing needs over time. (p. 1763)

Discussions of different models showing the relationship between PD, teacher learning and consequent student achievement suggests that the interaction of contextual features affects the outcomes from the PD and this complexity is difficult to incorporate into a static model. This is because the relationship between components changes as the PD progresses, making it difficult to predict what should be in focus at any particular moment.

As a result of these concerns, Coburn (2003) highlighted the need for "new research designs better suited to capture this more complex vision" (p. 8). To do this, we suggest that there is a need for better understanding of the complicated relationship between professional

development and student outcomes. This is particularly important if differences in student achievement, correlated to certain demographics, are to be overcome (Flores, 2007).

As professional development facilitators, we considered that it was important not just to understand the teachers' background and needs, but also the context in which they worked and how these interacted. To do this, we conducted a case study of three teachers to investigate what affected individual teachers to make changes to their teaching, so that they could support students' writing in mathematics with the intention of improving their results. This paper examines the case of Kay through analysing her narratives about her involvement.

PD on Language in the Learning of Mathematics

The focus of the professional development was about language in mathematics, particularly about writing in mathematics. As Joubert et al. (2010) indicated in their framework, the choice of focus came predominantly from two sources: our previous research experiences on language in mathematics education (see Meaney, 2006; Meaney, Trinick, & Fairhall, 2012); and the school leadership and the teachers who identified literacy issues as contributing to students' poor test results. Similar to Jorgensen's (2015) point, there was general agreement that there is a need for teachers "to be aware of the language demands of mathematics if they are able to successfully transition speakers whose home language is different from school mathematics instruction into successful learning of mathematics" (p. 314). In this school, the students were transitioning from non-standard dialect into learning to use standard Australian English in learning mathematics. For many teachers, attending to language issues is often not part of their professional awareness and even when they recognise that there is a need to attend to it, they are uncertain how to do it. For example, Jackson and Gibbons (2014) noted that classroom practices which supported students' reasoning and justifying skills "are complex to support and develop, for both teachers and students" (p. 3). The school and teachers in our project identified language issues as being important and welcomed the possibility of gaining input on this.

In summarising research on what is needed for students to develop deep understandings of mathematics, Jackson and Gibbons (2014) identified that "students need regular opportunities to justify, prove, and debate the accuracy of solutions and to compare solutions in an effort to identify mathematical connections between them" (p. 3). For this to happen, students need skills and fluency for discussing mathematical ideas and this usually requires a teacher to support them to gain these. If this support is not provided, then students may struggle to learn mathematics. Prediger and Krägeloh (2016) stated, "large scale studies show that many multilingual students and monolingual underprivileged students experience substantial language barriers resulting in limited school success and in particular achievement in mathematics" (p. 89).

Barriers can occur because of differences between the students' everyday communicative language and the standard academic language needed to participate in discussions about abstract mathematical ideas (Mushin, Gardner, & Munro, 2013). They also may arise from societal expectations about the potential of these students to learn mathematics, particularly

in regard to how fluency in the language of instruction may affect their learning (Svensson, Meaney, & Norén, 2014). Regardless of the reasons for the barriers, there is evidence that better understanding of the role that language plays can lead to improved student achievement in mathematics (Jorgensen, 2015). For example, one of Jorgensen's findings was that careful scaffolding by teachers and assistant teachers of the language requirements for working with mathematics contributed to the sustained provision of student learning opportunities.

Teaching and learning the mathematics register

The language used to work with mathematics has been labelled, the mathematics register (Meaney, 2005). Halliday (1978) stated:

> We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

Mathematical vocabulary is only a small part of the mathematical register, with grammatical structures being more important as they provide students with the logical structures needed to express the relationship between mathematical ideas (Meaney et al., 2012). Logical structures are important in interpreting typical mathematical learning tasks, such as word problems, and in producing acceptable responses. One way of supporting students to understand the structure of word problems is to have them write their own as this can raise issues, to do with interpreting word problems. For example, in a study of 509 Year 6 and 7 students' posing of mathematical problems, Silver and Cai (1996) found that 20 percent of the responses were statements rather than questions. In their analysis, 40 percent of students generated less than 20 percent of the mathematical questions. This suggests that many children have difficulty with the aspect of the mathematics register to do with structuring mathematical problems, which is likely to result in students struggling with interpreting word problems.

The PD program was based on previous work, where improvements in students' ability to explain and justify their mathematical understandings had occurred (Meaney et al., 2012). In that work, we had used the Mathematics Register Acquisition model (MRA) to raise teachers' awareness of the kind of scaffolding that students needed at different points when learning new aspects of the mathematics register. In each of the four steps, the contributions of both the teachers and the students to the learning is described (see Figure 3). The MRA model illustrates how students should gain increasing control over their production of new aspects of the mathematics register. As discussed in Meaney (2006), the MRA model uses understandings from second language acquisition to consider how teachers' scaffold students' learning of different aspects of the mathematics register. In Meaney et al. (2012), a large number of recorded lessons from a range of classes, across 11 school years, were analysed to identify the strategies used in teaching oral and written mathematical language. It was found that the teachers' strategies did not differ with the students' age but changed as a new topic was introduced and consolidated. This research also found that although teachers provided a range of scaffolding strategies connected to the first two stages of the

MRA model, they did not provide as many opportunities for students to take control of their language use, as required in the final two stages. This awareness provided the opportunity to work with the teachers to develop strategies that supported the students to work more with the last two stages of the MRA model.

For the PD with Kay, we designed it so that it combined a focus on writing in mathematics with an awareness of how to scaffold students' learning of the mathematics register. We considered that this had the best possibilities for supporting teachers to change their practices so that students had increased opportunities to work mathematically.

In this paper, we examine Kay's involvement in the PD. Her focus came to be on children's posing and responding to word problems. As noted in a later section, Kay's students seemed to increase their understanding of the structure of word problems as well as how to present their ideas orally to their classmates. However, this change in student outcomes does not indicate that Kay's involvement in the PD was straightforward. Our analysis describes how aspects of the messiness around her acceptance of input and her adoption of new practices, at different times supported or hindered their implementation and thus the possibilities for students to learn mathematics.

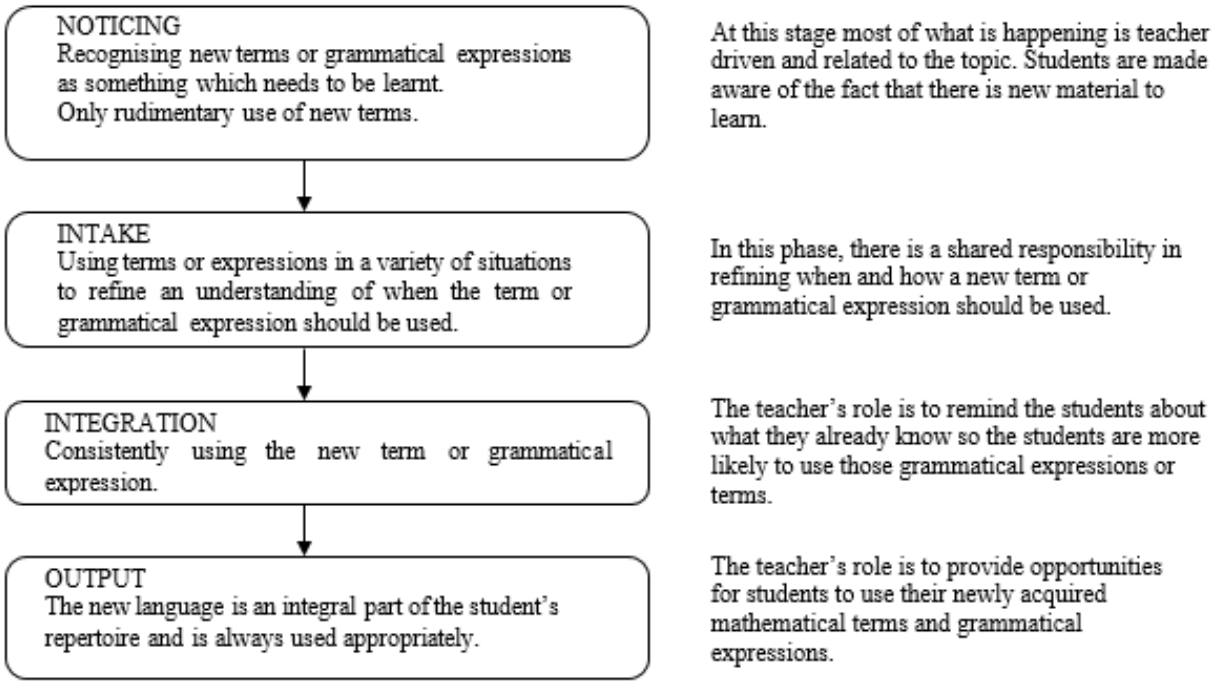

Figure 3. Mathematics Register Acquisition model (adapted from Meaney, 2006)

METHODOLOGY

The research on Kay is a case study, in that the events discussed are bounded by Kay's involvement in the professional development, both temporally and physically (Cohen, Manion, & Morrison, 2000). Hitchcock and Hughes (1995) identified the defining features of a case study as:

- It is concerned with a rich and vivid description of events relevant to the case.
- It provides a chronological narrative of events relevant to the case.
- It blends a description of events with the analysis of them.
- It focuses on individual actors or groups of actors, and seeks to understand their perceptions of events.
- It highlights specific events that are relevant to the case.
- The researcher is integrally involved in the case.
- An attempt is made to portray the richness of the case in writing up the report. (p. 317)

In the next sub-sections, we show how this project incorporated these features into the design of the study.

The Data

In order to gain the rich description of Kay´s involvement in the PD, we collected data in a range of ways. These included: the initial and final interviews of Kay (about half an hour each); initial and final group interview of her students (about 20 mins each); video recordings of four lessons (between 30 minutes to an hour for each lesson); audio recordings of Kay and Tamsin's discussion of the lessons made while watching the video recordings the following day (about an hour to an hour and a half for each meeting); and audio recordings of the five weekly meetings between the teachers and researchers (about an hour and half for each meeting). As well, student work samples were collected.

Data Analysis

The data was analysed in two ways. The first identified whether the students' use of the mathematical register had improved, by comparing how the students used language to describe their mathematical work in the first and fourth videoed lesson. The differences in topic and tasks did not allow for a systematic analysis of language differences, so the results are discussed in very broad terms and from Kay's perspective. Guskey's (2002) model suggests that teachers' attitudes and beliefs change only after they have identified improvements in students' learning outcomes. Therefore, it was important to identify whether Kay saw improvements in her students' learning outcomes and how this seemed to affect her attitudes and beliefs about language learning in mathematics education. In a case study, it is important to gain the main actor's perspective on events.

In case studies, it is important to identify key events. Hence, the second analysis determined what seemed to affect Kay's possibilities for changing her practices. It consisted of first identifying factors that appeared continuously over time in Kay's narratives and which seemed to influence her reflections on the tasks that she trialled in her lessons.

In doing these analyses, we were inspired by narrative enquiry, a methodology that has been much used in teacher education (Clandinin, Pushor, & Orr, 2007). As Connelly and Clandinin (2006) stated "narrative inquiry, the study of experience as story, then, is first and foremost a way of thinking about experience" (p. 375). In particular, we have used Clandinin et al.'s (2007) three commonplaces in regard to narrative inquiry research: temporality, sociality and place. Temporality recognises that events never just happen but that participants' future, present and past affect the events, which should be considered to be in transition. Across the PD, both previous and future events were described differently by participants at different times. We, therefore, identified when Kay seemed to tell different stories about the same event and what influence those differences. Sociality includes "environment, surrounding factors and forces, people and otherwise, that form each individual's context" (p. 23), including the relationships between participants and researchers. In case studies, it is important for researchers to acknowledge their own participation. As the PD facilitators, acknowledging our role in wanting to find out what was occurring was important. However, it was also important to see how Kay made use of her professional relationships in the narratives that she told. The commonplace of place was about the importance of where the PD was occurring. The context is described in the next section. In our analysis, we looked for how Kay discussed different aspects of the situation in which she was working. By iteratively enquiring into the data across these three commonplaces, we were able to identify the contextual features that affected Kay's possibilities for adopting different aspects of the PD.

The Context

The school where Kay worked was in a regional centre of New South Wales and serviced a low socio-economic population. It had a high Indigenous population as well as children from defence service families, which contributed to a turnover of up to sixty percent of students during the year. The students' poor academic results in national tests meant that the school received funding for teachers to attend PD. However, within a context of ongoing political discussion about what to do with schools that failed to show improvements, a non-negotiable result of the professional development was that national test results had to improve (see Lange & Meaney, 2013). The school funded the teachers' release time to participate in the professional development project that we offered. Our university at the time funded us to conduct a research project to identify the aspects of the PD, which supported or hindered the teachers to change their practices. The project began in September and finished in November 2009.

Unlike the other two teachers in the project who were full-time, permanent staff members, Kay was employed on a part-time, casual basis, from funding given to the school because of their poor test results. This use of short-term funding was common in Australia and so Kay's experiences provide insights into a group of teachers who carried the responsibility for

improving test results, but whose work is under-researched. In the interview before the project began, Kay described that as a casual teacher she was not usually considered eligible for PD as the permanent staff's needs had priority. She felt able to volunteer for our project when few other teachers wanted to participate.

Kay worked with a group of six children who were withdrawn from a multi-Year 3/4/5 class because they were identified as likely to do poorly on national tests. In her initial interview, she described her aim as altering the children's attitude from not enjoying mathematics, by tailoring her teaching to match their learning needs. She felt able to do this because she was not restricted by the syllabus for the school, which required teachers to teach topics according to a pre-determined schedule. Nevertheless, she considered important the long-term needs of the students, which included being able to use mathematics in high school and to function in society. From her perspective, the students needed short, hands-on lessons to match their attention span, and which were relevant to their lives. In a discussion during one of the PD meetings, she described how the Indigenous students, who were the majority of students in her group, particularly needed hands-on lessons.

She considered that teachers at the school would view working on writing in mathematics as difficult because of the children's literacy problems. She considered that it was possible to do this if the children heard mathematical language, as a first step to writing it.

In the initial interview, the children in the group verified the importance of language issues in mathematics by stating that in high school they would get hard stuff and the teacher would not read the question for them – "you've got to learn how to read yourself, and you have to figure it out yourself" (Student focus group interview, 15/09/2009). Thus, it seemed that the teacher and the students were in agreement that language issues were important in mathematics learning.

The PD

The three teachers participating in the PD were all working with different ages of children and taught different topics. Focusing on writing provided opportunities to discuss common aspects, but individualise the writing tasks to the different classes. To support the tailoring of the PD, we introduced Timperley, Wilson, Barrar, and Fung (2007) teacher inquiry and knowledge building cycle, shown in Figure 4 in the first group meeting. We anticipated that this would enable us to "build on what teachers already know, taking into account the voice of the teacher" (Joubert & Sutherland, 2009, p. 28). Using such a model also appeared to be in alignment with conducting a case study as it contributed to the teachers providing input about their perception of events (Hitchcock & Hughes, 1995).

After the teachers had agreed to participate, the teachers were provided with a copy of Meaney (2006) in which the MRA model was described. At the initial meeting, we discussed what quality writing in mathematics might be, which included making students aware of how to combine sentences with diagrams in order to explain their thinking through writing, while also considering different audiences for their writing. This discussion was followed by a discussion of the MRA model and the reading that the teachers had been given. Tamsin described the importance of the MRA model as "a meta language for teachers to be able to talk about what people were doing" (10 September 2009).

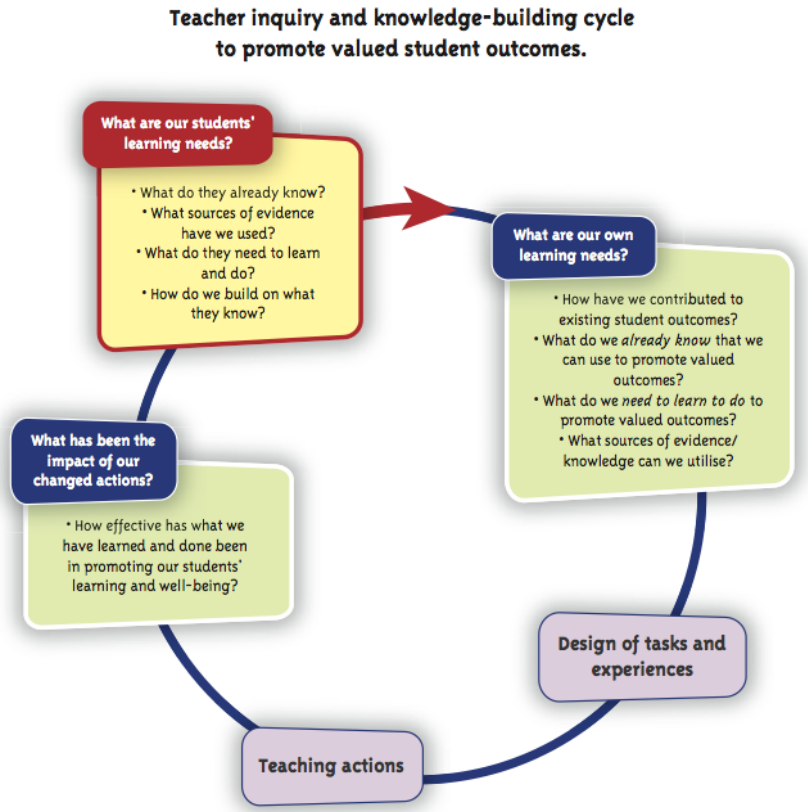

Figure 4. Teacher inquiry and knowledge building cycle from Timperley et al. (2007).

In order to support teacher reflections on their practices as required by the teacher inquiry model (Timperley et al., 2007), we video recorded a lesson from each teacher for four weeks and Tamsin discussed the lessons with each teacher individually on the following day. This approach had been viewed by the teachers in earlier research projects as being valuable in supporting their learning, as it involved them having to reflect deeply on their own teaching (Meaney et al., 2012). In the weekly group meetings, the teachers were expected to discuss, but not show, their lesson as we considered that showing their videos to other teachers may have been too confronting (Meaney et al., 2012; van Es, Tunney, Goldsmith, & Seago, 2014). Certainly, the requirement to be filmed reduced the interest of teachers in the school to participate in the PD.

In the weekly group meetings, the teachers had to describe what had happened that week, particularly in the recorded lesson. This supported them to discuss the steps in the teacher inquiry and knowledge building cycle (Timperley et al., 2007). The teachers discussed what they had done, not just in terms of whether the students had improved their writing in mathematics and how, but in regard to what they, the teachers, wanted to develop in future lessons. These discussions allowed us, as PD facilitators, as well as the other teachers to offer

suggestions about possible alternative teaching practices and to discuss the purposes for writing in the learning of mathematical ideas. It was during the group meeting, after the third set of filming, that Tamsin suggested having students write problems as a way of helping them understand the structure of word problems and, thus, be able to respond more appropriately to them. This suggestion had arisen from Kay's frustration with her students answering word problems.

Success of What Kind?

The first analysis was to determine if the students' mathematical writing was considered by Kay to have improved over the PD project. Identifying how Kay saw the relationship between what the students could do and the activities that she had implemented on the first and last filming days was part of the teacher inquiry model. This analysis contributed to understanding whether Kay considered that changing her teaching practices was effective in supporting students' learning.

In the first videoed lesson, Kay asked the students to provide a title to a mathematical game about rounding numbers to the nearest ten that they had played and describe the rules for it. Kay's idea was to model writing by writing the students' ideas on a flip-board. Although Kay felt that the children enjoyed playing the game, providing a title was difficult. In the joint meeting following the lesson, she said "they really struggled on, even the title, like, well what will we call it. So, we eventually got that" (17 September 2009). However, in her comments from the day following the video recording (15 September 2009), she acknowledged that the students seemed unaware of the needs of an audience who knew nothing about the game. In the video of the lesson, Kay channelled the children into providing a title that included "rounding", which was the mathematical skill that they had practiced. However, it is not clear if Kay, in stating the students "got that", meant that they did eventually offer what she felt was necessary in the title or that the students actually realised what an audience would need in order for the game title to make sense to them.

In the joint meeting, when she discussed what she would focus on the following week, she told a story about where the group had been when she first began to work with them, how far they had developed and what she wanted them to develop next:

> I said something like write a sentence about the picture or something, they wouldn't even pick up their pens. … We don't know how to spell it … and basically flatly refused. So, I've got them to the stage where they will give me some sort of written stuff, and I'm sort of wanting to move on to giving them something to work on and going away and thinking what are the different ways I can present that? … Produce that by themselves, like the maths, working mathematically, and then writing. Rather than just relying on the teacher all the time. Because they've got the ideas, haven't they? The ideas are there. It's just getting the confidence to write about it. (17 September 2009)

Kay's perceptions of the students' struggle with producing an appropriate title is placed in a chronological narrative of events (Hitchcock & Hughes, 1995), in which she described the students as gradually taking on more responsibility for their writing. Although it may be

somewhat naïve to consider that all that the students needed was more confidence, Kay's acknowledgement that students should be able to write mathematics independently was in alignment with the MRA model (Meaney, 2006).

This need for the students to become independent writers of mathematics who were not reliant of the teacher for input was also present in Kay's discussion of the final videoed lesson, but in this case she could use the lesson itself to indicate that the students had become more independent writers. Figure 5 shows the problems that the children wrote and how the other pair of students worked out their solution to it. In the final joint meeting, Kay stated:

> So we did a 'writing your own problem' which is what you've [Tamsin] been talking about, so I thought that's a really good idea so we'll have a whack at that now and see how it goes … we were really, really happy with the result, like they followed the format of what to do next and they wrote the algorithm.
>
> Yeah we [Kay and Tamsin] were just blown away basically both of us … it went much better than I thought it would … they came back and shared again and they had to explain how they actually got the answer – how did I write the algorithm, how did I write the answer and actually present the picture and the work. (30 October 2009)

From Kay's perspective, the activity of having the students write their own problem, which had been suggested by Tamsin in the previous joint meeting, was a success. She could see that they had used the model she had provided for constructing and solving their own problems. This kind of problem posing is known as "presolution posing" in that a stimulus, in this case the numbers, is provided to the students who then pose a problem based on that stimulus (Silver & Cai, 1996). The students were able to present first the problem and then the solution to each other, where they explained some of their reasoning about what they did. Although these students did not reach the level of reasoning and justifying advocated by Jackson and Gibbons (2014), which would contribute to them gaining deep mathematical understandings, from Kay's perspective the students were more willing to use language to

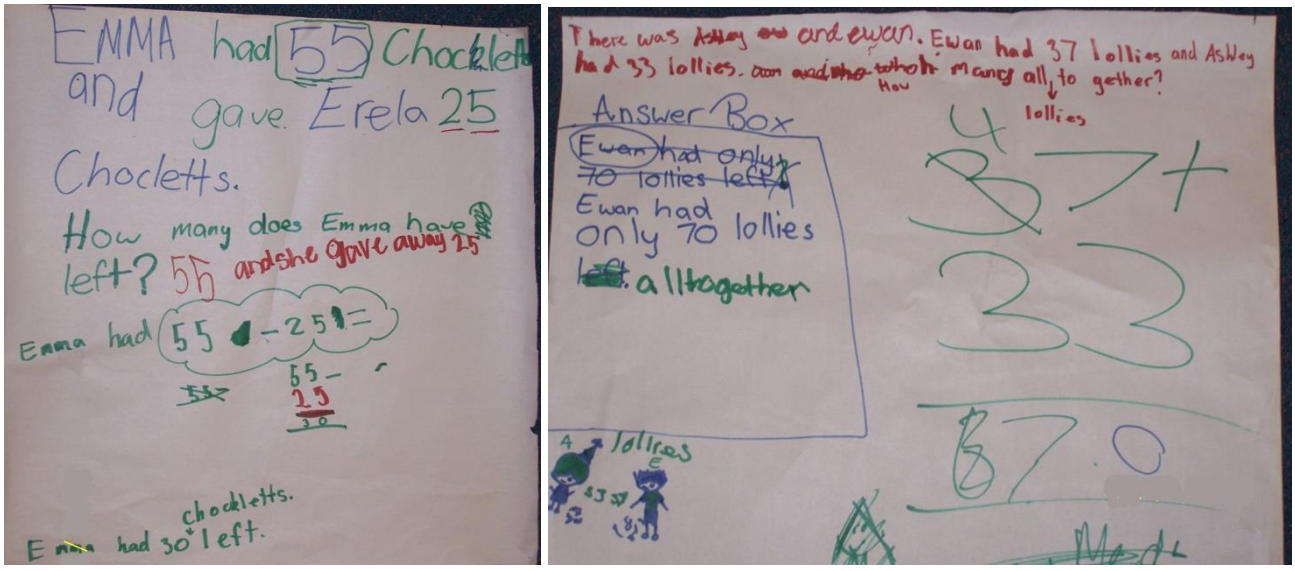

Figure 5. The problems and solutions of the two pairs of students

discuss mathematical.

Nevertheless, the question needs to be asked whether the increase in student achievement was because the students had gained skills that they did not have already or because the teacher had changed her practices, which allowed the students to show what they could do. As is discussed in the next sections, it was likely a combination of these that produced the student performance in the fourth lesson.

Clandinin et al.'s (2007) commonplaces of temporality, sociality and place can be seen in Kay´s stories about the two videoed lessons. Temporality was important in that it helped Kay to place her work with these students in an ongoing project. She related her teaching practices both to past experiences with these students and to future ones that she would like them to have. This can be seen specifically in the story from the first videoed lesson, but is also implicitly present when she referred to the suggestion for having the students pose their own problems as arising from a previous group meeting. Sociality was also important because Kay's relationship with the students is at the heart of her stories. In describing both the first and fourth videoed lesson, she situated the students as knowledgeable. Timperley et al.'s (2007) model (Figure 4) clearly indicates that knowing what students can do needs to be the basis for further teaching. Although perhaps naïve in her evaluation that the students just needed more confidence, she did adjust the activities she provided based on her reflections on previous experiences with this group of students. Kay's story about the students' success with the problem posing indicates that the relationship with us, as the PD facilitators, helped her identify alternative teaching approaches. Place also influenced the stories in that they were situated within a school environment, where there were certain expectations of teachers and students. As is discussed in the next section, identifying how Kay wove these commonplaces into her stories provided insights into the factors that affected her possibilities for changing her teaching to increase the students' mathematics results.

Factors Affecting Kay Changing Her Teaching Practices

In our second analysis, we identified four factors that interacted together to support or hinder Kay's possibilities for changing her practices. These were: Kay's beliefs about the need for children to be successful; the suggestions offered for alternative actions by the professional development facilitators; the responses of the students to the activities; and Kay's situating of herself as a good teacher. Describing the four components provides an analytical simplification of the messiness of being in a PD project, while still contributing to seeing the process as complex.

All four factors can be linked to aspects of Joubert et al.'s (2010) model (Figure 2) which deal with the teachers' motives, beliefs, knowledge and experiences. Clandinin et al.'s (2007) commonplace of sociality can be seen in all four factors as they are related to the sociality of the environment in which Kay was operating, her relationships to students, her colleagues and us as PD facilitators, who tried to enlarge the possibilities Kay saw for action. However, Joubert et al.'s (2010) model does not show the dynamic nature of how participating in the PD and trialling new tasks was affected by the context in which Kay worked. Although Putnam and Borko (2000) suggested that the context of the classroom contributes to teachers

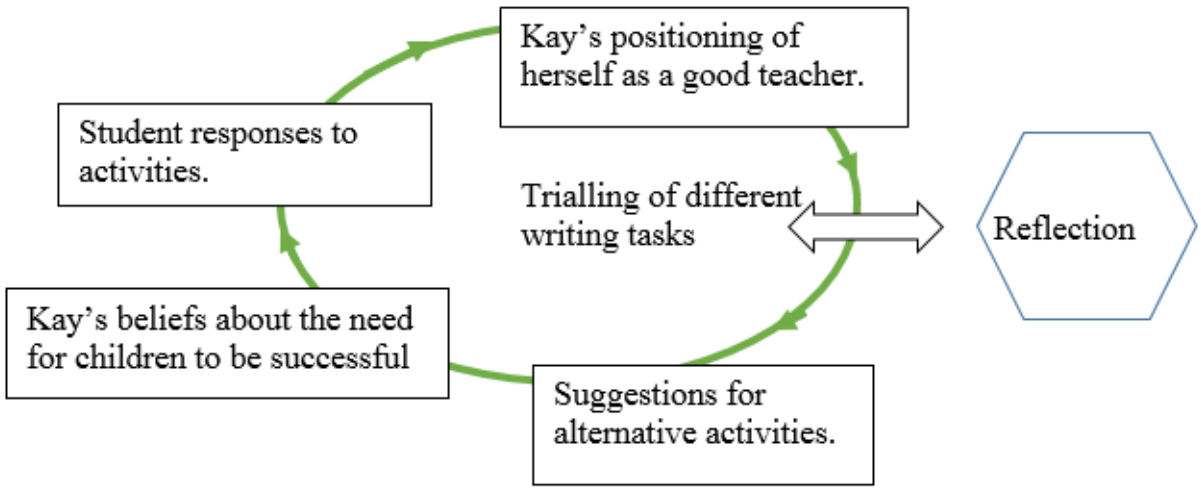

Figure 6. Reflection connected to trialling of different writing tasks through 4 factors

developing views about what they can do and these views are resistant to reflection and, therefore, also to change, we considered that the factors that affected Kay's reflections were connected to the trialling of the different writing tasks and thus connected to changes in her practices. Figure 6, although still a static representation, is an attempt to represent the dynamic nature of the relationships, connected to Kay's actions of reflecting and trialling of different writing tasks. In the next sections, we describe each of these factors using data from Kay's narratives, before providing a description of how the factors blended together in the second lesson.

Kay's beliefs about the need for children to be successful

Teacher beliefs were a component of all of the models discussed in the earlier section (Clarke & Hollingsworth, 2002; Guskey, 2002; Joubert et al., 2010). In these models, changing teacher beliefs were connected to changing teacher practices. In contrast, from the initial to final interviews, Kay reiterated a consistent set of beliefs about ensuring that children were successful. From her perspective, the students mostly hated mathematics because they were not good at it and her role was to change their view, by ensuring that they were successful. In the initial interview, as well as mentioning the children's literacy problems, she stated:

> Most of the children I'm teaching are finding difficulty with mathematics, so their attitude generally is that they don't enjoy it and that they don't want to actually do any of it at all, they find it very difficult, they find it boring, they find that they can't keep up in the classrooms, so that a lot of the time they may become behaviour problems because they're struggling with it all, I don't think that they understand

> that if I do that now that will help me later on in my life, I don't think they see it as relevant to their life …
>
> In the small groups the children are within reason of about the same ability, so if I can aim it at that and I think they have that sense of okay it's not so bad I can actually do some of this stuff. (Initial interview)

From Jackson and Gibbons' (2014) perspective, these comments include a mixture of both productive and unproductive views of learning mathematics. On the one hand, Kay positioned herself as the person who could affect students' learning. On the other hand, the students are ascribed an attitude which situated them as being responsible when they did not learn.

The need to have her students succeed contributed to Kay trialling out what Jackson and Gibbons (2014) would consider productive and unproductive instructional actions. An example of Kay's productive instructional actions was when she suggested, while watching the video of the final lesson, that the students needed to have control of mathematising the problem. She stated "I wanted them to be able to get to the algorithm and use what they've been taught in solving the problem without the teacher, which they did and I was really happy with that". This was in alignment with our promotion of the MRA model to support teachers to gradually remove their input, so the students could take control of their writing in mathematics and, therefore, their learning (Meaney, 2006).

Nonetheless, the need for children to experience success also led Kay to adopt unproductive instructional actions. As Jackson and Gibbons (2014) noted, teacher actions such as these tended to reduce the cognitive demands of the task. For example, in the first lesson when Kay was trying to have the students come up with a suitable title for the rounding game, she ended up sounding out the start of words she was expecting, such as "rounding", so that all the children had to do was to provide the final part of the word to be successful. By adopting this strategy, she seemed to focus her teaching on the first phase of the MRA model, Noticing, by highlighting for the students the importance of the term "rounding". If the children had been able to use this term appropriately in the title for the game as requested by Kay, they would either be acting in phase 3 or 4 of the MRA model. By instituting the phase 1 activity of having the children fill in the name, following her heavy prompts, Kay may have divorced the meaning of rounding, experienced in the game, from the term being used to describe it.

The need for the children to succeed also lured Kay into trying to produce a set of procedures for students to follow so they could be successful. For example, in discussing the video of the third lesson, Kay stated:

> What I'm trying to do with them is keep them in a structured way so that when they get out of the classroom they can carry that across, they've got to know: first thing; second thing; what do I do next?; where do I go next?; where do I write it?; what order do I do all that in. (27 October, 2009)

Although the intention was to have the children be in control of the process, this approach often did not support the children's mathematical writing because the children did not

understand the purpose of each step. As described in a later section, engagement in PD on writing in mathematics resulted in her rethinking the appropriateness of having the students follow a lock-step series of procedures.

Suggestions offered by PD facilitators

Timperley et al.'s (2007) teacher inquiry model (see Figure) requires that the teachers clarify their own learning needs. Therefore, in the first joint meeting before filming began, all the teachers were asked to do this. Kay considered her involvement to be "more about developing my teaching style rather than what I actually want the kids to gain from this" (First joint meeting, 10 September, 2009). However, as she began to reflect on her lessons and share her understandings in the joint meetings, she started to describe her aims for the students. For example, she stated in the third joint meeting that her aim for the students was to write responses to word problems using complete sentences:

> The other thing I've been trying to get them to do is actually answer the question, like write a sentence about the answer … they just wrote the question again, like when I said, write the answer. (1 October 2009)

Once she described her goals for the students' mathematical writing, it was possible for us as PD facilitators to offer suggestions for how she could support the students, both in the sessions discussing individual lessons and in the group meetings. These suggestions, such as having the students write their own word problems, were open-ended and required the teachers to determine for themselves how to implement them. They also provided opportunities for Kay to offer different kinds of scaffolding, in alignment with the four phases of the MRA model. At the end of the project, Kay commented that the PD was not just about listening to suggestions from the facilitators but being expected to implement them.

> I think this PD is more hands on. It forces you into looking at okay what sort of lesson am I going to do, how am I going to structure that, how am I going to plan for it, how am I going to critique it and what am I going to do with that information after, whereas the normal PD is just go and watch and then you may or may not get to actually put any of those things into your actual classroom or to share it. (Final interview)

Kay took our suggestions and also ones gained from listening to the reflections of the other teachers in the group meeting and incorporated them into her planning. This is in alignment with her original aim about improving her teaching style (September 10 2009), which was raised again as a concern when watching the first video (September 17 2009), but moved on to considering explicitly the opportunities that she could make available for students' writing. In so doing, she followed the steps of Timperley et al.'s (2007) model (Figure 4) by basing her planning on what she considered the students needed in order to improve their writing in mathematics. She was able to use her reflections, on past and present events to do with the students' mathematics writing, to consider how and why she adopted new teaching practices to support students to gain more control over their use of different aspects of the

mathematics register. Her reflections seemed to contribute to her perceiving what was offered in the PD as something that she could and should try with her students (Joubert et al., 2010).

The responses of the students to the activities

Kay's relationship with her students also affected the environment in which she operated and these were evident in the narratives that she told about being in the PD. In Guskey's (2002) model (Figure 1) but also in Joubert et al.'s (2010) (Figure 2) model, changes in student outcomes led to changes in teacher attitudes and beliefs. In contrast in Kay's case, it was often what she perceived that the students could not do which caused her to think about her teaching practices. For example, in watching the third lesson with Tamsin, Kay commented on the students' difficulties with interpreting numbers, which she considered to be a language issue:

> Those students need the same language and repetition from the teacher, so they understand what you're asking, 3 tens, what is it really and they didn't know. We went back and we built it, there it is. What is it really? (27 October 2009)

In discussing the student's actions, she often described what she considered to be alternative actions that the students should be able to do. In this case, Kay decided to provide intake activities, phase 2 of the MRA model, about interpreting the place value of different numerals. Sometimes, as a result of seeing how the students responded to a lesson, she made changes while teaching:

> When I actually planned the lesson, this whole bit in it was not planned. I hadn't actually thought of doing it as a small group first, then as I was teaching I thought, hang on a minute, they hadn't actually been asked to do this before. I don't want to whack them into it and see what happens. Let's do a group example first, so that sort of happened as I was teaching. (3 November 2009)

Her reflection on the students' previous experiences combined with what she saw happening in the lesson made her adapt her teaching to better fit what she felt were the students' learning needs. In this example, the adaptation was in alignment with the third phase of the MRA model (Meaney, 2006) (Figure 3), in that the students were expected to know how to write mathematical problems, but by modelling it as a whole group activity first, she could remind them of the features that they should be paying attention to. However, as noted in the previous section, when the students were unsuccessful Kay's aim of ensuring that they were successful led her to adapt her teaching so that it restricted the students' possible behaviours, without necessarily providing them with a clear understanding of why it helped their writing in mathematics.

Kay's situating herself as a good teacher in discussions

The final factor that appeared consistently in Kay's narratives about the PD was her need to situate herself as a good teacher. In the narratives that she told, this provided information on the temporality, sociality and place commonplaces in which she operated. Needing to see

herself as a good teacher seemed to affect her willingness to evaluate her changes to teaching practices as required by the Timperley et al.'s (2007) model (Figure 4) and instead led her to blaming the students for their lack of learning. As identified in Joubert et al.'s (2010) (Figure 2) model, contextual aspects of the situation were likely to influence teachers' motivation as well as their attitudes and beliefs. The contextual features, such as her situation as a casual teacher, seemed to result in Kay being unwilling to show her uncertainty which affected how she adapted her teaching and evaluated new practices.

In the discussions about the videoed lessons and in the group meetings, Kay rationalised her decisions about adopting new activities so that she appeared as a good teacher. This rationalisation could be seen in how Kay changed from noting that she had asked the children the wrong question in the commentary on the first lesson, to describing the same situation in the joint meeting as an activity, which brought out a lot of language in the children. Her shifting of the narrative over time indicated how temporality provided opportunities to describe what had happened to different audiences at different points in time. This seemed to be because she did not have the same possibility for displaying her uncertainty as the other teachers in the PD. Kay´s uncertainty is in contrast to other research in which teachers gained confidence to change their teaching by discussing it in collaborative groups (Horn & Little, 2009) and with such reflection leading to growth (Day, 1999; Pitsoe & Maila, 2013). For example, Hardy and Rönnermann (2011) advocated professional development that included:

> A broader conception of education, involving robust, collaborative inquiry amongst teachers into their work, not only results in much more sustained and substantive student learning, but also leads to improved outcomes on more standardised measures of student assessment. (p. 464)

It may be that Kay's situation as a casual teacher, employed specifically to ensure that low-achieving students improved their mathematics achievement, meant that she felt unable to discuss her struggles with teaching writing in mathematics. The other two teachers were permanent staff who at times admitted that their videoed lessons were not successful and that they were responsibility for what occurred (see for example Lange & Meaney, 2012). Kay may have felt that if she showed too often what she could not do, it could affect whether her contract was renewed, which would have serious implications for her financial situation. This uncertainty seemed to result in her not being able to take advantage of the support that a network of teachers has been documented as providing (Coburn, Russell, Kaufman, & Stein, 2012).

However, one outcome of not being able to discuss problematic aspects of her teaching was that she was restricted to blaming the students. For example, in the third joint meeting, Kay said:

> I'm trying to get them to be able to answer me verbally and written, you know, and really they struggle and need to do it verbally, you know, I'm almost like, there's the answer, look at the bottom of the algorithm you've just got to write it, you know (laughing) but they still don't know that's where the answer is. … I think they're starting to get it but … (1 October 2009)

In the initial interview, Kay had distanced herself from the other teachers at the school who felt that the children's literacy problems made it inappropriate to teach writing in mathematics. Yet, when she failed to see the students being successful, she ended up blaming them. This blaming of students restricted Kay's possibilities for reflecting on her own teaching practices. In the example above, she did not recognised that the students needed to be scaffolded into identifying how the answer to the algorithm was also the answer to the word problem, a phase one activity on the MRA model, but instead seemed to expect them to be fluent in interpreting what they had in relationship to the answer to the word problem, phase 4 in the MRA model. Without having the possibility to reflect on the mismatch between her expectations and the level of support on the MRA model that the students' responses indicated they needed, it was more difficult for Kay to provide appropriate activities for the students.

Kay often situated Tamsin into her narratives as an independent evaluator of what she had done well. This can be seen in her description of the students' success in writing, solving and presenting problems in the fourth lesson that was provided earlier. The relationship between Kay and us, as the PD facilitators, was delicate. The professional relationship between teachers and facilitators is complicated by personal relationships as well as societal ones (Meaney, 2004). In order to work with teachers, it is necessary for facilitators to develop trusting relationships, which can only be based on good personal relationships. However in an insecure working environment, Kay used her developing personal relationship to show that an external evaluator supported her teaching approach. It is unlikely that Kay was conscious of situating herself as a good teacher and using us in the process. So although this seemed to affect her possibilities for reflecting on her own teaching, it was difficult to make her aware of how this was affecting her interactions with the children.

Reflection and Teacher Change

Kay's narratives illustrate the messiness of the relationship between PD and improving student outcomes that Joubert et al. (2010) described. Kay's reflections on the students engaging in the tasks she implemented were affected by the factors, outlined in the previous sections, sometimes in isolation and sometimes together. Day (1993) suggested that teacher reflection is often limited to planning or evaluating the actions that occur in a lesson. However as indicated previously, it seemed that Kay could reflect more broadly about what she wanted to achieve. In this section, we look at Kay's reflection in regard to the second lesson in which the children did not read and respond appropriately to word problems. The difficulties with the lesson provided Kay with much to reflect on. However, not all of this reflection contributed to her changing her practices or to improved student outcomes.

In the joint meeting after the second videoed lesson, Kay described what had happened:

> What do you do when you're trying to solve a problem, because I've been doing lots of lead up into dissecting the problem, how do you read it, how do you get the numbers out of it, so we did that first, and actually did like a flow chart, I guess, on the board. We're going to read it twice, we're going to look at the numbers, we're going to look at the question, we're going to look at, what does the question want us

> to do, what are we going to do, we're going to draw it, we're going to write an algorithm. … Then we split off into groups and the problems were probably harder, well, they were harder than I'd been doing before with them. Because I've been wanting to keep it really simple, so they could learn process, rather than be challenged by the maths of it, and so this was the first time they'd done something that was challenging and the first time that they'd worked in small groups by themselves, with just a partner. So a lot happened in a very short period of time and it showed me a lot about where the kids are at, and what parts of the process that they understand and what they had trouble with. And I think the main thing which was just emphasised again today, is that they don't understand the question, they can't decode the question, they can read the question, but then they can't, they don't know what it is that they want you to do. So they had all sorts of problems in different ways and the groups, one group just couldn't do it at all, one group went pretty well, and the other group got off on a tangent, and just couldn't get back to the original story. (1 October 2009)

In this description, Kay situates herself as a good teacher by describing how this lesson fitted her focus. When it did not go to plan, she blamed the children for not being able to complete the activities. In this quote, Kay's reflection stayed at the level of identifying who was to blame for why the lesson was not a success. Her reflections on the unproductive instructional techniques (Jackson & Gibbons, 2014) she had used were not shared with us, as facilitators, or the other teachers. She fell back on the normalising discourse around the children having literacy problems (Horn & Little, 2009). She did not seem to gain support from being part of a teacher network.

In the video of the lesson, the children followed the steps in the list, but seemed unsure why they did them. For example, one step was for them to find the numbers in the problem. The pair of students described by Kay as those who "got off on a tangent" worked on the following problem:

> Miss Butcher has chickens at home. If the chickens laid 2 eggs per day for a week, and Miss Butcher saved them up, how many eggs would she have?

Following Kay's list of steps, this pair of children identified the "2". The next step was to draw a picture, so the children drew some chickens. At this point Kay sat with them and went over the problem, repeating that the chickens laid two eggs on Monday, two eggs of Tuesday etc. The children asked how many chickens there were and eventually Kay stated that there were only two chickens, so they laid one egg each, every day. Kay then reminded them that the next step was to write an algorithm. When Kay moved to another group, the children counted the chickens that they had drawn and wrote a number sentence where they added the 12 chickens to the 2 eggs. Consequently, the answer that they arrived at was 14. When Kay realised what the children had done, she spent time trying to get them to see that their interpretation of the steps was wrong. In the discussion of the lesson with Tamsin, Kay stated:

> Done some work on key words like: how many altogether? What's the difference between? How many were left? The most common ones that you see. They still

> haven't got it, I've done some of but I know they haven't got it yet. (29 September 2009)

In this comment, Kay situated the students as having possibilities for learning by adding "yet" to her description. The productive view (Jackson & Gibbons, 2014) that students could be successful, one of the factors that influenced her reflection, gave Kay possibilities to considering different ways to move the students forward. In this case, the reflection allowed her to think about alternative actions.

In the two days between discussing the video with Tamsin and the joint meeting with the other teachers and researchers, Kay had the children redo the problem with the chickens, focussing on different representations and their connections to the meaning of the word problem. The solutions were brought to the joint meeting (see Figure 7) and allowed Kay to talk about the difficulties she was facing, while also showing that her students had ultimately been successful. She could then receive suggestions for alternative practices, while still appearing to be a good teacher. It gave her a small space to take on the "experimentation, risk taking, and reflection required to transform practice" (Putnam & Borko, 2000, p. 10). Reflection on the lesson combined with the wish to be seen as successful led to Kay trying out new tasks with the students. Reflection on the outcomes of these tasks provided her with an opportunity to discuss both the difficulties with the original lesson and the success of the following lesson.

The focus of the professional development was on writing in mathematics. The complexity that Kay encountered when trying to support her students through modelling and scaffolding how to interpret and produce word problems made her reflect more generally

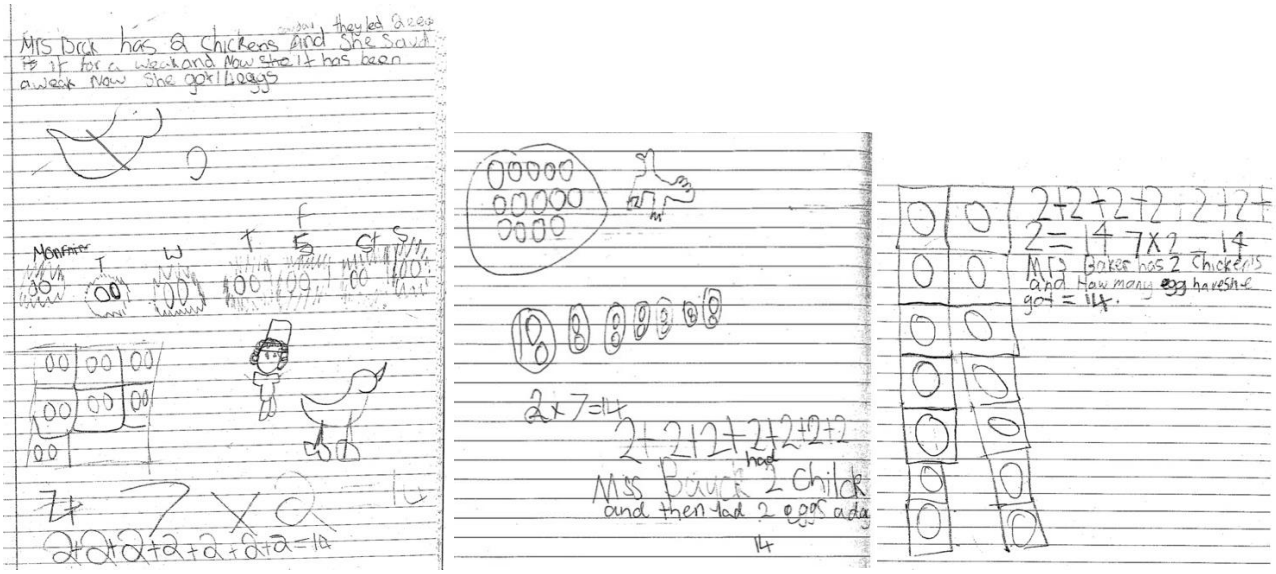

Figure 7. Children's solutions to the chicken problems

about mathematics and language learning.

> As I work with this group more, it's sort of like Pandora's box, just as you think of what I need to teach them, then that leads into other things they don't know and in order to teach them that and there's another. It's like unpacking a suitcase. (29 September 2009)

Thus the reflection that Kay engaged in about the trialling of the tasks was complicated. Her assumptions about the different aspects of writing and interpreting word problems were tested regularly as she found that the students had not noticed the importance of some term or expression and therefore were unable to use them meaningfully to make sense of what they were doing. This often forced her to reconsider what aspects of the word problems she should work on with the students.

The four factors operated together to affect what she reflected on and the outcomes of the reflection. Sometimes the reflection made her focus on specific incidents, where her need to be seen as a good teacher clashed with her aim for the students to be successful. If the aim for the students to be successful was at the fore, then suggestions from the PD were considered in regard to how the previous lesson could be improved. On the other hand, if the need to be seen a good teacher could not be achieved easily when discussing the results of an activity, then the students tended to be blamed and it was difficult for alternative practices to be suggested by us, as the facilitators, or to be adopted by her into the new lessons. However, Kay's reflection would sometimes give her a broader understanding about the teaching/learning of how to write and interpret word problems. When this happened the factors seemed to align in a positive manner. This then provided her with opportunities to take more risks with her teaching and not always expect students to experience immediate success.

CONCLUSION

In this case study, we investigated how one teacher, Kay, perceived her participation in a short PD programme. From Kay's narratives, it was possible to see how she reflected on the trialling of different tasks through the four factors of: her beliefs about the need for children to be successful; the suggestions offered for alternative actions by the professional development facilitators; the responses of the students to the activities; and her positioning of herself as a good teacher. As the PD facilitators, an understanding of Kay's reflections gave insights into why some of our suggestions were not adopted in regard to providing students with better opportunities to improve their writing and interpreting of mathematical problems.

Recognising the role of language in mathematics learning requires knowledge about how children learn to listen, speak, read and write mathematics (Meaney, 2006; Meaney et al., 2012). Although many of the teachers at this school considered that the students' poor literacy results meant that asking them to write in mathematics was not possible, Kay's work with the students suggests that both mathematics and literacy understanding can be improved when students engage in tasks that are meaningful for them. Kay found that accepting the complexity of "unpacking the suitcase" required her to deal with more than

one aspect of the students' learning at a time. Assuming that students had fluency in regard to one aspect of interpreting and writing word problems led Kay to become aware that students struggled with at least one other aspect that was important for solving word problems. Dealing with this complexity made her to some degree re-think what was involved in the teaching and learning of word problems.

From our perspective as PD facilitators, the research indicated that it might have been useful both to Kay and the other teachers in the project, if examples of difficulties were discussed in relationship to mismatches with the MRA model. For example, the circumstances in which Kay assumed the students were fluent when in fact they did not show they had even noticed essential aspects could have provided Kay with possibilities for re-structuring the activities during the lesson. This may have provided her with increased reflection possibilities.

Taking a broader perspective, there will always be teachers, like Kay, who because of a range of factors interacting together, may not produce the improvement in student results that large scale professional development often promises. Kay's status as a casual teacher who relied on being able to show that the students were increasing their possibilities for improved test results affected how she could interact with others. Yet, in countries such as Australia, funding specifically provided to improve students test results, generally goes to employing teachers on short-term contracts. Such teachers have reduce opportunities to indicate openly that they struggle with aspects of their teaching. In Kay's case, the complexity of the language issues connected to mathematical learning did seem to support deep reflection, even if she rarely discussed all aspects of that reflection in the group. The outcomes of this deeper reflection led to her trialling a variety of activities and reflecting on what was helpful about them for supporting students' learning.

This small study describes some interesting results, especially about how teachers who are often given the responsibility to raise students' test scores but who are not permanent staff need to situate themselves as good teachers. As was the case with Kay, this may make it difficult for teachers to take on PD suggestions about how to support students' mathematical learning. Further research could contribute to identifying how using models such as MRA can provide discussion starters for reflection on why students are only sometimes successful with learning how to use mathematical language. Although the MRA was the foundation for this PD project, it was not used explicitly to discuss why activities were successful or not. Further work should consider how this model could be used to raise discussions above individual experiences to reflect at a meta level about students learning of mathematical language.

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Revisiting Early Research on Early Language and Number Names

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ABSTRACT
This article addresses the relationship between language and mathematical thinking by reconsidering early work on language and number names. The analysis examines theoretical assumptions, later empirical data, and critiques of those early studies. Researchers, practitioners, and curriculum designers in mathematics education working in multilingual settings need to develop an updated view of this early work on number names across languages, carefully considering what early research actually showed, how it has been critiqued, and how to theoretically frame claims about language and mathematical thinking. The analysis presented here suggests several ways to frame such an updated perspective, including work on linguistic relativity and ecological approaches to the relationship between language and mathematical thinking.

Keywords: cross-cultural research, language, mathematical thinking, multilingual classrooms

INTRODUCTION

Integrating language into research on mathematics learning is an important goal for both practical and theoretical reasons. Although this integration is important for all learners, it is crucial for improving mathematics learning and teaching for students who are bilingual, multilingual, or learning the language of instruction. This integration is also relevant to theory: since research in mathematics education uses language to provide one window into thinking, the role of language is central in theorizing about mathematical thinking and learning.

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State of the literature

- Early empirical studies explored how the structure of number names in different languages impacts mathematical thinking.
- These studies generated multiple claims about what phenomena differences in the structure of number names explain, from differences in strategies young learners use for base-ten problems to overall achievement in mathematics beyond early counting and operations with whole numbers.
- Recent work provides updated views of early research and includes contextual factors.

Contribution of this paper to the literature

- The paper reconsiders early work on language and number names.
- The paper provides an updated perspective of this research, using work on linguistic relativity and ecological approaches to frame the relationship between language and mathematical thinking.
- The analysis explores the theoretical assumptions for early studies, considers later empirical data, and examines critiques of early work.

Addressing the relationship between language and mathematics learning presents several challenges. The most significant challenge is that research examining language and mathematics learning must be grounded not only in current theoretical perspectives of mathematics cognition and learning, but also in current views of language. This article examines early work on the relationship between language and mathematical thinking, in particular, how differences in the structure of number names impact mathematical thinking, in particular about place value and, more generally, mathematics achievement.

This work on language and mathematical thinking has generated or influenced a constellation of claims regarding how differences in the structure of number names in different languages impact mathematical thinking. These range from the specific claim that differences in the structure of number names can predict or explain differences in the strategies very young children use for base-ten problems, to more general claims about early mathematical thinking, learning, or understanding, and, in the extreme, to explain differences in overall achievement in mathematics beyond early counting and operations with whole numbers.

This article will examine the theoretical assumptions for such claims and consider whether current empirical data supports or contradicts them. In closing, the paper suggests alternative approaches to mathematics education research and practice in multilingual or cross-cultural settings based, not only on empirical findings, but also theoretically grounded assumptions about the relationship between language and mathematical thinking.

In order to provide a theoretical grounding, the paper uses work by Lucy (1996, 1997) on linguistic relativity. The main section of the paper provides a critical review of early studies that examined the relationship between language and thought in the context of mathematics by focusing on number names (e.g., Miura, Chang, & Okamoto, 1988). This early work is often cited when summarizing what we know about the relationship between language and

mathematics learning. For example, it was cited in a review of the research literature on mathematics learning in early childhood published by the National Academies Press (Cross, Woods, & Schweingruber, 2009). It is difficult to summarize the complex relationship between language and mathematics learning in a few sentences (or even a few paragraphs). Any summary of a set of research studies with complex and sometimes contradictory results can be misunderstood, especially if interpreted through a reductionist theoretical lens.

This work and early results are not to be simply accepted as fact, but, instead, have been critiqued and considered in light of later research and competing claims, results, and theoretical perspectives. Researchers have also considered some of the nuances involved in reaching conclusions based on this research. For example, Ng and Rao (2010) provided a thorough review and critique specifically on the research relevant to Chinese number words and mathematics learning. It is crucial that researchers, practitioners, and curriculum designers in mathematics education working in multilingual settings develop an updated view of the early work on number names across languages, carefully considering what early research actually showed, how it has been critiqued, and how to theoretically frame claims about language and mathematical thinking. The analysis presented here suggests some ways to frame such an updated perspective.

Theoretical Framing

The first theoretical issue to examine is the term language. Many commentaries on the role of academic language in teaching practice focus on the structure of language and ignore the functions of language or reduce the meaning of the term language to single words (vocabulary or lexicon) and the proper use of grammar (e.g., Cavanagh, 2005). In contrast, work on the language of academic disciplines provides a more complex view of mathematical language (e.g., Pimm, 1987) as not only specialized vocabulary (new words and new meanings for familiar words) but also as extended discourse that includes syntax and organization (Crowhurst, 1994), considers the functions of the mathematics register (Halliday, 1978), and includes broader sociocultural constructs such as Discourse practices (Moschkovich, 2007).

Theoretical positions in the research literature in mathematics education range from asserting that "mathematics is *a universal* language," to claiming that "mathematics is *a* language," to describing how mathematical language is problematic for learners. Rather than joining in these arguments to consider whether mathematics is a language or reducing language to single words, I use a sociolinguistic framework to frame this essay. From this theoretical perspective, language is a socio-cultural-historical activity not something that can either be mathematical or not, universal or not. I use the phrase "the language of mathematics" not to mean a list of vocabulary words or grammar rules but the communicative competence (Hymes, 1972) necessary and sufficient for competent participation in the mathematical Discourse practices of a variety of communities. I sometimes use the term "language(s)" to remind us that there is no pure unadulterated language and that all language is hybrid.

The relationship between language and thought has been a long-standing object of study. One perspective can be summarized by the Sapir-Whorf hypothesis, the conjecture that one's

thoughts are determined by the language one speaks. The strong version of this hypothesis would claim that all human thoughts and actions are constrained by language. The weaker (more accepted) version, sometimes referred to as linguistic relativity (Lucy, 1996, 1997), claims that language only somewhat shapes one's thinking. When reviewing and framing studies on linguistic relativity, Lucy distinguished between research projects interested in exploring the question of linguistic relativity and those that simply focus on providing accounts "for the noteworthy (often 'deficient') behavior at issue" (1996, p. 302). According to Lucy, if a study focuses on providing an account that addresses only noteworthy behavior, instead of considering more broadly the multiple and varied linguistic and cognitive phenomena of a community, that study does not explore issues of linguistic relativity. Also, if a study focuses only on deficient behavior, it should be suspect as providing evidence for linguistic relativity. This distinction seems fundamental for revisiting early work on number names.

According to Lucy, common defects of research that claims to address linguistic relativity "include" working within a single language, privileging the categories of one language or culture in comparative studies, dealing with a relatively marginal aspect of language (e.g., a small set of lexical items), and failing to provide direct evidence regarding individual cognition" (1997, p. 37). Instead, Lucy proposes that any study on the question of linguistic relativity should:

> 1) Distinguish between language and thought (examine outcome behavior that can be observed independently of language use),[1]
> 2) Elaborate the mechanisms by which language influences thought, and
> 3) Explore the extent to which other contextual factors affect the influence and the operations of those mechanisms (1996, p. 306).

I use these criteria to examine the set of studies claiming evidence for the effect of number names on young children's mathematical thinking, because these studies raise more general issues related to linguistic relativity (Lucy, 1996, 1997) and they have been cited repeatedly.
Below is one example taken from a volume produced by the National Research Council in the United States that summarizes research on mathematics learning in early childhood. Cross et al. (2009) cite work by Miura and others (Miura, 1987; Miura & Okamoto, 1989, 2003; Miura, Kim, Chang, & Okamoto,1988; and Miura, Okamoto, Kim Steere, & Fayol, 1993) and conclude:

> They have found that speakers of languages whose names are patterned after Chinese (including Korean and Japanese) are better able than speakers of English and other European languages to represent numbers using base-ten blocks and perform other place value tasks (p. 108).

Statements such as this can be misinterpreted, misunderstood, or used to reduce the complex relationship between language and mathematics learning.

Revisiting and Reframing Work on Language and Number Names

Early studies by Miura et al. have been used to support claims about how the structure of number names impact mathematical thinking, in particular about place value and, more generally, mathematics achievement. Miura et al. (1988) claimed to have provided evidence that the structure of number names in several languages (Korean, Chinese, and Japanese) makes it easier for young children to develop number concepts such as the base-ten structure. In these languages number names for the teens follow the structure of the base ten system: ten-one, ten-two, ten-three, and so on. In contrast, in English the names for 11-19 (and in Spanish for 11-15) do not correspond as transparently to the base-ten structure.

The results of these early studies have been critiqued, are contested, and can be interpreted in multiple ways. Later work critiqued the methods used, showed that children can be influenced by the instructions provided by interviewers, and found that differences disappear when changing from oral to written modes. A study by Towse and Saxton (1998) concluded that children's representations of numbers can be heavily influenced by experimental conditions and that the influence of language on the cognitive representation of number was less direct than had been suggested in earlier studies. Another study (Brysbaert, Fias, & Noel, 1998) examined differences between French and Dutch (in Dutch the order of tens and units is reversed). Although they found differences in naming latencies for the solutions to simple addition (two-digit plus one-digit numbers), these differences disappeared when participants were asked to type instead of say the answer.

How do these studies fare when using Lucy's framing to assess the defects and requirements for studies on linguistic relativity? Most studies worked across more than one language not within a single language, a common defect pointed out by Lucy. Whether any one study privileges the categories of one language or culture over the categories of other languages can be debated, and the conclusion depends on the nuances of how these categories are framed in each study. If a study uses nonverbal data to document mathematical thinking (or even to complement the analysis of verbal data), then it would fulfill Lucy's requirement that data "distinguish between language and thought and examine outcome behavior that can be observed independently of language use" (1996, p. 306). However, these studies clearly share two common defects described by Lucy, since they deal with a relatively marginal aspect of language (e.g., a small set of lexical items). Do the studies provide direct evidence regarding individual cognition? The critique provided by Towse and Saxton (1998) seems to address this defect, since one could argue that they fail to provide direct evidence regarding individual cognition. The results do not reflect individual cognition but instead individual cognition that is limited to mathematical thinking during a particular kind of interview context.

More recent studies provide further empirical evidence for the complex nature of the relationship between language and mathematical thinking, even for this particular small set of lexical items. Competencies in counting or arithmetic most likely consist of several components (Dowker, 1998), and some of these components are not linguistic, such as cardinality (Sarnecka & Carey, 2008) or using a number line for estimation (Muldoon, Simms, Towse, Menzies, & Yue, 2011). These competencies follow complex development paths (Dowker, 1998; Kimura, Wagner, & Barner, 2013; Muldoon et al., 2011; Sarnecka &

Carey, 2008). Even when findings suggest that a counting system can have some influence on arithmetic performance, the "effects tend to be limited to rather specific areas of arithmetic" (Dowker, Bala, & Lloyd, 2008, p. 536). Lastly, and perhaps most importantly for teaching, since studies have shown that cues provided by the interviewer can change those effects (Alsawaie, 2004; Towse & Saxton, 1998), any imagined or possible deficit for learning the base-ten system related to language structure can be addressed through instruction.

Miura (1987) also linked the differences in number names to students' mathematics scores. This study of American and Japanese children residing in the United States found evidence of a "differential cognitive organization of number resulting from differences in primary-language characteristics." These results may support the limited claim that the names for the numbers from 10 to 20 in three languages (Chinese, Korean, and Japanese) may make the base-ten system more transparent than the number names in non-Asian languages (English, Spanish).

However, the findings in these studies cannot be used as evidence for the much broader claim that Asian children's advantage in mathematical reasoning originates in or is caused by those linguistic differences. A jump from a limited claim about early counting to broad causal arguments about the mathematics achievement of Asian children disregards the ecological nature of mathematical reasoning and learning. Extending these studies to more general and broader explanations for differences in mathematics achievement use simplistic views of both culture and language.

These results only support the limited claim that the names for the numbers from 10 to 20 in Asian languages (Chinese, Korean, and Japanese) make the base-ten system more transparent than the number names in non-Asian languages (English, Spanish), and this claim holds only in particular circumstances. The findings in these studies cannot be used as evidence for the broader claim that Mandarin or Japanese linguistic differences cause, hinder, or facilitate the learning of mathematical notions beyond early understanding of place value or mathematics achievement in general. We cannot make broad claims about the role of language with respect to children who, in addition to using different languages, may also have had different school experiences, different mathematics instruction, as well as different experiences outside of school, such as after school training in using the abacus. There are important contextual aspects to include that impact learning about number, such as the effect of schooling, tutoring, out of school experiences with numbers, cultural differences in beliefs about the importance of mathematics, and parental instruction on numbers. These cannot be reduced to one single and over simplified factor such as the names for numbers.

Claims that mathematics achievement or success can be attributed causally to early facility with the names for numbers in different languages are suspect for many reasons. In general, claims that language differences are the single explanation for cognitive differences are suspect, based on research on cross-cultural cognition that describes the complex relationships between language, cultural practices, and thinking (i.e., Cole, 1996a, 1996b; Gay & Cole, 1967; Glick, 1975). In terms of Lucy's list of requirements, research would need to explore other contextual factors before making such claims.

Ng and Rao (2010) conducted an extensive review of the literature and critique of the research relevant to Chinese number words and mathematics learning that provides an

updated view of the research on this topic and includes other contextual factors. The review makes several important points. First, in particular for Chinese, NG and Rao (2010) report that Miller, Kelly, and Zhou (2005) "found that the influence of language on mathematics achievement was limited to the aspects of counting that involved number naming and the base-ten concepts" (Ng & Rao, p. 188). Second, "whereas number word systems and the application of more sophisticated strategies have a significant role in accounting for early cross-national differences in mathematics achievement, the relationship between number word systems and later achievement is more complex" (Ng & Rao, p. 186).

Multiple other factors need to be considered for later achievement including school curriculum, instructional methods, and strategies used in textbooks and some factors may vary across different Asian[2] language communities. For Korean children in second and third grades, other factors suggested by researchers included textbook presentation and the highly competitive educational system (Fuson & Kwon, 1992a, 1992b). Murata (2004) suggested that Japanese children's learning of addition may be facilitated by the relevance of the mathematics curriculum. Stevenson and Stigler (1992) compared preschool through fifth grade children in the United States, Japan, and Taiwan and concluded that differences in achievement could have many possible reasons including children's learning in and out of school, teacher training, lesson organization as well as parental beliefs, expectations, and practices.

Later studies of mathematics instruction, for example in Hong Kong and Korea (Leung & Park, 2002), of Chinese teachers' beliefs and conceptions (Correa, Perry, Sims, Miller, & Fang, 2008; Ma, 1999), and of Japanese textbook representations (Murata, 2008) have documented the multiple and varied experiences that might influence achievement in those countries. One detail is that studies of curriculum and pedagogy have been conducted in early elementary settings, but not in pre-school. One word of caution is that "no studies have focused on cross-cultural differences in mathematics curriculum and pedagogy during the pre-school years, which makes it difficult to determine whether the number system or cultural support for learning of basic number concepts is more important in explaining early mathematics achievement" (Ng & Rao, 2010, p. 195)."

Lastly, there may be historical aspects to consider. For example, since achievement differences were not documented among older Chinese and American adults, these differences may not have existed 60 years ago and may reflect long term changes in the United States (Geary, Bow-Thomas, Liu, & Siegler, 1996). Therefore, the differences cannot be attributed solely to language. Another possibility is that mathematics instruction has changed in one or more of these countries over a longer time period than research has been able to consider. Overall, separating the influences of language from other factors is difficult and perhaps impossible.

Implications for Mathematics Education Research and Practice in Multilingual Settings

Having revisited and reframed this early work, in this section I consider the implications for mathematics education research and practice, especially in multilingual settings. One important implication is that curriculum and instruction for very young children can develop ways to compensate for the irregularities of English or other languages in naming numbers. Such

pedagogical strategies can help students overcome any early difficulties they might face. Another implication is the need to use theoretical frameworks from outside mathematics education, as exemplified by Lucy's requirements for work and claims regarding linguistic relativity. Rather than suggesting any one particular approach, in this section I describe how ecological perspectives are relevant for developing broader approaches to frame the study of language and mathematical thinking.

As alternatives to reductionist frameworks, researchers in the learning sciences have proposed using ecological frameworks for studying thinking, especially with students from non-dominant communities. These frameworks shift the focus to a) examine cognition across different situations, not only in school; b) document resources, as opposed to only obstacles, and c) consider learners' *repertoires of practice* (Gutiérrez & Rogoff, 2003), to avoid ascribing essential cultural practices to any one group or to individual traits. In particular, the construct *repertoires of practice* reminds us that individuals have access to a variety of practices. This perspective assumes that learners have access to multiple practices, that individuals develop, and communities change. Therefore, we should "neither attribute static qualities to cultural communities nor assume that each individual within such communities shares in similar ways those practices that have evolved over generations" (Lee, 2003).

Work on language and mathematical thinking cannot assume that language practices, communication styles, or home cultural practices are homogeneous throughout any community. For example, Gutierrez, Baquedano-Lopez, and Alvarez (2001) describe language practices as "hybrid" and based on more than one language, dialect, or practice. We cannot assume that *any* cultural group has "cultural uniformity or a set of harmonious and homogeneous shared practices" (González, 1995, p. 237). González decries perspectives that "have relegated notions of culture to observable surface markers of folklore, assuming that all members of a particular group share a normative, bounded, and integrated view of their culture" and suggests that "approaches to culture that take into account multiple perspectives can reorient educators to consider the everyday experiences of their student" (p. 237). Researchers should keep in mind that learners from any community can and do participate productively in a variety of roles, responsibilities, communication styles, and mathematical activity that includes hybrid linguistic and thinking practices.

By necessity, researchers in mathematics education who address issues of language in multilingual settings have used work from fields outside of mathematics education to inform research on the relationship between language and mathematics thinking and learning. Work outside of mathematics education contributes theoretical frameworks for studying discourse in general, methodologies (e.g., Gee, 1996), concepts such as registers (Halliday, 1978) and Discourses (Gee, 1996), and empirical work on classroom discourse (e.g., Cazden, 1986; Mehan, 1979). This work has provided crucial concepts necessary for studying the role of language in doing and learning mathematics.

In considering what work might be relevant to research and practice on language and mathematics learning, it is important to distinguish between psycholinguistics and sociolinguistics, because these two perspectives differ in how they conceptualize language. While sociolinguistics stresses the social nature of language and its use in varying contexts, psycholinguistic studies have been limited to an individual view of performance in

experimental settings. From a sociolinguistic perspective, psycholinguistic experiments provide a different view of how people use language. As Hakuta and McLaughlin (1996) explain:

> The speaker's competence is multifaceted: How a person uses language will depend on what is understood to be appropriate in a given social setting, and as such, linguistic knowledge is situated not in the individual psyche but in a group's collective linguistic norms. (p. 608)

Research in mathematics education provides theoretical frameworks for integrating language into research on mathematics learning without privileging formal mathematical discourse. As an example, Brenner (1994) provides useful distinctions among different kinds of communication in mathematics classrooms and describes three components of a "Communication Framework for Mathematics."

> Communication About Mathematics entails the need for individuals to describe problem solving processes and their own thoughts about these processes. . . . Communication In Mathematics means using the language and symbols of mathematical conventions . . . Communication With Mathematics refers to the uses of mathematics which empower students by enabling them to deal with meaningful problems. (p. 241)

In general, work on language and mathematical thinking needs to avoid using deficit models of learners and their communities (Moschkovich, 2002). Many deficit models stem from assumptions about learners and their communities based on race, ethnicity, socio-economic status, and other characteristics assumed to be related in simple, and typically negative ways to thinking and learning in general. Deficit models are so pervasive and insidious that we may sometimes fail to recognize them. Focusing on the mathematical activity that occurs in a community, in homes, or outside of school is an important way to avoid using deficiency models.[3] If research does not examine the mathematical activity in local communities, then it may seem that these learners do not engage in mathematical activity outside of school thus further contributing to seeing them as deficient. It is crucial to uncover the mathematical thinking learners use, both in and out of school settings. In order to uncover the mathematical meanings learners construct rather than the mistakes they make, researchers need frameworks for recognizing the mathematical meanings that learners are constructing in and with language.

A situated and sociocultural perspective (Moschkovich, 2002) is one framework for shifting the focus from looking for deficits to identifying the mathematical discourse practices evident in student contributions (e.g., Moschkovich, 1999). This framework assumes that mathematical Discourse is complex, grounded in practices, and connected to mathematical concepts. Discourses occur in the context of practices and practices are tied to communities. Discourse practices are constituted by actions, meanings for utterances, focus of attention, and goals; these actions, meanings, focus, and goals are embedded in practices. Instead of focusing on deficits, research using this perspective has documented how bilingual students communicate "about mathematics" using hybrid resources, a combination of everyday and

formal language as well as using gestures, objects, tables, graphs, and symbols (Moschkovich, 2015).

Since mathematical discourse is multimodal and multi-semiotic (O'Halloran, 1999), in order to document all the possible resources that learners use to think mathematically, research focused on language and mathematical thinking should include more than language and use a multimodal and multi-semiotic perspective of mathematical activity. Instructional practice needs to include opportunities for mathematical thinking that include multiple modes of communication, sign systems, and types of inscriptions.

FOOTNOTES

1. For examples, see Boroditsky (2000, 2001) and Boroditsky & Gaby (2010).
2. I use the term "Asian" although it is misused to refer to one language, population, or cultural category, when, in fact, there are multiple differences among "Asian" languages and cultural practices.
3. See, for examples, work on "funds of knowledge" in Latino/a working-class communities in the United States (Civil, 2002; González, Andrade, Civil, & Moll, 2001; Moll, Amanti, Neff, & Gonzalez, 1992).

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School Academic Language Demands for Understanding Functional Relationships: A Design Research Project on the Role of Language in Reading and Learning

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ABSTRACT

Acquiring conceptual understanding of functions is far from being trivial for most students, especially language learners. The article reports on a design research project with students in Grades 8-11 (n = 94) that fostered academic language learners' development of conceptual understanding in the interplay of different semiotic representations. Theoretical and qualitative analyses of students' learning pathways and obstacles allowed the specification of school academic language demands based on concept demands for dealing with functional relationships. The strong interplay between concept and language demands can be described by the correspondence of conceptual compaction of conceptual facets and the language-related condensation of their verbalizations.

Key words: communicative and epistemic role of language, conceptual understanding, design research, functions, topic-specific academic language demands

INTRODUCING the PRACTICAL PROBLEM and the THEORETICAL QUESTIONS

Language proficiency is well known to influence mathematics achievement, but not only due to reading demands. In this article, the role of language in processes of developing conceptual understanding is investigated for the mathematical concept of functional relationship. Figure 1 shows an example from a high stakes test in grade 10 (MSW NRW 2012, p. 2) that illustrates interconnected reading and concept demands in a concrete way. Of course, this item contains reading challenges in the lexical dimension (e.g., the meaning of mileage and condensed expressions such as "mileage for a speed of"), but its main challenge is the conceptual understanding of functional relationships:

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State of the literature

- In general, inadequate school academic language has been identified as an obstacle to learning for both monolingual and multilingual students with low language proficiency, especially with regard to conceptual understanding. However, the specific academic language demands emerging during conceptual development in mathematics learning have not yet been well specified.
- As many empirical studies show students' difficulties with the mathematical concept of functional relationships, teaching approaches for fostering its conceptual understanding have been developed. So far, however, the role of language in the processes of conceptual development has not been sufficiently investigated.
- Teaching approaches for fostering language learners have been criticized for being confined to the lexical dimension rather than supporting the syntactical or discursive dimensions.

Contribution of this paper to the literature

- In a design research project, school academic language demands for dealing with functional relationships are specified empirically.
- A teaching approach is developed for fostering language learners' conceptual understanding of functional relationships and investigated empirically with respect to the interplay of the topic-specific concept and language demands in the learning processes.
- In this way, the paper contributes to theorizing on the role of academic language for mathematics learning and to empirically grounding design principles for language-sensitive classrooms.

In order to mathematize the problem, students need to know that a function always connects two variables. Once the first variable and the dependent second variable are identified, the challenge in items (1) and (2) is reduced to finding out which quantity is given and which one is wanted and solving the given equation. However, many multilingual and monolingual students with low language proficiency could not activate this conceptual understanding in order to solve items of this type (Prediger, Wilhelm, Büchter, Benholz, & Gürsoy, 2015a).

This phenomenon was the starting point for a design research project that intended to foster

Mileage problem. "The average mileage of the Wacker family's car (in liters/100 km) can be approximately calculated in dependency of its speed (in km/h) by the following equation:
$f(x) = 0.0005 \cdot (x - 40)^2 + 4.5262$.
(1) What is the average mileage for a speed of 150 km/h?
(2) What is the speed when 9.0 l for 100 km are needed?"

Figure 1. Reading demands and concept demands interrelated in a high stakes test item (Grade 10) ("In dependency of" is literally translated from German, it refers to functional relationships).

students' conceptual understanding of functional relationships in a content- and language-integrated teaching-learning arrangement. In order to develop a theoretical and empirical foundation for this practical need, the role of language in students' learning processes towards functional relationships has to be understood deeply, including the specification of topic-specific language demands. Thus, the intent of the design research project was not only to solve a practical problem (how to foster students' understanding) but also to contribute to two theoretically important overall research questions (to be refined in Section 2):

- Which language demands appear in processes of developing conceptual understanding?
- How can students be enabled to master both, the concept and the language demands?

In approaching these research questions, this article introduces the theoretical background on the roles of the school academic language register for conceptual understanding (Section 1) and then sketches the specific mathematical topic of functional relationships (Section 2). The research methodology of the project is briefly outlined in Section 3. Section 4 presents selected results of the qualitative analysis of concept and language demands in dealing with functional relationships while reading and solving word problems. Section 5 provides insights into processes of enhancing students' conceptual understanding based on a content- and language-integrated teaching-learning arrangement.

THEORETICAL BACKGROUND: THE ROLES OF SCHOOL ACADEMIC LANGUAGE FOR CONCEPTUAL UNDERSTANDING

Language Gaps in Conceptual Understanding and Conceptual Development

When achievement gaps between privileged and underprivileged students are reported, researchers mostly choose socio-economic status and immigrant background or family language background as indicators of privilege (Haag, Heppt, Stanat, Kuhl, & Pant, 2013; Mullis, Martin, Foy, & Arora, 2012; OECD, 2007; Secada, 1992). These indicators can easily be used to measure the issue of privilege using students' self-reports or existing school data, such as free school meals; thus, they are used more often than language proficiency. This trend also applies to the recent PISA report on low performers' backgrounds (OECD, 2016). However, when language proficiency in the language of instruction is also controlled, it turns out to be the factor with the strongest connection to mathematics achievement, stronger than multilingualism, immigrant background, or socio-economic status (Prediger et al., 2015a; Heinze, Reiss, Rudolph-Albert, Herwartz-Emden, & Braun, 2009). We thus agree with Hirsch (2003) that a "chief cause of the achievement gap between socio-economic groups is a language gap" (p. 22). This language gap occurs for multilingual as well as monolingual learners. Hence, for this article, the term *language learner* refers not only to second language learners but also to all students with low academic language proficiency in the language of instruction (which in this study is German). This focus is in line with Moschkovich's claim that "studies should focus less on comparisons to monolinguals and report not only differences between monolinguals and bilinguals but also similarities" (Moschkovich, 2010, p. 11).

The strong connection between mathematics achievement and language proficiency is often investigated with respect to language biases in tests (Abedi, 2006; Haag, Heppt, Roppelt, & Stanat, 2015) and constraints in reading proficiency (Abedi & Lord, 2001; Hirsch, 2003). In these studies, language is mostly treated in its *communicative role* and tends to be considered as external to the core of mathematics.

However, beyond reading challenges, many students with low language proficiency encounter other serious obstacles: in two recent studies, items that provided statistically unexpected difficulties (i.e., differential item functioning) for students with low language proficiency were those with high concept demands, such as conceptual understanding, not those items with reading obstacles (Ufer, Reiss, & Mehringer, 2013; Prediger et al., 2015a). This finding resonates with many qualitative studies, which show possible language obstacles in the processes of conceptual development (Moschkovich, 2010; Prediger & Krägeloh, 2015).

These findings call for taking into account not only the communicative role of language, but also its *epistemic role* in the processes of knowledge construction as a medium of thinking (Heller & Morek, 2015; Vygotsky, 1978). Students with low language proficiency might not only be hindered by reading obstacles (*communicative role*) in showing their competencies in tests but also be constrained throughout their individual school history, especially with respect to developing conceptual understanding (Prediger et al., 2015a; Moschkovich, 2015).

Three Roles of the School Academic Language for Conceptual Understanding

In order to explain the statistically evident connection between language proficiency and conceptual development, we draw upon the sociolinguistic distinction between school academic register and everyday register (Cummins, 2000; Snow & Uccelli, 2009; Schleppegrell, 2004). A register is defined as a "set of meanings, the configuration of semantic patterns that are typically drawn upon under the specific conditions, along with the words and structures that are used in the realization of these meanings" (Halliday, 1978, p. 23). Hence, registers are characterized by the types of communication situations, their fields of language use, the discourse styles, and modes of discourse. The school academic language is the register "that is used by teachers and students for the purpose of acquiring new knowledge and skills . . . , imparting new information, describing abstract ideas, and developing students' conceptual understanding" (Chamot & O'Malley, 1994, p. 40). Thus, the school academic language register has the *role of an important learning medium*, used in the mode "communicate to learn" (Lampert & Cobb, 2003; Pimm, 1987).

The sociolinguistic relevance of the school academic register lies in its second role, as an *unequally distributed learning prerequisite:* Whereas all children can acquire basic communication skills in the everyday language in their families, only socially privileged families also provide learning opportunities for aspects of the academic register (Snow & Uccelli, 2009).

An educational consequence can be drawn immediately: If the school academic register serves as a learning medium, it is a learning prerequisite for all students. If this prerequisite

cannot be taken for granted for all students, it is a matter of equity to treat it as *learning goal* in classrooms (from "communicating to learn" to "learning to communicate"; cf. Lampert & Cobb, 2003; similarly Schleppegrell, 2004; Thürmann, Vollmer, & Pieper, 2010).

For treating the school academic register as a learning goal in mathematics classrooms, its relevant features have to be well specified. Linguists have described the general differences between everyday language and school academic language in the *lexical dimension* (e.g., by specialized vocabulary, composite or unfamiliar words, and specific connectors) and in the *syntactical dimension* (e.g., long and syntactically complex sentences, passive voice constructions, and long noun phrases and prepositional phrases). Beyond the lexical and syntactic dimension, the school academic register can be characterized on the *discursive dimension* through specific discursive practices (e.g., arguing and explaining why), which are also not equally offered in all families (Bailey, 2007; Heller & Morek, 2015; Thürmann et al., 2010).

Although there is a consensus on these lexical, syntactical, and discursive features in general, there is still a research gap in specifying the specific school academic language demands that are *most relevant for learning specific mathematical topics,* for instance, the development of a conceptual understanding of functional relationships examined in this study (Moschkovich, 2015; Bailey, 2007). As each mathematical topic requires specialized language means to think and communicate about it, this specification needs to be topic specific. This article intends to contribute to this specification, because it provides a theoretically grounded and empirically based foundation for a focused language support. As topic-specific academic language demands are not separable from technical language on the micro level, we subsume both under academic language demands.

In order to *specify academic language demands*, most existing studies choose the method of analyzing textbooks and other curriculum materials (e.g., Bailey, Butler, Stevens, & Lord, 2007; Thürmann et al., 2010). Although this approach is insightful (and is also used in our preparatory work in Section 1.3), it risks the tendency to prioritize written language over oral communication and to restrict the focus mainly to the communicative role of language. In order to take into account the epistemic role of language in the three functions of (1) learning medium, (2) unequally distributed learning prerequisite, and (3) learning goal that requires further topic-specific specification, we extend the approach to analyzing (oral) learning processes on the micro level. As most regular classrooms do not provide conceptual learning opportunities, these learning processes have to be initiated by specifically designed learning arrangements. Thus, the research methodology of choice for this research is *topic-specific design research with a focus on learning processes,* which allows the researchers to overcome the deficit focus on language learners' obstacles by focusing on subtle resources in processes (see Section 3).

Moschkovich (2010) pleads for a research focus on students' processes of developing conceptual understanding and claims that "in order to focus on the mathematical meanings learners construct, rather than the mistakes they make, researchers will need frameworks for recognizing the mathematical knowledge, ideas, and learning that learners are constructing in, through, and with language" (Moschkovich, 2010, p. 12). In order to provide a systematic base for these empirical tasks, we briefly report on the language demands as far as they

could be specified theoretically.

First Specification of Lexical, Syntactical, and Discursive Demands for Functional Relationships

The first step of the study involved specifying academic and technical language demands in the *language reception* on functional relationships in a preliminary textbook analysis (Zindel, 2013). Table 1 shows excerpts from the (incomplete) collection of used phrases for functional relationships that occur in word problems. The *lexical variety* of three different phrases for the same concept (three lines in the table) appears to be less critical than the *syntactical complexity* given by the German grammar with at least two to four grammatical variations for each phrase (in the six cells). Subtle syntactical constructions (grammatical cases) allow different orders for subject and object in the sentence without changing the sense. This is challenging for many students (even for those with high language proficiency) because the subtle syntactical differences and commonalities require language awareness.

All phrases in Table 1 describe functional relationships in a very condensed way and have to be interpreted by the students in order to decode the texts. However, many students do not even identify their relevance in a problem text (Zindel, 2013), as this discursive demand of identification requires conceptual understanding of functions. This conceptual understanding can become visible when students are able to relate different representations (in word problems, mainly the verbal and symbolic representation), which again requires their interpretation.

Table 1. First steps towards receptive language demands: German phrases for functional relationships (Zindel, 2013)

f(A) = B	Active Sentence Structure	Passive sentence structure
Dependency B of A	*The function indicates B in dependency of A.* • Die Funktion gibt B in Abhängigkeit von A an. • Die Funktion gibt das von A abhängige B an. • Die Funktion gibt B an, das von A abhängig ist. • Die Funktion gibt B an, das von A abhängt.	*In the function, B is given in dependency of A.* • B wird in Abhängigkeit von A angegeben. • Es wird das von A abhängige B angegeben. • Es wird B angegeben, das von A abhängig ist. • Es wird B angegeben, das von A abhängt.
Assignment A→ B	*The function assigns each A to a B.* • Die Funktion ordnet jedem A ein B zu. • Die Funktion ordnet ein B zu jedem A zu.	*Each A is assigned to a B.* • Jedem A wird ein B zugeordnet. • Ein B wird jedem A zugeordnet.
Implicit description by prepositions	*The function gives a B for* [*to*] *each A.* • Die Funktion gibt **für** jedes A ein B an. • Die Funktion gibt **zu** jedem A ein B an.	*For* [*to*] *each A, a B is given.* • Es wird **für** jedes A ein B angegeben. • Es wird **zu** jedem A ein B angegeben.

Summing up, the theoretical analysis of previous research and the textbook analysis allowed the specification of four main discursive demands (denoted by capital letters) in dealing with word problems of functions in language production and reception, which are strongly intertwined:

- READING COMPLEX TEXTS (in this study: word problems involving functions) is the discursive demand that requires managing the presented syntactical complexity.
- It first involves IDENTIFYING the relevant but highly condensed phrases in which the information about the functional relationship is coded.
- Decomposing the condensed phrases then involves the language production with the discursive demand of INTERPRETING TEXTS OR SYMBOLS.
- Of course, interpreting the texts is only possible after having developed conceptual understanding of the core concept functional relationship, and most important to the development of this understanding is the productive discursive demand of EXPLAINING THE MEANING of concepts (see Prediger & Wessel, 2013).

Because each of these discursive demands also requires conceptual understanding of functional relationships, the next section focuses on conceptual understanding.

CONCEPTUAL UNDERSTANDING OF FUNCTIONAL RELATIONSHIPS: STATE OF RESEARCH AND RESEARCH NEEDS

State of Research on Functional Relationships: Perspectives and Representations

The functional relationship is considered "one of the most fundamental and significant" concepts, applied in many inner- and extra-mathematical situations (Niss, 2014, p. 239). Although the approaches for specifying necessary elements for its conceptual understanding vary (see Niss, 2014; Carlson & Oerthmann, 2005; Leinhardt, Zaslavsky, & Stein, 1990), there is a common core related to representations and basic meanings, which are distinguished, for example, by the following perspectives:

- The *correspondence perspective* on functions conceptualizes how each value x in a functional relationship $y = f(x)$ is assigned to a unique value y (Vollrath, 1989; Confrey & Smith, 1994). Thompson refers to this perspective as a kind of static perspective, explained as seeing an "invariant relationship between two quantities whose values vary" (Thompson, 2011, p. 46).
- In contrast, the *covariation perspective* focuses on the way in which two varying quantities change together (Vollrath, 1989; Confrey & Smith, 1994; Carlson & Oerthmann, 2005). Thompson (2011) outlines covariational reasoning as "the very operations that enable one to see invariant relationships among quantities in dynamic situations" (p. 46).
- The *holistic perspective* on the function mainly focuses an encapsulated object perspective (Vollrath, 1989).

Besides these perspectives, some scholars have suggested other distinctions (e.g., the action, process, and object perspective by Dubinsky & Harel, 1992), while others have suggested distinctions that are bound to single types of functions (e.g., linear and exponential) or single representations (e.g., algebraic representation in equations, numerical representation in tables, graphical representations in graphs, and verbal descriptions). In this paper, we try to consider the core of functional relationships relevant in all these four representations, and we focus on the correspondence and covariation perspective and on the need for students to coordinate them (Vollrath, 1989; Thompson, 2011).

Conceptual understanding of functional relationships has often been described as the ability to adopt different perspectives flexibly in all four representations and to coordinate them (as summarized by Niss, 2014). Since the 1980s, *connecting four representations* has been identified as a key activity for understanding functional relationships (Swan, 1985; Leinhardt et al., 1990; Duval, 2006), but often with some shortcomings: In spite of the claimed symmetry, most design and research projects have focused either on the relation between qualitative graphs and verbal descriptions (e.g., Swan, 1985) or on graphs, equations, and tables (e.g., Leinhardt et al., 1990; Moschkovich, Schoenfeld, & Arcavi, 1993; Romberg, Fennema, & Carpenter, 1993). Less attention has been spent so far on the *connection between equations and verbal descriptions*, such as in mathematizing word problems in functions expressed either in the everyday, school academic, or even technical register. Another shortcoming concerns the "translation" metaphor, which does not imply a one-to-one-translation: Even if all three perspectives (correspondence, covariation, and holistic) are relevant in each representation, the shift between representations mostly implies modifications of meanings (Moschkovich et. al., 1993, p. 72); this also applies to shifts in the language registers.

Reacting to students' documented difficulties with activities involving flexibly moving between representations, a huge variety of teaching approaches have been suggested (see Leinhardt et al., 1990; Carlson & Oerthmann, 2005). These findings all show that enhancing students' conceptual understanding is a possible but complex task with many different aspects: "The desired outcomes are not likely to occur by default with most students . . . and they come at a price: time and effort" (Niss, 2014, p. 240; more details in Carlson & Oerthmann, 2005).

This fact raises the need to specify the conceptual core demands for functional relationships common to all types of functions and in all representations. The empirically grounded facet model of this core is presented in Section 2.2 and examples are investigated for the connection between verbal and algebraic representations in Section 4.1.

Facet Model for Specifying Concept Demands for Functional Relationships

Because the wide consensus about relevant perspectives and representations for functions has turned out to be too general for the purpose of specifying language demands in dealing with functional relationships, we have constructed a refined model of conceptual facets for

functional relationships that provides a language for a finer-grained analysis of elements of students' conceptual understanding of the core of functional relationships. In the empirical part of this article (Section 4), this model will be used to identify the language demands when dealing with different facets of functional relationships.

In order to construct this model, we refer to Hiebert and Carpenter's (1992) definition of understanding as related to learning with meanings. A concept "is understood if it is part of an internal network. . . . The degree of understanding is determined by the number and the strength of the connections" (Hiebert & Carpenter, 1992, p. 67).

This conceptualization of understanding as consisting of a dense network of pieces of knowledge calls for refining the pieces of knowledge we call facets of knowledge. The construct of *understanding as a network of facets* was fruitfully combined with Aebli's (1981) construct of *compacting into denser concepts*: When learning new concepts, single facets of conceptual understanding have to be acquired and then related to each other. Once the network is mentally constructed, it can be compacted into more condensed facets. A deep understanding of a concept is reached when learners are able to flexibly switch between the compacted facets and to *unfold* them again into their more elementary facets (Drollinger-Vetter, 2011).

Based on the theoretical construct of Hiebert and Carpenter (1992) and Aebli (1981) of understanding as a *network of facets that are compacted into denser concepts* and on the preliminary empirical results of our research, we constructed the model of conceptual facets of understanding the core of functional relationships in Figure 2. It provides the language for describing and comparing students' resources, processes, and obstacles (see Section 4.1).

In order to explain the facet model, we refer to the mileage problem in Figure 1. In this

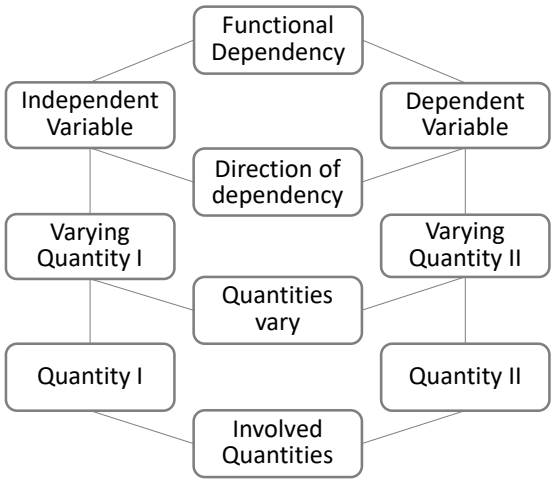

Figure 2. Model of conceptual facets of understanding the core of functional relationships

In this problem, the (compacted) symbolic equation $f(x) = 0.0005 \cdot (x - 40)^2 + 4.5262$ has to be related to the (condensed) phrase "the average mileage . . . can be approximately calculated in dependency of its speed. . . ." The successful coordination of both representations is considered an indicator for understanding the compacted concept ||functional dependency|| (our denotation || . . . || marks a facet of the model in Figure 2 or additional facets that students address).

Students who understand this compacted concept can unfold it into the conceptual facets required for succeeding in this coordination of representations: Students need to know that each functional relationship connects two ||involved quantities|| and that these ||quantities vary||. The ||direction of dependency|| matters, so it is important to identify the speed as the ||independent variable|| and mileage as ||dependent variable||. This analysis resonates with Thompson (2011), who emphasized the high relevance of quantities as mental entities for understanding functional relationships and of quantitative reasoning. The facet model is the base for the following definition:

> Conceptual understanding of the core of functional relationships is defined as the ability to adopt different perspectives in different representations and to coordinate them by addressing the facets from Figure 2 flexibly and adequately. This requires flexible compacting and unfolding of conceptual facets, thus moving upwards and downwards in the facet model.

In this definition, "flexibly" marks the need to find different ways in different situations, and "adequately" refers to the specific situations given by a context problem, a teacher question, or a task. As the empirical analysis will show, the model allows unpacking of concept demands for compacting and unfolding complex concepts along with the specific language demands.

Fostering Language Learners' Conceptual Understanding

Once having specified the network of conceptual facets to be acquired by students, the question arises as to how this acquisition can be fostered, especially for language learners. Moschkovich (2013) has articulated four general recommendations for multilingual language learners that apply also to monolingual language learners:

- #1: Focus on students' mathematical reasoning, not accuracy in using language.
- #2: Focus on mathematical practices, not language as single words or vocabulary.
- #3: Recognize the complexity of language in mathematics classrooms and support students in engaging in this complexity.
- #4: Treat everyday and home languages as resources, not obstacles. (Moschkovich 2013, p. 50)

The main mathematical practices we focus on are sense making and modelling, for which Moschkovich (2013) recommends "keep[ing] tasks focused on high cognitive demand, conceptual understanding, and connecting multiple representations" (ibid, p. 52). Thus,

connecting multiple representations is not only a learning goal but also an important design principle for achieving the goal.

The design principle of connecting algebraic, numerical, verbal, and graphical representations (e.g., Bruner, 1967) can be extended to the idea of relating language registers (everyday register, school academic register, and technical register). This has been theoretically justified (Prediger, Clarkson, & Bose, 2016) and investigated for the case of fractions (Prediger & Wessel, 2013). Rather than planning a unidirectional process from the everyday register and graphical representations to the technical register and symbolic representation, the design principle of relating registers and connecting representations pleads for repeatedly moving forward and backward, without assuming a hierarchy between the representations or registers.

Cognitive activities for connecting representations and registers have been described by Duval (2006): following Piaget's operative principle, he emphasizes the effectiveness of the activity of *systematic variation* in one representation and investigating its effects in a second representation (Duval, 2006, p. 125). In our teaching approach, we apply the activity of systematic variations of phrases, i.e., in the verbal representation (see Sections 4.3 and 5).

Research Questions

Based on these theoretical considerations and preliminary specifications, the research questions on specifying demands (RQ1) and on possible approaches for fostering students' conceptual understanding (RQ2) can be refined as follows:

(RQ1) Which concept and language demands arise for students when dealing with functional relationships and how are they interrelated while connecting representations?

(RQ2) How can the designed teaching learning arrangement with the design principle "relating registers by systematic variation of phrases" support students' learning processes towards mastering the interrelated concept and language demands?

METHODOLOGİCAL FRAMEWORK OF THE DESİGN RESEARCH PROJECT

Methodology of Topic-Specific Design Research with a Focus on Learning Processes

Since for this project, specifying the demands and learning goals is as important as investigating effects of design approaches, we choose the methodological framework of Topic-Specific Design Research.

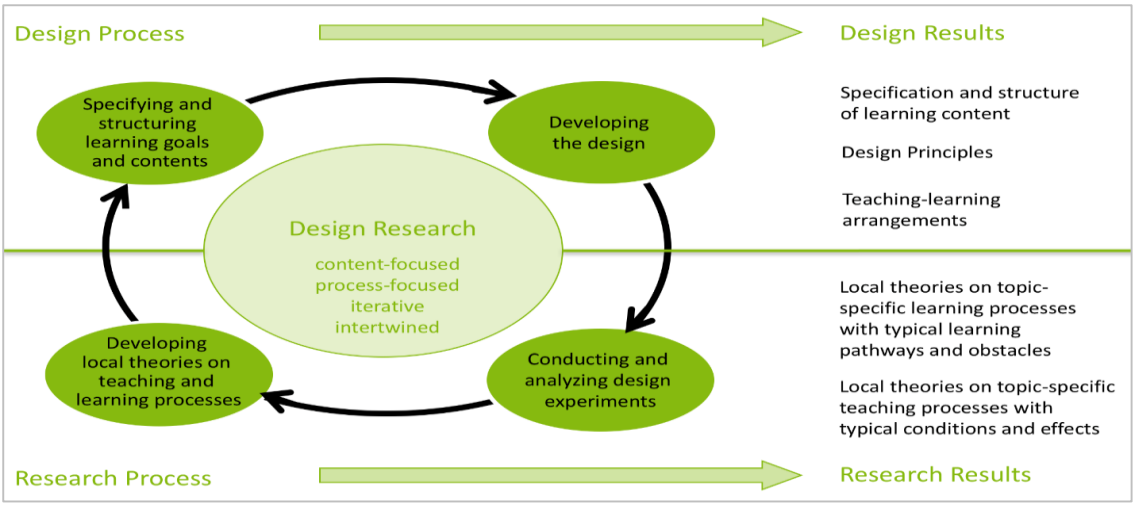

Figure 3. Four working areas for Topic-Specific Design Research (Prediger et al., 2012)

Like many approaches within the methodology of design research with a focus on learning processes (Gravemeijer & Cobb, 2006; Prediger, Gravemeijer, & Confrey, 2015b), our framework of Topic-specific Design Research relies on the iterative interplay between designing teaching-learning arrangements, conducting design experiments, and empirically analyzing the processes. Its four working areas and typical design and research results are depicted in Figure 3 (Prediger et al., 2012).

The design outcomes of the reported project comprise a further elaboration of the specified and structured learning content (in this case, concept and language demands for developing conceptual understanding of functional relationships), refined design principles (in this case, connecting representations and systematic variation of phrases; see Prediger et al., 2016), and a prototype learning arrangement. The research outcomes consist of empirical insights and contributions to local theories on learning and teaching processes of the treated topic (in this case, the role of the school academic language in processes of developing conceptual understanding of functions).

Design Experiments as the Method for Data Collection

Design experiments are considered the methodological heart of design research studies as they allow in-depth investigations of learning *processes* rather than only learning *outcomes* (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Gravemeijer & Cobb, 2006).

In the overarching project, we conducted three design experiment cycles (19 design experiments in 1-3 sessions each) in laboratory settings with 18 pairs of students and one single student (one student's partner was ill) in Grades 8-11 (14-17 years old). The fourth design experiment cycle took place with students in three whole classes in three classroom settings (n = 57), with each class lasting for 45 minutes each. In total, 42 design experiments each lasting 30-60 minutes were completely video-recorded (1890 minutes of video) and

partly transcribed. At the beginning of the first cycle, a textbook analysis and clinical interviews with think-aloud protocols were conducted in order to identify typical obstacles with problems such as the one in Figure 1. Based on this material, the teaching-learning arrangement was developed and iteratively elaborated using design experiments in four cycles.

The case studies presented in the following chapters use data from Cycle 1 (clinical interviews dealing with RQ1) and Cycle 3 (design experiments dealing with RQ2) in which the design experiments in laboratory settings were led by the second author. The students in the case studies reported from Cycle 1, Manuel, Luisa und Dennis, were in Grade 10 and were 15-16 years old. The case study from Cycle 3 involved Fynn and Svenja, who were 15 years old and from a Grade 9 class in a comprehensive school. These students were selected as cases because they had shared monolingual German language backgrounds with further language learning needs in the school academic language register, but had contrasting profiles in mastering the concept and language demands.

Qualitative Methods for Data Analysis

The qualitative analysis of selected transcripts of interviews and design experiments was conducted with the aim of specifying concept and language demands in the processes of problem solving or acquiring conceptual understanding.

By employing a turn-by-turn analysis of the selected transcripts, students' conceptual thinking was captured in Vergnaud's (1996) framework of students' individual concepts- and theorems-in-action. Vergnaud defines a theorem-in-action as "proposition that is held to be true by the individual subject for a certain range of situation variables" (Vergnaud, 1996, p. 225). Theorems-in-action are indicated using "< . . . >", e.g., <For identifying the dependent quantity, it suffices to consider the unit of rate of change.> These theorems-in-action are shaped by concepts-in-action, defined as "categories . . . that enable the subject to cut the world into distinct . . . aspects and pick up the most adequate selection of information" (ibid.); in this study they are ||involved constants|| and ||dependent variable||. In the first step of data analysis, students' theorems-in-actions were inferred from their utterances and actions. Vergnaud's framework allows extrapolation of the underlying concepts-in-action. In a second step, categories for concepts- and theorems-in-actions were built by systematically comparing and contrasting the different cases of students' thinking. In the preliminary work, the systematization of concepts-in-action resulted in the model of conceptual facets (as presented in Section 2.2). Thus, facets of the model are typical concepts-in-action, but other concepts-in-actions can also be inferred by the open data analysis procedure. In the third step presented here, the model was used as an open categorical scheme, and the extrapolated uses of facets were related to the language forms in which they appeared.

Together, these analytic procedures allowed the researchers to unpack the conceptual and language-related sides of demands in both situations of reading word problems (Section 4) and design experiments for developing conceptual understanding (Section 5).

CONCEPT AND LANGUAGE DEMANDS IN DEALING WITH FUNCTIONAL RELATIONSHIPS WHILE READING AND SOLVING WORD PROBLEMS

The empirical specification of concept and language demands started with analysis of three cases with respect to concept demands (Section 4.1) and language demands (Section 4.2).

Revealing Concept Demands in the Interplay of Representations

The three cases show the processes of three students, Manuel, Luisa, and Dennis, when trying to solve the mileage problem from Figure 1. The case of Manuel represents a successful process in connecting the given symbolic and verbal representation. After reading the mileage problem (in Figure 1), he immediately thinks aloud:

7 Manuel In each case, you have the function, which anyway assigns . . . the mileage to the speed—here . . . .

20 Manuel When one factor changes, . . . that the other factor changes . . . The function [*tells*] you only . . . for which speed you have which mileage.

The analysis of Manuel's thinking process is visualized by the facet model in Figure 4 in which adequately addressed facets or connections are framed by green lines and inadequately addressed facets by red dashed lines.

In Line 7, identified the ||functional dependency|| adequately and reformulated the text of the problem: "anyway assigns . . . the mileage to the speed." He seemed to transform "in dependency of" into the alternative (but equally condensed) phrase "assigns to" (Table 1). We interpret his flexible descriptions for the highest level in the facet model as an indicator

Lines 7, 20: Symbolic Representation ←→ Verbal Representation

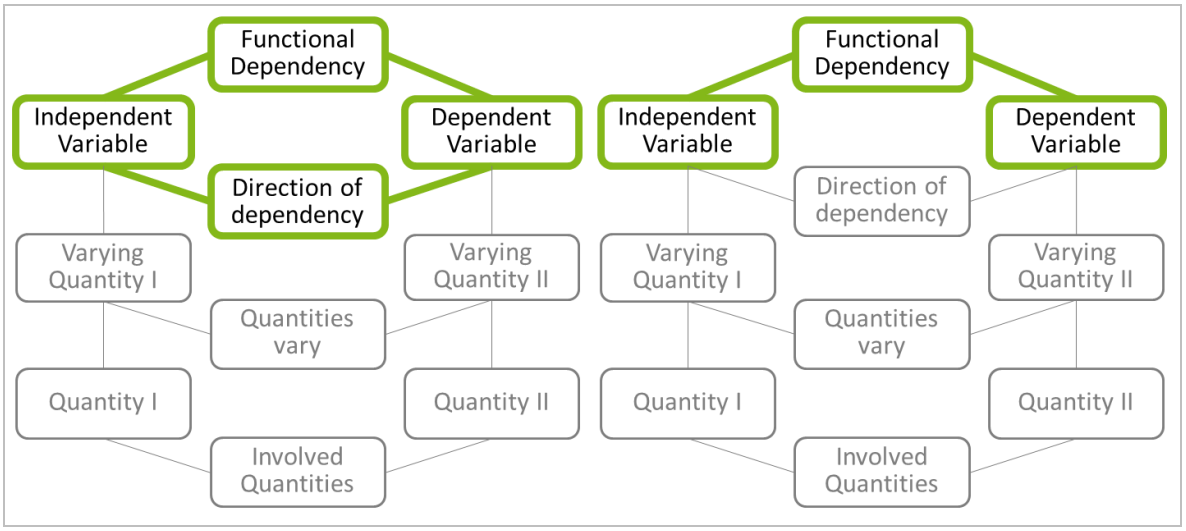

Figure 4. Reconstruction of Manuel's addressed conceptual facets

of his highly developed conceptual understanding.

The analysis of Line 20 supports this interpretation. Building on the insight that there were two varying quantities, Manuel realized that the ||direction of dependency|| mattered: "when one factor changes, . . . the other factor changes" (Line 20). This allowed him to reformulate the verbal representation in a highly condensed form: "The function [*tells*] you only . . . for which speed you have which mileage." For this translation, he implicitly identified the ||independent variable|| and the ||dependent variable|| adequately. Hence, he unfolded the functional relationship on the medium level of the facet model (marked in green in Figure 4) successfully with respect to the symbolic representation. One can assume that he would have been able to unfold it also on the lowest level, but this was not necessary for him.

In contrast, many other students encountered serious difficulties in the design experiments. The facet model allows the identification of sources of their obstacles, as it did for Luisa (15 years old).

17 Luisa Thus, we have here three numbers [*hints to the constants in the equation*].
19 Luisa But here [*in the text*], there are only two, driven kilometers [*per hour*] and mileage. Any [*of the three*] must be of something completely different.

Luisa's theorem-in-action, <The three parameters in the equation belong to the quantities in view>, indicates a deviant coordination of the ||involved quantities|| in the verbal representation with the ||involved constants|| of the symbolic representation, without focusing the phrase "in dependency of."

Her attempt to coordinate the ||involved constants|| in one representation and the ||involved quantities|| in the other representation is visualized in the facet model in Figure 5. It indicates the urgent need to enhance her conceptual understanding of functional relationships beyond decoding the text.

In Dennis's (15 years old) case, the model allows identifying his understandings that are not yet conceptually viable and capturing his successive process of cracking the word problem. Dennis started as follows:

5 Dennis They have only given the information for the mileage and the speed.
6 Dennis That is now the question; if there is x, x is probably the mileage, because "in dependency of the speed" is then—oh, probably simply the 40 or the 4.5462.

In clarifying the meaning of the problem, in Line 5 Dennis identified the two ||involved quantities|| (see Figure 6). So far, this facet was treated in an isolated way, without yet addressing the ||direction of dependency||, for example.

In Line 6, Dennis identified an inadequate ||independent variable|| in the symbolic representation and constructed a deviant meaning for it in the verbal representation: His implicit theorem-in-action, <In order to identify the value of the independent variable, one can search among the constants of the equation>, led him to consider a single value rather than a (possibly varying) quantity. This is interpreted as an indicator of a not yet accomplished understanding of the facet ||dependent variable|| and as a reason why he related the phrase "in dependency of the speed" to an appropriate part of the equation.

Lines 17,19: Symbolic Representation ←→ Verbal Representation

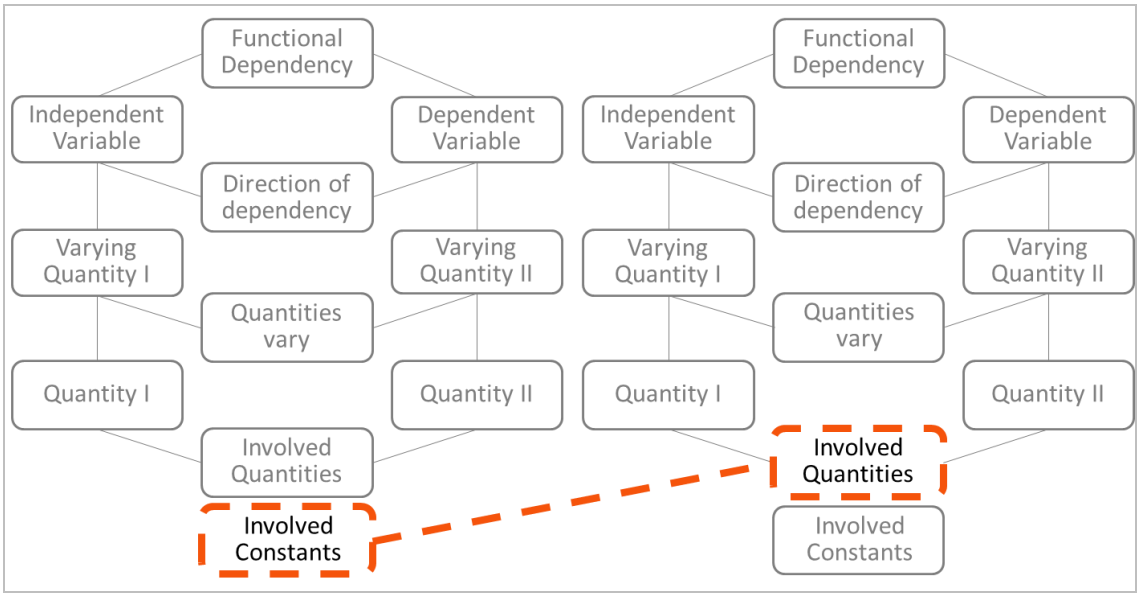

Figure 5. Reconstruction of Luisa's addressed conceptual facets

In a much later step, he corrects himself:

101 Dennis x is the speed, because—the mileage is now—don't know exactly what this will be—but x is the speed, so that you can always insert something else.

For the ||independent variable||, he activated an appropriate theorem-in-action: <The independent variable is the one that can be evaluated for different values>. Using this theorem, he unfolded the ||functional dependency|| but isolated the ||independent variable|| from the ||dependent variable||. This isolation was the source of the difficulty in identifying the role of the mileage.

With some more support of the design experiment leader, he could finally succeed in relating the different conceptual facets to each other (compacting) and thereby in decoding the problem.

These small excerpts from the cases of Manuel, Luisa, and Dennis show the *concept demand of coordinating and connecting the different facets in both representations*: all conceptual facets can become relevant for succeeding in coordinating the symbolic equation and the phrase "in dependency of" (literally translated from German), as they have to be adequately addressed, combined, and related between representations. Obstacles appear when students:

(a) focus too exclusively on one facet (e.g., as Dennis in Line 5),
(b) address a mismatching facet (e.g., as Luisa referring to the constants),
(c) mismatch one facet in different representations (e.g., as Luisa in Line 19), or
(d) show structural misunderstanding of a facet (e.g., as Luisa or Dennis in Line 6).

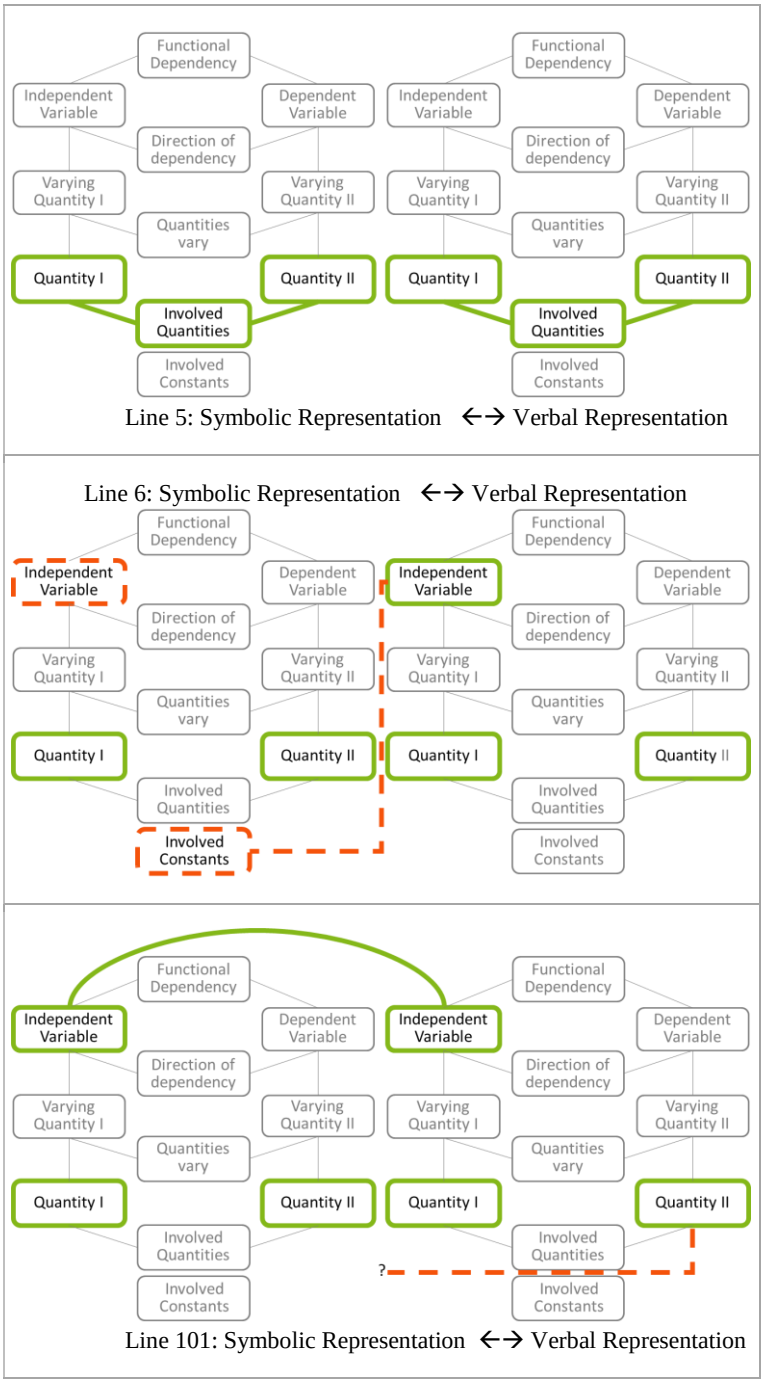

Figure 6. Reconstruction of Dennis' addressed conceptual facets

Whereas mode (d) indicates conceptual misunderstandings, modes (a)-(c) could also only indicate a strategic flaw in decoding the concrete text in spite of existing understanding of

the concept. In either conceptual obstacles or strategic reading obstacles, the model of conceptual facets (Figures 4-6) allows the empirical unpacking of the complex underlying cognitive phenomena.

Revealing Receptive and Productive Language Demands

These case studies can now be discussed with respect to the occurring language demands: the case of Luisa exemplifies the *receptive language demands* anticipated in Section 2.2 in the communicative role of language: Luisa failed in READING COMPLEX TEXTS as she missed IDENTIFYING the condensed phrase "the mileage in dependency of the speed."

Beyond that, the empirical analysis in Section 4.1 provides insights into *demands in students' language production* occurring with the demanded language decomposition of the highly condensed phrase for ||functional relationships|| that refer to the epistemic role of language: As the complex phrase contains all other conceptual facets in a compacted form, condensing syntactically (e.g., by nominalizations or prepositional constructions; see Jorgensen, 2011) can be considered the language-related counterpart of the conceptual process of compacting in Aebli's sense (1981).

This *correspondence of conceptual compacting and language-related condensing* is visualized in Figure 7. Thus, for INTERPRETING and UNDERSTANDING the phrase, it must be unfolded into its facets on the lower levels of the model, and this process of unfolding requires language production on the lower levels. The corresponding de-composing of nominalizations brings much longer sentences for the four facets. Manuel's decomposed explanation activates if-then clauses (Lines 7-20, typical for the covariation perspective) and expresses the the ||direction of dependency|| as well as the two ||varying quantities||. Isolated identification of quantities on the lowest level, as in Dennis's case, sometimes goes along with language challenges to express the dependency in relational words; this is another prototypic example

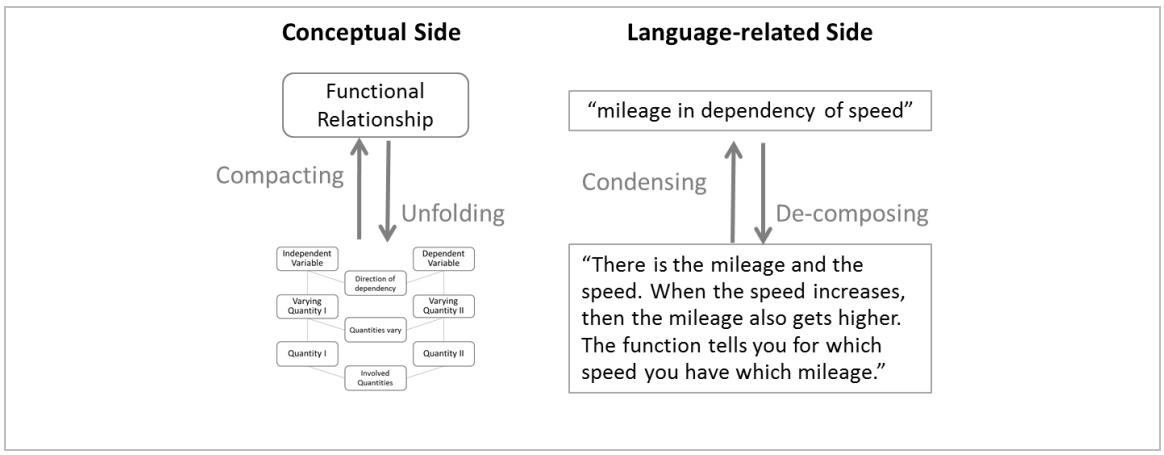

Figure 7. Correspondence of conceptual compaction and language-related condensation

for the epistemic role of academic language. In addition, having conceptual understanding is necessary to be able to address conceptual facets verbally. We summarize the main findings for this topic:

Receptive and productive demands occur in the communicative and epistemic role of language. The strong interplay between concept and language demands can be described by the correspondence of conceptual compaction of conceptual facets and the language-related condensation of their verbalizations.

Consequences for the Teaching-Learning Arrangement for Understanding Functional Relationships

The refined specification of concept and language demands outlined in Section 4.2 constituted the starting point for redesigning the learning arrangement. Due to the findings on the necessity of relating conceptual facets, the redesign followed a new design principle: *focus on coordinating and relating the conceptual facets*. This coordination of conceptual facets is triggered by the design principles of relating registers and systematic variation of texts (see Section 2.3).

Figure 8 shows one central activity from the designed learning arrangement in Design Experiment Cycle 3. In Question 1, students are asked to compare two offers for online streaming: DreamStream and Streamox3. When working on such tasks, students usually refer to the ||rates of change|| and the ||start values|| for the comparison. In order to answer Question 2, students calculate values in the table. The tables can be read vertically (in a covariation perspective) or horizontally (correspondence perspective). The covariation perspective emphasizes the meaning of the ||involved constants||, while the correspondence perspective underlines the ||involved quantities||. The intent of Questions 3 and 4 about the equation is to enhance students' focus on the ||involved quantities||.

In order to find the equation, students need to coordinate all facets, ||involved quantities||, ||quantities vary||, and the ||direction of the dependency||, and, in this case, even the ||involved constants|| are important. Question 5, by deciding and explaining which phrases match or mismatch to the equations, addresses different facets explicitly, because they vary systematically in one of these facets.

In this way, the activity is designed to foster conceptual understanding by dealing with unfolded facets and re-compacting them. This is especially necessary for those students who did not understand single facets structurally or those who are not able to compact them without prompts. Furthermore, comparing the descriptions aims at initiating reflection and sensitizing for details in the formulations (thus enhancing some language awareness).

Table 2 summarizes some of the decisions in the design of Cycle 3 that were made based on consequences from previous cycles. Without assuming any automatism in how design elements can enhance the overcoming of obstacles, Table 2 roughly sketches hypothesized connections. Empirically, the potential of the principle of systematic variation of phrases for overcoming conceptual obstacles will be shown in the next section.

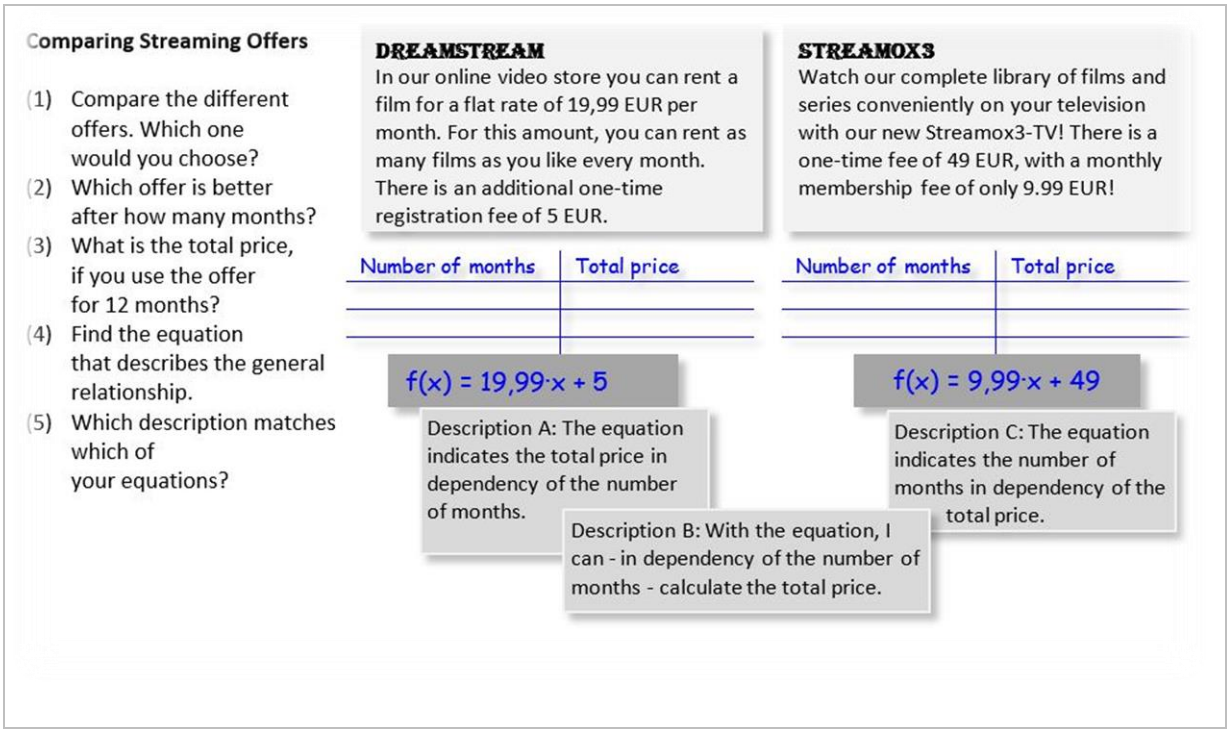

Figure 8. Activities from the learning arrangement in Design Experiment Cycle 3, realizing the design principles of relating registers and systematic operative variation of phrases (Descriptions A-C literally translated from German)

Table 2. Overview of consequences of previous cycles' effect on the design of Cycle 3: Design elements for different obstacles

Potential conceptual obstacles	**Design elements in the learning arrangement**
(a) Focus too exclusively on one facet	Systematic variations of phrases triggers focus on other facets
(b) Address a mismatching facet (constants)	Structure of the intended learning pathway shifts the attention from the constants to the involved quantities
(c) Mismatch of one facet in different registers	Enhance language awareness by reflecting the systematic variations of phrases
(d) Show structural misunderstanding of a facet	Develop conceptual understanding by working on the missing facets
Potential language obstacles	**Design elements in the learning arrangement**
(e) No attention to key phrases such as "in dependency of"	Enhance language awareness by reflecting on the systematic variation of phrases
(f) Phrase is focused, but inappropriately interpreted due to missing strategic focus on relations	Finding equations triggers to search for the quantities, thus fix meaning of variables

CONCEPTUAL AND LANGUAGE-RELATED PROCESSES WHILE DEVELOPING CONCEPTUAL UNDERSTANDING OF FUNCTIONAL RELATIONSHIPS

The following two transcripts from Svenja's case offer empirical insights into how the redesigned learning arrangement in Section 4.3 helps students to master the intertwined concept and language demands (RQ2).

Svenja (15 years old) worked with Fynn and the design experiment leader (in this case, the teacher) in Cycle 3 in attempting to reflect the meaning of Description A (in Figure 8). They provide insights into the intertwinement of students' conceptual and language-related learning pathways. The first transcript shows how the receptive and productive language demands are interrelated. The transcript starts when Svenja's partner Fynn tried to explain whether Description A matched the streaming offer from DreamStream (Question 5 in Figure 8).

340	Fynn	Uh. First, the equation doesn't indicate anything [*reading Description A*]. Well, in the end it does, but [*simultaneously*] one shall calculate it.
341	Svenja	[*simultaneously*] . . . It doesn't indicate a price. So.
342	Fynn	Exactly.
343	Svenja	But . . . what one, uh, has to pay.
344	Teacher	[*approvingly*] Mhm.
345	Svenja	It isn't a fixed price; um, well, so total price, because one doesn't know now how many months, because . . . see as months. That's why . . . .
346	Teacher	. . . does it match?
347	Svenja	Um. "In dependency of the months." So this here [*points to the functional equation of the DreamStream offer*]. This . . ., let's say here, dependency are five months.
348	Teacher	[*approvingly*] Mhm.
349	Svenja	So that one is able to calculate the price—the total price one has to pay after five months.

Fynn had difficulty identifying the phrase that was relevant to deciding whether the description matched or not. He justified his first judgment that it mismatched by saying that "the equation doesn't indicate anything" (Line 340).

Svenja (for whom the analysis is depicted in Figure 9) elaborated Fynn's utterance with respect to the ||dependent variable|| and asserted that the equation did not indicate one fixed price (Lines 341-345). She approximates this idea in three steps: "it doesn't indicate a price" (Line 341), "but what one has to pay" (Line 343), and, finally, "it isn't a fixed price . . . because one doesn't know now how many months" (Line 345).

After an incoherent utterance in Line 343, she started to address several facets, with more language coherence in Line 345: She compacted the ||varying quantity II|| and explained the ||dependent variable|| by relating it to the ||independent variable||. With her utterance "one doesn't know now how many months," she addressed the ||direction of dependency||.

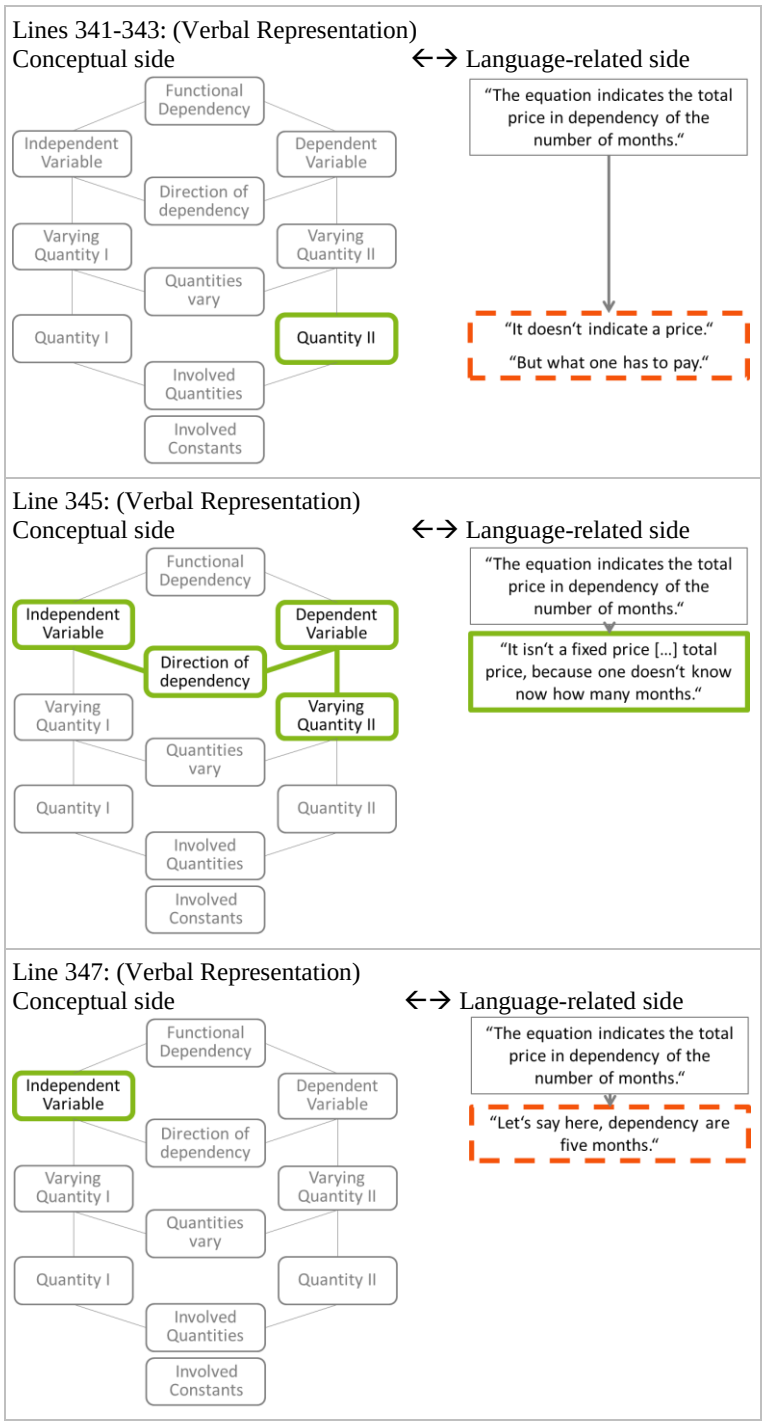

Figure 9. First part of the reconstruction of Svenja's learning pathway in the model of conceptual facets

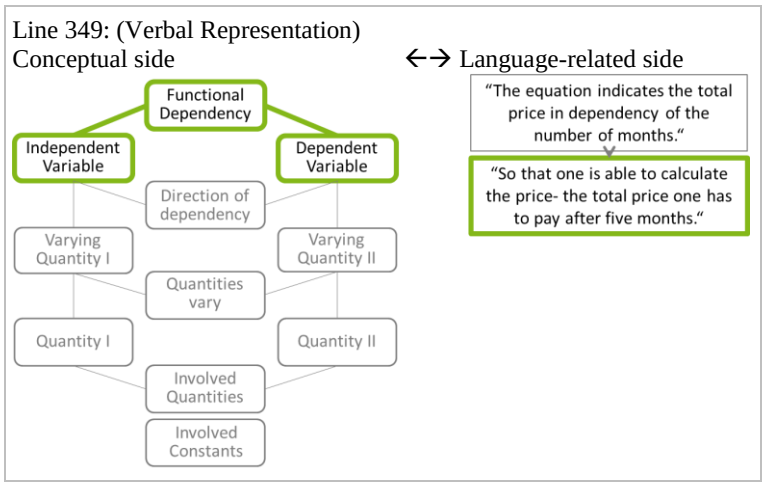

Figure 9. contiuned

Having unfolded the necessary conceptual facets in this way, she condensed them again to the given phrase "in dependency of the months" (Line 347). When she intended to evaluate the function for value 5 she chose as an example, she articulated this intention by saying "dependency are five months" (Line 347) as a not yet completely adequate but highly condensed phrase. The last utterance, "the total price one has to pay after five months" (Line 349), is a perfect description of her example. Thus, she adequately addressed the ||direction of dependency|| with reference to the ||independent variable|| and the ||dependent variable|| on a high level of compaction.

Within Lines 341-349, Svenja decomposed the condensed phrase and successively described its facets. The process shows how much language production is necessary for the conceptual process of unfolding. All four discursive demands (READING, IDENTIFYING, INTERPRETING, and EXPLAINING) are involved here and mastered with respect to the relation between verbal and symbolic representation.

Some minutes later, Svenja and her partner Fynn have assigned all matching phrases to the DreamStream offer. The transcript below starts when Svenja explained why the same phrases match Streamox3 (in Figure 8):

377 Svenja Because these 9,99 € are per month
378 Teacher Mhm
379 Svenja [*Points to the Streamox3 offer*] and there one time this unique [the basic rate] as here these unique 5 € [*looks at the DreamStream offer*], one has to pay.
Hence, these as well [*points to the phrases they have assigned to the DreamStream offer*] . . .
for both the same.

Svenja (Figure 10) activated the deviant concept-in-action ||involved constants|| in order to justify matching both descriptions. This hindered her from justifying the match using the

||involved quantities||, which were compacted in the ||functional dependency||. Svenja also changed her reading strategy:

385 Svenja Let's say we have five months again [*points to the Streamox3 offer*], then one calculates five times this amount that one has to pay per month.
386 Teacher Mhm
387 Svenja Uh. Plus this starting amount that you have to pay generally when you buy this box.
388 Teacher Mhm
389 Svenja That's the same as here, um, when one subscribes, one has to pay always these five Euros [*points to the DreamStream offer*] and . . . five months—we take, um, this 20 multiplied by 5.
390 Teacher Mhm
391 Svenja Then . . . this price per month multiplied by five plus this one-time five Euros. That is exactly the same as here [*points to the Streamox3 offer*], so to speak.

In Lines 385-391, Svenja connected the two ||involved constants|| and the ||independent variable|| by the calculation rule and thereby justified the match of the phrases using the

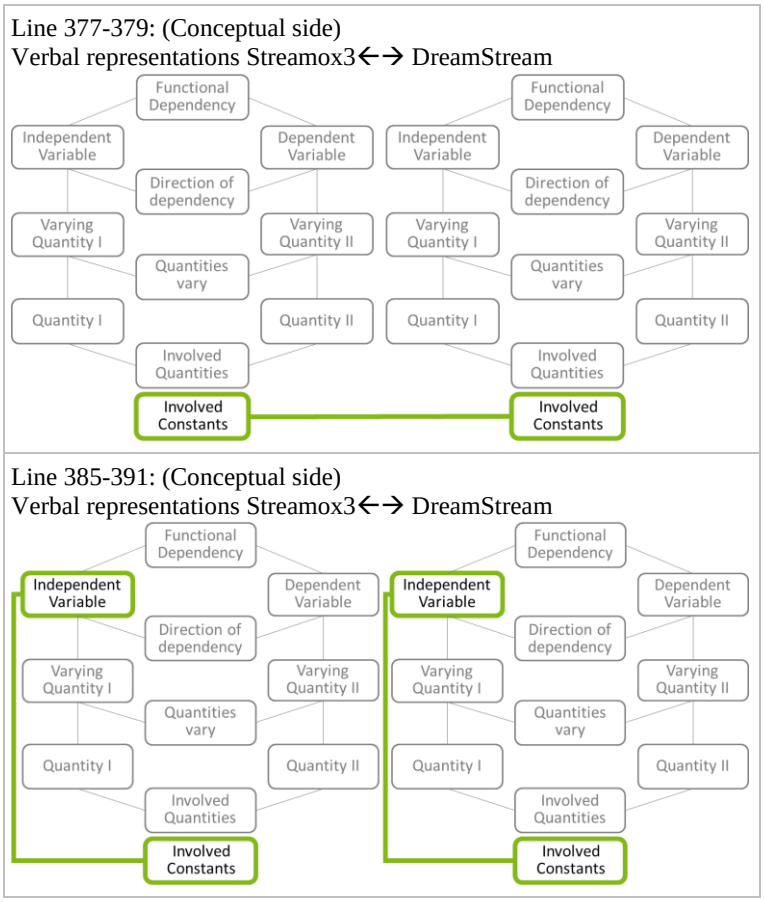

Figure 10. Second part of the reconstruction of Svenja's learning pathway in the model of conceptual facets

theorem-in-action, <For controlling the match of phrases to two different equations, the meaning of the start value, rate of change, and the independent quantity can be compared>. Nevertheless, so far, she has not related the ||independent variable|| to the more compacted facet ||functional dependency||, as she has not referred to the ||dependent variable||. Moreover, interestingly, the phrases she used all referred to calculation rules, not directly to the conceptual facets and their meaning in the context. The teacher asked her to reconsider the tables.

392 Teacher Uh, so, what does that description have to do with the tables? You have always looked at the equations . . . How can I find something in the tables that matches this here well [*hints to Description A*] . . .

393 Svenja [*4 sec break*] Well, here we have the months [po*ints to the left column of the DreamStream table*] and here we have the total price, [*points to the right column of the DreamStream table*] we have to pay every month. . . . again an example of five months.

394 Teacher Mhm

395 Svenja Having multiplied this . . . by 20, we have these 500 Euros, which we have to pay for five months [*points to the DreamStream table*]. This means we have already calculated the price here. And here it is also the same [*points to the Streamox3 table*]. One has here the total price when one would pay for five months.

394 Teacher Yes

397 Svenja That is why that matches somehow, because it is in dependency of the months. When you subscribe for two or five months, it is thus always the total price [*points to the right column of the Streamox3 table*].

The table headers seemed to steer Svenja's attention to the ||involved quantities||, even if only implicitly addressed by the deictic "here" in Line 393 (analyzed in Figure 11). Including the ||involved constants|| in her calculation led her to think of the total prices as ||dependent variable|| in Line 395. Svenja activated the phrase "in dependency of" in order to justify the match between the two phrases using the identical ||functional dependency||. For doing so, she also addressed the independent variable|| and the ||dependent variable||.

This analysis of Svenja's pathway shows how the successive activities with varying phrases can support Svenja in addressing several conceptual facets and relating them to each other. Without going to the highest level, she succeeded in unfolding the compacted concept. The tables played a key role not only in Svenja's but also in other students' learning pathways as they scaffolded the comparison of texts with respect to the involved quantities.

For Svenja as well as for the 36 students in the other design experiments, the empirical analysis of students' learning pathways has proven the analytic power of the facet model as an analytical tool for extrapolating students' conceptual pathways in dealing with functional relationships and the connected language demands.

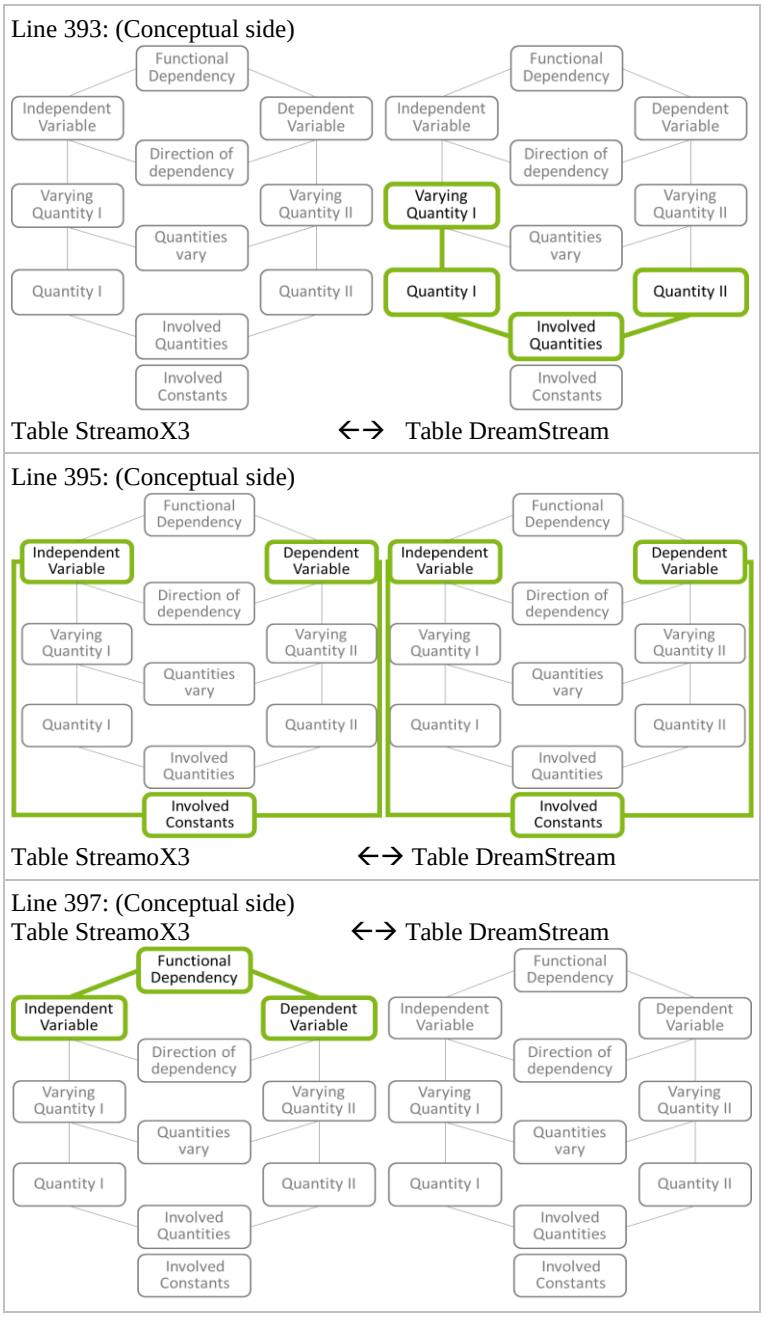

Figure 11. Third part of the reconstruction of Svenja's learning pathway in the model of conceptual facets

Furthermore, the analysis has provided empirical insights into the functioning of two highly important design principles: connecting representations and systematic variation of phrases, which both have the potential to initiate students' discursive activities and deepen their conceptual understanding (Prediger et al., 2016).

DISCUSSION

Statistical results showing that social achievement gaps can be traced back to language gaps have shifted the attention from specific challenges of multilingual learners to the wider demands of the school academic language register (including the technical register) for multilingual as well as for monolingual students (Hirsch, 2003; Prediger et al., 2015a). Taking into account the epistemic role of language, three functions of the academic language register must be taken into consideration: (1) as underestimated learning medium, (2) as unequally distributed learning prerequisite, and (3) as learning goal, which thus has to be specified more concretely (e.g., Lampert & Cobb, 2003; Thürmann et al., 2010).

In order to enhance language learners' pathways towards language learning goals, the relevant academic language demands have to be specified in more detail and for different mathematical topics (Bailey, 2007). It was the aim of this design research project to contribute to this topic-specific specification of academic language demands in both, the lexical, and the syntactical and discursive dimension (Moschkovich, 2002). The empirical analysis of students' reading processes (in Cycle 1) and then students' learning processes (in Cycle 3) provided insights into the complexities of academic language demands in their lexical, syntactical, and discursive dimensions.

For the analysis of students' reading processes, the often described activity of connecting representations (Duval, 2006; Swan, 1985) was differentiated in detail in order to locate the obstacles on the micro level. The resulting model of conceptual facets (Figure 2) follows Aebli's (1981) idea of concepts being flexibly unfolded or compacted (see Figure 7). Like every specification of demands, the model can be used analytically to describe typical processes, learning pathways, and obstacles (Sections 4.1, 4.2, and 5). Beyond this, it serves as a prescriptive orientation for designing the learning arrangement; in our case, the activities were focused on coordinating specific conceptual facets with each other (see Section 4.3). The conceptual facet model also allows for the analysis of language demands, as they were revealed to be strongly connected to the facet model: In order to be able to address conceptual facets from the model, language means that describe these facets on each level are necessary; in this way, the facet model allows differentiating the language means:

Language Demands Arising When Dealing with Functional Relationships (RQ1)

Four discursive demands were specified theoretically when dealing with functional relationships (always marked in capital letters): READING COMPLEX TEXTS (in this study, word problems of functions), IDENTIFYING the relevant but highly condensed phrases in which the information about the functional relationship is coded, INTERPRETING TEXTS OR SYMBOLS, and, for developing the necessary conceptual understanding, EXPLAINING THE MEANING of functional relationships. The empirical analysis shows that these demands occur simultaneously, together with the simultaneous relevance of the communicative and the epistemic role of language. Syntactical demands are mainly receptive ones, appearing with highly condensed phrases in given texts. The productive language demands appear in the processes of making sense of the texts as well as in situations of conceptual development.

Most important with respect to the epistemic role of language are the findings of systematic parallelism of conceptual learning processes of both unfolding and compacting with the language-related processes of decomposing and condensing on the micro level of students' processes. This observation has immediate practical consequences as it leads to specifying possible scaffolds in the lexical and syntactical dimension in the processes for different facets. Beyond this, it might be theoretically relevant as it coordinates density (as a typical characteristic of academic language) with a typical characteristic of mathematics: Mathematical concepts are highly compacted constructs of complete networks of facets. However, once compacted they receive a new ontological status as reified objects. In this way, we found an empirical foundation of Sfard's (2008) theoretical assumption of inseparability of concept reification and language condensation. As argued in Section 4.2 and empirically illustrated by the case studies of Luisa (Section 4.1) and Svenja (Section 5), these processes of unfolding and compacting on the conceptual side as well as decomposing and condensing on the language-related side are intertwined. This gives another empirically grounded explanation as to why and which kind of academic language is required for learning mathematics.

Along the investigated learning pathways (e.g., in the case of Svenja), students meet various language demands in the lexical, syntactical, and discursive dimensions when trying to verbalize their emerging insights, and the parallel processes of compacting/unfolding on the conceptual side and condensing/decomposing on the language-related side are striking. Additionally, the increasing precision and explicitness of students' language could be found as relevant for explicitly addressing each facet.

In these case studies, language appears in all three functions: as learning prerequisite and learning goal. At the same time, it is a learning medium for students' reflection on topic-specific phrases, which sensitizes students for language demands and for mastering concept demands. The analysis of the case of Svenja illustrates how she successively mastered the decoding of the complex texts and how she successively elaborated her language production for this purpose. In particular, the reflection of systematically varied phrases leads her to address and combine all relevant conceptual facets before compacting them again to one concept.

Effects of the Design Principle "Relating Registers by Systematic Variation of Phrases" to Support Students in Mastering the Interrelated Concept and Language Demands (RQ2)

In a similar way, the design principle of systematic variation of phrases also proved its situative potential for deepening the conceptual understanding in other cases (not presented here). From these findings, we draw the first evidence for the hypothesis that reflecting topic-specific complex phrases and their relation to each other can be an appropriate support for deepening students' conceptual understanding. The classroom design experiments of Cycle 4 (not reported here) suggest that this also seems to apply for more language-proficient students.

Limitations and Further Research Needs

Necessarily, the case studies presented here have limitations, especially with respect to the small sample of students (n = 4). Even when taking into account all 97 students in the sample of the overarching design research project, the results are limited by the specific topic-specific activities in view. Further research will be required to expand the scope of the results to more students, to other activities with functions (especially in the covariation perspective), and to other mathematical topics beyond functions.

However, the study substantially contributes to the research discourse on content- and language-integrated learning of functions by offering practical solutions for classrooms and enhancing the theoretical discourse on the role of academic language for mathematics learning. As emphasized by Barwell (2012), Moschkovich (2015), and others, language demands cannot be reduced to the lexical or syntactical dimension, as its discursive dimension always shows the most relevant complexities in the learning process. Research designs that allow for the investigation of learning processes can provide insights into the complex intertwinement of the communicative and epistemic role of the school academic language, being both learning medium and learning goal at the same time. Further research will be required to unpack these complexities for further mathematical topics.

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Subject-Specific Genres and Genre Awareness in Integrated Mathematics and Language Teaching

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ABSTRACT
The increasing attention devoted to the role of language in the different school subjects calls for approaches of integrated subject matter and language teaching and learning. In this article we argue for the importance of subject-specific genres for integrated mathematics and language teaching. Based on an exemplary analysis of geometric construction texts we show that subject-specific genres in the context of schooling might be influenced by different academic and institutional contexts. In a case study of a classroom discourse in 7th grade about geometric construction texts we show how these different contexts pose a challenge for teaching this genre. As a result, genres in school mathematics might appear as blended genres. Based on our findings we refine the notion of genre awareness as an important aspect of teacher knowledge in order to better prepare teachers for the challenges of integrated subject matter and language teaching.

Keywords: genre, genre awareness, geometric construction, geometric construction text, geometry, subject-specific genre

INTRODUCTION

In the past decades an increasing awareness of the important role of language as a communicative and cognitive tool is noticeable in subject matter teaching. Smit (2013) even proposes a linguistic turn in educational research, which puts new demands on educators, designers and researchers in order to foster students' development of their language awareness when learning subject matter content.

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State of the literature

- Recent research emphasizes the important role of language for mathematics learning and calls for approaches integrating language and mathematics learning and teaching.
- So far, research regarding linguistic properties of mathematical language tries to identify special features of mathematical language and acknowledges the existence of different genres in mathematics.
- Genre-pedagogy and genre-based approaches show that taking genres as basis for subject matter learning is a fruitful way for integrated subject matter and language teaching and learning.

Contribution of this paper to the literature

- The analysis of the genre of geometric construction texts from different perspectives (disciplinary mathematics, didactics of mathematics, and mathematics education) will contribute to an epistemological understanding of the notion of subject-specific genres.
- The analysis of an episode of classroom discourse shows that dealing with mathematical genres in school is a specific challenge, because genre features have to be implemented in a consistent way in order to foster an integrated subject matter and language learning.
- The notion of genre awareness as an important aspect of teacher knowledge is refined.

It is widely acknowledged that subject matter and language learning have to be integrated, because teaching language as something separate from content prevents learning in authentic contexts and does not provide access to subject-specific language learning. For second language learning, Gibbons (2002) suggests the integration of language learning with curriculum content and argues: "*From a language-teaching perspective, then the curriculum can be seen as providing authentic contexts for the development of subject-specific genres and registers*" (p. 119).

In the literature, we find many claims that relate to mathematical discourse, mathematical texts or the mathematical register in general without taking the particularities of the specific mathematical topic or activity into account. Based on a comprehensive literature survey of demands and properties of mathematical language in general and of mathematical texts in particular Österholm and Bergqvist (2013) conclude that "*it seems difficult to make claims that are valid for all mathematical texts or for mathematical language in general*" (p. 752). Scholars in the field call for more differentiated views of mathematical texts, discourses and genres. Prediger and Wessel (2013) argue for multiple mathematical registers. Moschkovich (2010) suggests that "*mathematical discourse is not a single, monolithic, or homogeneous discourse*" (p. 153). Whereas she thinks of a spectrum of discourse practices in different contexts such as "academic, workplace, playground, home, and so on" (p. 153), Morgan (1998) also draws attention to different genres of text: "Just as there are a number of varying social practices that may be labelled as mathematics […] there are a variety of genres of text that may be called mathematical (e.g. research paper, textbook, examination question and answer, puzzle, etc.) (p. 8)."

The notion of genre has been recognized by several researchers in mathematics education as a useful way to think about different kinds of texts within mathematics education. Until now, scholars mostly refer very broadly to the notion of genre in order to acknowledge the existence of a variety of texts within mathematics and mathematics education. Very few in-depth analyses of mathematical genres have been presented so far. Among them are the analyses by Gerofsky (1999) and Smit (2013). Gerofsky (1999) uses genre pedagogy as a theoretical framework "*to uncover hidden cultural meanings, assumptions and intentions inherent in the generic forms of schooling*" (p. 36). Smit (2013) designs learning opportunities, which "facilitate pupils' development of the language required for learning content" (p. 102) building on genre pedagogy. Both approaches contributed to a better understanding of subject-specific genres in mathematics education and led up to implications for teaching and learning mathematics. Whereas Gerofsky (2004) provides a comprehensive analysis of mathematical word problems as a well-known mathematical text type, Smit (2013) used genre pedagogy in order to design, enact and evaluate a 'new' pedagogical genre that she calls interpretative description of a line graph.

However, until now there is a lack of theoretical understanding of the subject-specific genres in school mathematics. On the one hand, a general and comprehensive survey of subject-specific genres in school mathematics is still missing. On the other hand, there is a dearth of research into the features of subject-specific genres in school mathematics.

In this article, we address subject-specific genres as a way to integrate subject matter and language learning. We focus on one particular genre of school mathematics, namely geometric construction texts. In German secondary education, a geometric construction comprises a drawing and a verbal step-by-step description of the single construction steps, which we call geometric construction texts. These are labeled with a fixed and unique technical term in the German speaking context ("Konstruktionsbeschreibung") and always occur in a well-defined mathematical context, namely geometric constructions.

The aim of this paper is twofold: On the one hand, our in-depth analysis of genre features will contribute to the understanding of another particular subject-specific genre in mathematics education. On the other hand, we will use geometric construction texts as an exemplary case to generally show how school mathematics genres are influenced by different contexts. Thus, our analysis will also contribute to an epistemological understanding of the notion of subject-specific genres. Analyzing a particular instance of teaching geometric construction texts in a 7th grade German mathematics classroom, we will show how genre features derived from different contexts of situation might be blended in instruction and lead to the implementation of inconsistent genre features.

Our analysis is guided by the following questions:

1) What are the functional and language features of geometric construction texts in the different contexts of situation, namely disciplinary mathematics, didactics of mathematics, and school mathematics education[1]?
2) Are geometric construction texts a single genre or do they have to be considered as different genres depending on the context of situation?

3) How do the different contexts of situation influence the teaching of geometric construction texts in mathematics classrooms?
4) What are the consequences for teaching geometric construction texts in mathematics classrooms?

In order to answer these questions, we first clarify the meaning of genre and discuss the role of the context of genre. This results in a differentiation of genres in terms of their relevant contexts (section 2). To analyze features of genres in different contexts of situation, we develop a model of genre (section 3). Methodologically, we use this model as a tool to analyze genre features from three perspectives: 1) particular text exemplars, 2) didactical teacher education literature and 3) classroom discourse. We describe our methods in section 4. The analysis of geometric construction texts in three different contexts of situation (disciplinary mathematics, didactics of mathematics and school mathematics education) and the discussion whether geometric construction texts have to be considered as one genre or as different genre is set out in section 5. Section 6 explores how genre features are implemented in classroom discourse. Based on this analysis, we describe how different contexts of situation influence the teaching of geometric construction texts and lead to the implementation of blended genres. As a consequence of our analysis, we refine the notion of genre awareness as a way to prepare teachers for integrated mathematics and language teaching (section 7).

GENRES AND THEIR CONTEXTS

In this section we clarify the meaning of genre and discuss the role of the context of a genre. This results in a differentiation of genres in terms of their relevant context.

Learning at school comprises learning with and from texts. In doing so, "particular text or discourse types", so called *genres* are used (Schleppegrell, 2004, p. 82). Having a genre available means possessing a schema of particular texts which serve a similar communicative function.

The origins of genre pedagogy lie in "the inequality between students with respect to participation in the learning activities of the school, including both classroom learning and individual learning from reading" (Rose & Martin 2012, p. 304). Therefore, it has been widely adopted in second language learning (Hyland, 2007). Nevertheless, focusing subject-specific genres is not merely a fruitful way to integrate language learning with curriculum content for second language learners, but rather for all learners in terms of developing their academic language. Since thinking about the educational experiences that promote the development of language proficiency is a crucial task for educators of all students (Snow & Uccelli, 2009), we will neither make an explicit distinction between first (L1) and second language (L2) learners, nor will we address the issue of L2 learners in mathematics in particular. Participating in a school mathematics culture requires reading and writing mathematical genres for all learners.

Genre is an abstract concept of using language in texts. "It is based on the idea that members of a community usually have little difficulty in recognizing similarities in the texts they use frequently and are able to draw on their repeated experiences with such texts to read,

understand, and perhaps write them relatively easily" (Hyland, 2007, p. 149). According to Schleppegrell (2004), the „ability to realize the genres that are characteristic of particular social contexts allows participation in and mutual understanding of those contexts" (p. 83). In summary, genres are patterns of cultural-social interaction in a specific context.

What is regarded as the relevant context of a genre differs among scholars in the field. Referring to the metaphor of context as "that which surrounds", Cole (1996) distinguishes different layers of context of a learning situation: the task, the lesson, the classroom, the school, the community. Depending on the scope of the context, which is in focus, different genres might be relevant. With regard to genre pedagogy, Martin and Rose (2008) and Gibbons (2002, p. 2) differentiate between two kinds of contexts: the context of culture and the context of situation. Context of culture means that "speakers within a culture share particular assumptions and expectations, so they are able to take for granted the ways in which things are done" (Gibbons, 2002, p. 2). Martin and Rose specify the relationship between context of culture and the context of situation. They model the genre at the stratum of culture beyond the context of situation. According to this, the context of culture entails the context of situation, which in turn entails the text in context: "So patterns of social organization in a culture are realized ('manifested/ symbolized/ encoded/ expressed') as patterns of social interaction in each context of situation, which in turn are realized as patterns of discourse in each text" (Martin & Rose, 2008, p. 10). It is important to emphasize that the context of situation of a text determines the genre. Variations in the context of situation lead to different kinds of genre. To understand this relationship, it is necessary to specify the context of situation. Therefore, Martin and Rose (2008) as well as Gibbons (2002) refer to Halliday's (1985) three social functions of language: field, tenor and mode. Field, tenor and mode form the context of situation of a text.

> **Field** refers to what is happening, to the nature of social action that is taking place: what it is that the participants are engaged in, in which language figures as some essential component.
>
> **Tenor** refers to who is taking part, to the nature of participants, their statuses and roles: what kinds of role relationship obtain, including permanent and temporary relationships of one kind or another, both the types of speech roles they are taking on in the dialogue and the whole cluster of socially significant relationships in which they are involved.
>
> **Mode** refers to what part language is playing, what it is that the participants are expecting language to do for them in the situation: the symbolic organisation of the text, the status that it has, and its function in the context. (Halliday, 1985, p. 12)

Halliday uses the term 'register' to refer to these three dimensions. Register comprises field, tenor and mode and contextualizes language (Martin, 1997). Distinguishing register from genre makes it possible "to model genre at the stratum of culture, beyond register, where it could function as a pattern of field, tenor and mode patterns" (Martin & Rose, 2008, p. 16).

Research related to genre focuses different kinds of contexts of situation and thus identified and described a variety of genres. On the one hand, Systemic Functional Linguistics distinguishes elemental genres such as narrative, recount, arguments, exposition etc. These are contextualized in the culture as a whole and do not take into account any specific layers of context of a learning situation as described by Cole. On the other hand, genre-pedagogy (Schleppegrell, 2004; Gibbons, 2002; Hyland, 2007) focuses on the school as the relevant context. Their representatives argue: "Because school is a culture with its own expectations for particular ways of using language, students need to learn about the genres of schooling and the purposes for which they are useful" (Schleppegrell, 2004, p. 83). Focusing on the school-level context, Schleppegrell (2004) and Gibbons (2002) identify a number of school-specific written genres: recount, narrative, procedure, report, account, explanation, exposition, discussion and argument. Gibbons uses the term 'text type' to refer to these school-specific genres (Gibbons, 2002, p. 54) and to delineate them from elemental genres. Referring to Schleppegrell (2004) we prefer the term 'genres of schooling'.

Schleppegrell (2004) goes one step further and focuses on one particular subject, namely science education, as a particular part of the context of schooling. Referring to the context of science education, she refines the list once more to common genres in science education, namely procedure, procedural recount, science report and science explanation.

In this paper we will focus on another subject, namely mathematics. Our interest is to apply genre theory to mathematics education. We argue that besides the genres of schooling every school subject has evolved its own genuine subject-based genres either reflecting the culture and context of the discipline in question or serving particular didactical/pedagogical purposes of the subject. We call these genuine subject-based genres subject-specific genres.

Figure 1 shows the relationship between different kinds of genres depending on the determining context. It is important to note that the figure only refers to the above explanations and contains contexts and genres relevant for this paper. In principle, the figure can be extended by different contexts of situation. Furthermore, contexts of situation are not always clearly separable and sometimes overlapping.

A MODEL FOR ANALYZING GENRE-FEATURES

The common method to analyze genre features is to derive them from particular text exemplars that realize the genre. An empirical analysis of the teaching of genre features is not common in the field of genre pedagogy. Therefore, we have to develop an appropriate analytic tool for analyzing genre features from different sources, namely given exemplars of the text that realize the genre, didactical literature for teacher education that refer to genre features (question 1) and classroom discourse that focusses on genre features (question 3).

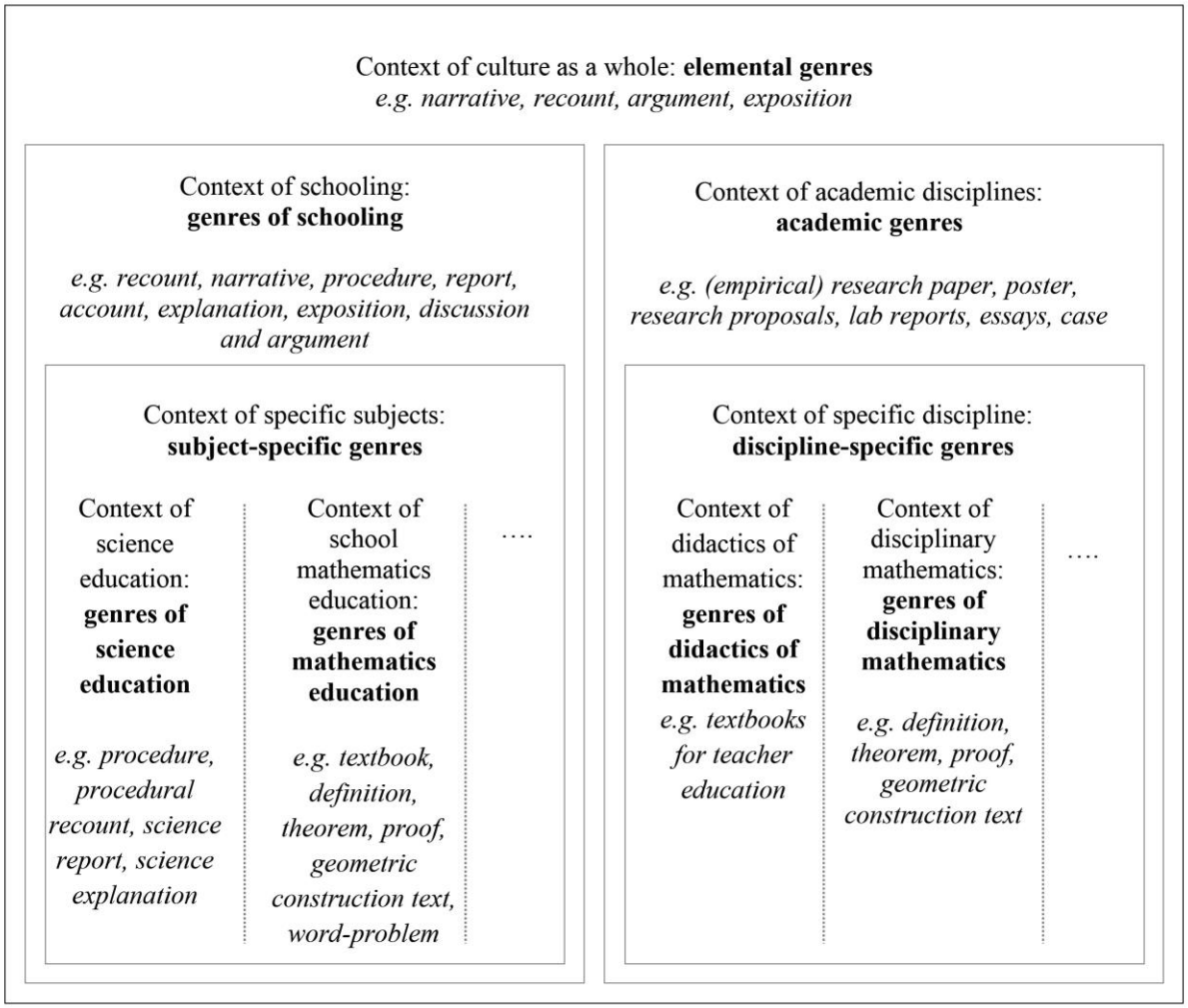


Figure 1. Genres depending on the determining context

Our model integrates three approaches: the characteristics of a genre stated by Gibbons (2002), Sandig's (1997) "model of a text convention", which helps to specify what constitutes a genre, and the modeling of context by Martin and Rose (2008).

Gibbons mentions four characteristics that make a genre different from another genre: 1) a specific purpose, 2) a particular overall structure, 3) specific linguistic features, 4) shared by members of a culture. Sandig (1997) refers to analogous characteristics, but in line with a functional view on language Sandig adds a general distinction between the linguistic function and the linguistic form of a genre. According to Sandig (1997), a genre is essentially constituted by a type of act and a text type. While the type of act refers to the properties of a genre in the sense of communicative functions and the context of situation (social function, the context in which it is used, the involved parties), the text type refers to the features of the genre in the sense of the language structure and the corresponding linguistic means to

realize the genre (speech acts, sequence pattern, formulation pattern etc.). Finally, it is the relationship between type of act and text type that constitutes the genre. The type of act directs the expectation of the text type while the features of the text type, especially the formulation patterns, indicate the type of act.

For our analysis, we draw on Sandig's general distinction between type of act and text type. It is possible to match Gibbons' characteristics of a genre to the text type and type of act: the specific purpose and the fact that a genre is shared by members of a culture are aspects of the type of act while a particular overall structure and specific linguistic features are aspects of the text type. We integrate these characteristics into the model of Sandig. For that, it is necessary to explain what it means that genres are shared by members of a culture. Following Gibbons (2002), this means that "genres are cultural" (p. 54) and knowing the context of culture is the basis for understanding genres. As already mentioned, Gibbons (2002) as well as Martin and Rose (2008) refer to Halliday's three social functions of language *field*, *tenor* and *mode*. Compared with Gibbons (2002), Martin and Rose (2008) specify the relationship between genre and context of situation (field, tenor and mode). They elucidate the role of field, tenor and mode in relation to the linguistic features of a genre and show that the context of situation varies according to the three register variables field, tenor and mode. Therefore, we integrate the modeling of context of situation by Martin and Rose (2008) in our genre model. Thus, we emphasize that the features of the text type vary dependent on the type of act and underline the interplay of type of act and text type features that makes a genre different from another.

Our synthesis of the characteristics of a genre stated by Gibbons (2002), of Sandig's (1997) "model of a text convention" and the modeling of context by Martin and Rose (2008) results in the model of genre features summarized in Table 1.

Table 1. Model of genre features

Type of act	**Text type**
Field: topic of the text *Mode*: Specific social purpose/function of the text *Tenor:* relationship between involved parties (writer and reader)	*Structure of the text* • constitutive and facultative speech acts • sequence pattern/organizational structure *Language features* • lexical features, e.g. technical language, style • grammatical features/grammatical constructions (e.g. connectives, adverbs, tense,…)

METHODOLOGY

Methodologically, we approach geometric construction texts from three perspectives: 1) We examine written text exemplars in terms of their genre features; 2) we collect descriptions of genre features from the didactical literature; 3) we analyze an audio-recorded and transcribed episode from a classroom discourse, in which geometric constructions and their descriptions were taught in a 7th grade mathematics class, in terms of explicit statements of genre features. For this end, we are using our model of genre features developed in the previous section. Our overall methodological approach is to use this model as a category system within qualitative content analysis (Mayring, 2015) in order to identify genre features from all three perspectives. Qualitative content analysis provides the appropriate rules and procedures for a methodological controlled analysis of text in order to categorize the textual material according to the categories of our model of genre features.

In the analysis of the classroom episode we complemented this method by conversation analysis (Sacks, Schegloff, & Jefferson, 1974) in order to understand the organization of interaction when teaching geometric constructions, particularly how text features are introduced and justified.

GENRE-FEATURES OF GEOMETRIC CONSTRUCTION TEXTS

In this section we analyze geometric construction texts in three different contexts of situation, namely disciplinary mathematics, didactics of mathematics andschool mathematics education. For analyzing geometric construction texts in the context of school mathematics education, we use geometric construction texts in mathematics textbooks. We regard mathematics textbooks as cultural artifacts that present the mathematical content potentially implemented into the classroom (Valverde et al., 2002).

Our analysis will reveal that geometric construction texts are not homogenous entities, but multifaceted with sometimes contradictory characteristics that are dependent on the context of situation.

Geometric Construction Texts in the Context of Disciplinary Mathematics

The combination of drawing and construction text of the geometric construction can actually be traced back to Euclid and has epistemological reasons: While the verbal description describes and defines the geometrical figure – the theoretical object with its properties – the drawing is a graphical representation of the figure (Parzysz, 1988; Sträßer, 2015). Compared to the drawing, the verbal description captures the history of the construction and thus allows to check whether each step is consistent with the axioms and theorems of geometry or not. Therefore, geometric construction texts are a special case of mathematical proof, namely an existence proof of the constructed geometrical objects. Consequently, their discourse function within the mathematics community is justification and their type of text corresponds to that of mathematical proof. Considering geometric approximations, the importance of the distinction between drawing and figure becomes most apparent. While the solution of the problem on the level of the drawing might yield an acceptable solution, the geometric construction text reveals that the solution is ‘only’ an approximation and not an exact construction of the geometrical object in question (Kadunz & Sträßer, 2007).

Geometric construction texts can also be regarded from an algorithmic perspective. From this perspective, the construction problem defines the given initial configuration and the (unknown) target configuration, while the geometric construction text is the algorithm, which yields a (unique) target configuration for every initial configuration (Holland, 2007, p. 80). Some Dynamic Geometry Systems (DGS) even use geometric construction texts as the user interface.

Geometric Construction Texts in the Context of Didactics of Mathematics

In order to analyze the genre of geometric construction texts in didactics of mathematics we refer to the relevant literature for teacher education in Germany. Switching the context from mathematics to didactics of mathematics yields further and different properties of geometric construction texts. Weigand et al. (2009) state that there are no definite norms for geometric construction texts. However, the authors name two principles, which are relevant for geometric construction texts: 1) They are supposed to provide a comprehensible and comprehensive account of the single constructions steps for someone else. 2) The language of geometric construction texts is supposed to adjust to learners' language and should develop from everyday to technical (mathematical) language, i.e. the didactical literature does not convey any norms of language features of geometric construction text in the school mathematics context.

Geometric construction texts serve several didactical functions (Weigand et al., 2009): They provide a good opportunity to verbalize actions, to write about mathematical procedures, and to communicate in the classroom. For learners they are supposed to serve as a report of the problem solution. Furthermore, they are a means to understand and evaluate the solution of the problem on the level of the drawing for learners and teachers. These functions are supposed to be realized by a comprehensible (verbal) description of the single steps of a geometric construction, which is sequenced in the order of the (basic) construction steps.

Geometric Construction Texts in the Context of Mathematics Textbooks

Figure 2a (translation in Figure 2b) provides an example of a geometric construction with geometric construction text from the textbook that the teacher and the students used in our case study.

A qualitative content analysis of widespread German mathematics textbooks for grade 7 based on the categories of our model of genre features (Table 1) reveals that the geometric construction text in Figure 2a is a prototypical example of a geometric construction text in terms of its text type features. Whereas it is expressed in the mathematics teacher education literature that the language features of geometric construction texts allow for variability, prototypical language features can be found in mathematics textbooks. Our analysis of this geometrical construction text reveals the following text type features:

1. Each step of the construction is numbered (sequence pattern/organizational structure).
2. Only main clauses (grammatical feature).
3. Short imperative sentences (grammatical feature).
4. Technical terms (line, circle, radius, intersect) (lexical features).

5. Symbols (c=5.5 cm, b=4.4 cm, a=3.6 cm, A, B, C) (lexical features).

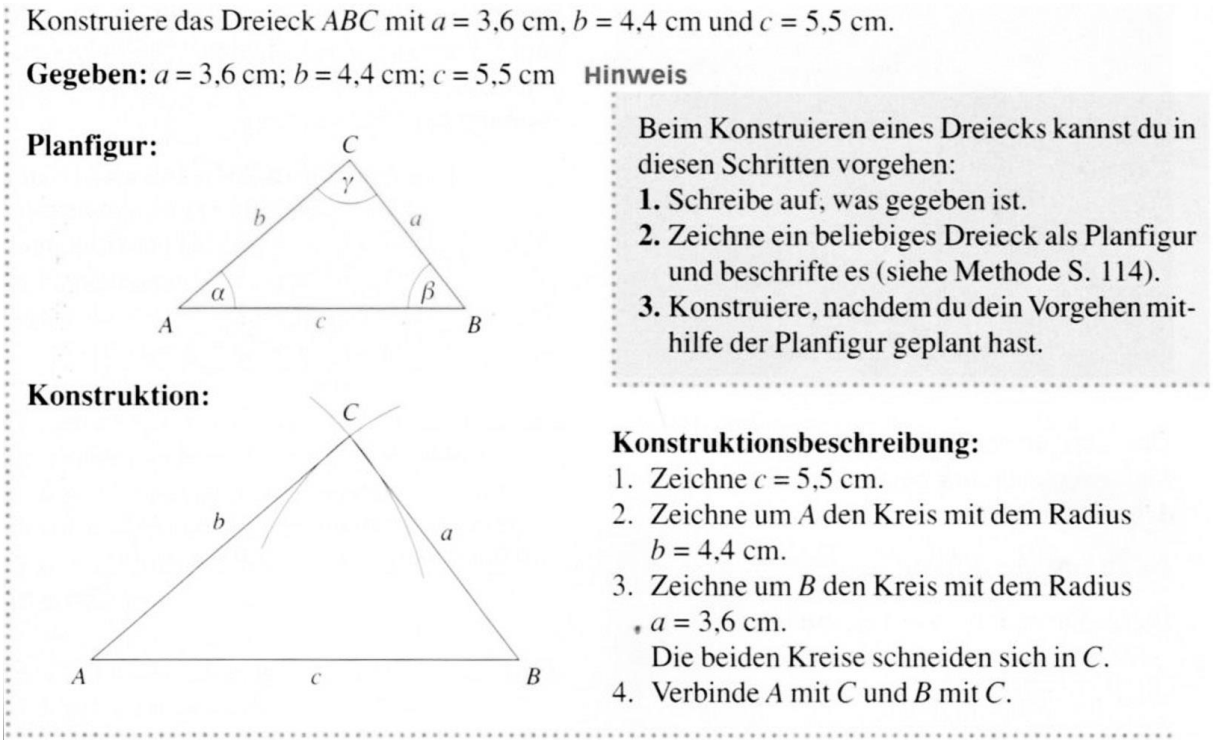
Characteristics of the type of act are commonly not explicated in mathematics textbooks.

Despite the fact that the mathematics teacher education literature promotes that the language of geometric construction texts is supposed to adjust to the learners,it is likely that the geometric construction texts in mathematics textbooks (and in automatically provided geometric construction texts in DGS) act as normative models (Dowling, 1996; Luke, de Castell, & Luke, 1989; Olson, 1989).

Table 2 summarizes the features of geometric constructions texts in different contexts and answers our first question. The features are categorized according to our model of genre features (Table 1). Our analysis was not able to unveil the manifestation of every category in every context of situation. Nevertheless, it is apparent in Table 2 that the genre features vary according to the context of situation.

Geometric Construction Texts as Blended Genre

Our analysis of geometric construction texts in three different contexts of situation (disciplinary mathematics, didactics of mathematics, school mathematics education) reveals


Konstruiere das Dreieck ABC mit $a = 3{,}6$ cm, $b = 4{,}4$ cm und $c = 5{,}5$ cm.

Gegeben: $a = 3{,}6$ cm; $b = 4{,}4$ cm; $c = 5{,}5$ cm

Planfigur:

Hinweis

Beim Konstruieren eines Dreiecks kannst du in diesen Schritten vorgehen:
1. Schreibe auf, was gegeben ist.
2. Zeichne ein beliebiges Dreieck als Planfigur und beschrifte es (siehe Methode S. 114).
3. Konstruiere, nachdem du dein Vorgehen mithilfe der Planfigur geplant hast.

Konstruktion:

Konstruktionsbeschreibung:
1. Zeichne $c = 5{,}5$ cm.
2. Zeichne um A den Kreis mit dem Radius $b = 4{,}4$ cm.
3. Zeichne um B den Kreis mit dem Radius $a = 3{,}6$ cm.
   Die beiden Kreise schneiden sich in C.
4. Verbinde A mit C und B mit C.

Figure 2a. Example of a geometric construction with geometric construction text from a German mathematics textbook (Koullen, 2006). (Transtlation in Figure 2b)

Construct triangle ABC with a = 3.6 cm, b = 4.4 cm, and c = 5.5 cm

Given: a = 3.6 cm, b = 4.4 cm, c = 5.5 cm

Tip
You can follow these steps in order to construct the triangle:
1. Write down the givens.
2. Sketch an arbitrary triangle and add the labels (see method p. 144).
3. Carry out the construction of the triangle after having planned how you will proceed with the help of your sketch.

Sketch

Construction

Geometric construction text:
1. Draw line c=5.5 cm.
2. Draw a circle around A with radius b=4.4 cm.
3. Draw a circle around B with radius a=3.6 cm. Both circles intersect in point C.
4. Connect A and C as well as B and C.

Figure 2b. Translation of Figure 2a

that the genre features vary dependent on the context of situation. These variations are especially visible in the mode and in the linguistic features. While the mode in disciplinary mathematics is that of justification, the mode in didactics of mathematics is that of reporting or describing.

As explained in section 3, a genre is constituted by the relationship between type of act and text type. The type of act directs the expectation of the text type while the features of the text type indicate the type of act. Consequently, differences in the type of act (e.g. in mode) also yield differences in the text type features of the genre and vice versa. Regarding the case of geometric construction texts this means that on the one hand, it is possible to derive language features that are related to the reporting mode explicated in the didactical literature: Geometric construction texts are usually written after the construction problem was solved on the level of the drawing. This retrospective perspective is typical for procedural recounts (Martin & Rose, 2008), which reflect (experimental) activities that have been done and thus provoke the use of the past tense. On the other hand, we can infer the type of act from the use of the imperative mood that is typically realized in mathematics textbooks. A prospective perspective and the use of imperative commands as in the example from the mathematics textbook is typical for the genre 'procedure', which aims at directing the actions of a possible reader (Martin & Rose, 2008).

Due to the variations of genre features in the different contexts of situation, geometric construction texts do not appear as a homogeneous and consistent genre. Nonetheless, it is

Table 2. Genre features of geometric construction texts in different contexts of situation

Type of act	**Text type**
1. Geometric construction texts in disciplinary mathematics	
Field: step-by-step-description of geometric construction *Mode:* • justification/existence proof *Tenor:* • mathematicians	*Structure of the text* • sequence of single basic constructions in the order of the construction
	Language features
2. Geometric construction texts in didactics of mathematics	
Field: step-by-step-description of geometric construction *Mode:* • comprehensible (verbal) description of single steps in geometric constructions • report of problem solution verbalization of actions • classroom communication *Tenor:* • learner and teacher	*Structure of the text* • sequence of single basic constructions in the order of the construction
	Language features • adjusts to language proficiency of learners • develops from everyday language to technical (mathematical) language
3. Geometric construction texts in textbooks	
Field: step-by-step-description of geometric construction *Mode:* *Tenor:* • learner and teacher	*Structure of the text* • sequence of single basic constructions in the order of the construction
	Language features • main clauses • short imperative sentences • technical terms • symbols

questionable, if geometric construction texts have to be considered as different genres. We argue that geometric construction texts will be easily recognized due to the occurrence of geometric construction texts in the context of geometric constructions (field) together with the step-by-step verbalization of construction steps (structure of the text). Furthermore, Feilke (2012) argues that the function and form of disciplinary genres is sometimes transformed in the context of schooling serving pedagogical and didactical functions. This also seems to be the case with geometric construction texts. While geometric construction texts have to be considered as a case of argumentative genre (existence proof) in disciplinary mathematics, their function in the pedagogical context is changed. Consequently, we consider geometric construction texts as a **blended genre** with varying features due to transformation in the context of schooling.

This fact poses particular challenges on teaching this genre. In the next section we will show how genre features of geometric construction texts from different contexts of situation are blended in the teaching of the genre.

GEOMETRIC CONSTRUCTION TEXTS AS A MATHEMATICAL GENRE IN CLASSROOM DISCOURSE

In this section we analyze one episode from a classroom discourse, in which geometric constructions and their descriptions were taught in a 7th grade mathematics class with 29 students. The questions that guided our analysis were, how the teacher implemented geometric construction texts in mathematics classroom and how the different contexts of situation influence the teaching of geometric construction texts (question 3).

The teacher has neither been introduced to genre-pedagogy before nor is he familiar with the recent efforts to foster language learning in the subjects. Here, we present an in depth analysis of one episode, where the geometric construction text is introduced for the first time. The methods of our analysis were described in section 4. This episode captures exemplarily what we have found in the whole lesson.

The Episode

After introducing the general problem of constructing triangles from only a few given properties – in this case the length of the three sides – the teacher develops a sketch of a triangle on the black board in order to mark the given magnitudes. During this activity he also repeats how sides and vertices of a triangle are labeled. The episode starts with the teacher developing a solution of the problem together with the students.

147	T:	Now we can start with the actual drawing. Any suggestions how we
148		carry out the construction? Frank?
149	S:	First we draw the base. So, AB 3.2 cm
150	T:	Ok. Let's do it. We use our 'Geodreieck'[2] and draw the base with 3.2 cm. It is important
151		that you immediately add the labels of the vertices at the ends of your side,
152		so you don't get confused. Instead of labeling the vertices

153		you can also add the measure of the side. This way, you've got
154		all the important information. And since I know how much you like writing and
155		how much you like to work neatly, we will write
156		a description of our construction right next to our drawing. We will do that
157		exemplarily today and will come back to it later, so that everyone
158		knows how we proceeded. The description of the geometric construction next to the drawing
159		and now it is important to be mathematically precise
160		in verbalizing and describing what we did, Sophie?
161	S:	We have drawn the base from A to B with 3.2 cm.
162	T:	Exactly, but we won't use such a complicated phrasing
163		'we have drawn the base', but the imperative mood
164		‚draw line segment', because it is a line segment from one point to the other, AB
165		with length 3.2 cm. This is the first step. This is
166		easy. Now it's going to be a little more difficult. We have two more sides given
167		and the triangle is supposed to look exactly as requested in the end. What will be
168		the second step? What do we have to do next?

In the first sequence (line 147-154) the teacher and the students develop the actual drawing of the construction. In the first turn the teacher initiates the activity and asks the students for suggestions how to carry out the construction (lines 147-148). One student answers by suggesting a first step of the construction. The teacher affirms (line 150) and starts to explain how to draw the triangle by using the 'Geodreieck' and how to label the vertices respectively adding the measure of the side. While explaining the procedure, the teacher carries out the drawing. When finishing the explanation and the drawing, the teacher leads over to the next sequence: the writing of the geometric construction text (lines 154-165).

The second sequence starts with justifying writing the text by referring ironically to the students' general motivation to write *(…since I know how much you like writing andhow much you like to work neatly*, lines 154-155) and by referring to the function of the text (*so that everyone knows how we proceeded*, lines 157-158), followed by the stipulation that the text has to be located next to the drawing. Then the teacher explicates the requirement "*to be mathematical precise*" when verbalizing and describing what they did (lines 159-160). Sophie frames the beginning of the text by saying "*We have drawn the base from A to B with 3.2 cm*." (line 161). He evaluates Sophie's answer as being too complicated and revises and rephrases her answer by stating to use the imperative mood "*draw line segment*" (line 164). The first sequence ends with the teacher's conclusion: "*This is the first step*." (line 165)

Qualitative Content Analysis of the Episode

The second sequence offers several instances in which the teacher relates to features of geometric construction texts. In the first place, the teacher explicitly refers to one important social function of geometric construction texts in the second sequence: "*so that everyone knows how we proceeded*". The indefinite pronoun "*everyone*" is used to denote an indefinite addressee. In the second place, the function is explicated that every step, i.e. the

chronological genesis of the construction, has to be described. However, a comprehensible rationale for writing geometric construction texts is not given. The teacher does not provide the students with reasons that give insight into the mathematical significance of a geometric construction text. Furthermore, he introduces language features of a geometric construction text explicitly by referring to a mathematical precise language and the use of the imperative mood. This is in accordance with the style of the geometric construction description from the textbook that is used in the class.

The teacher explicitly addresses type of act-as well as text type features of geometric construction texts. Table 3 summarizes the utterances that explicitly refer to the categories of our model of genre features.

Conversation Analysis of the Episode

In the second step of the analysis, our aim is to focus the relationship between the organization of sequences and language structure in order to understand how the teacher teaches geometrical constructions and writing geometrical construction texts and which role the use of language plays in it.

Analyzing the sequential organization of this episode shows that the teacher at first focuses on the actual drawing of the construction (= first sequence). After that he introduces how to write a geometric construction text (= second sequence).

Table 3. Synopsis of genre features explicitly referred to by the teacher

Type of act	**Text type**
Mode „Since I know how much you like writing and how much you like to work neatly we will write down a construction script next to our drawing“ „... so that everyone knows the way we proceeded.“ *Tenor* „everyone“	*Structure of the text* sequence following the sequence of the construction steps each step numbered
	Language features „it is important to be mathematical precise in verbalizing and describing what we did“ responding to student's suggestion: „Exactly, but we will not use such a complicated phrasing, but the imperative mood ‚draw line segment'“

This succession of steps implicates that the students use the present tense when developing the construction (line 149). Against this, the writing of the geometrical construction text is accomplished from a retrospective perspective. This retrospective perspective becomes apparent in the way language is used. When justifying writing the text the teacher refers to the function of geometric construction texts and explains this function using the past tense ("*how we proceeded*", line 158). Furthermore, he uses past tense to refer to the activity of drawing in the first sequence "*it is important to be mathematical precise in verbalizing and describing* ***what we did***", line 159-160). Accordingly, Sophie gives her answer using the present perfect (*We have drawn the base from A to B with 3.2 cm.*, line 161). With reference to the organizational structure of this episode this is a coherent answer and indicates how aptly the student replies to the teacher's language.

The teacher's revision of Sophie's answer by using the imperative mood can only be explained referring to the language features of geometrical construction texts in the mathematics textbook. The teacher does not provide any arguments why the imperative mood should be used. The students can only infer from the teacher's reaction ("*we won't use such a complicated phrasing*", line 162) that the imperative mood seems to be less complicated than the present perfect. This is likely to be confusing since the teacher himself provoked the use of the present perfect due to his own use of the past tense.

Furthermore, we find two instances in our analysis where the teacher adjusts students' wordings. In the first instance, the teacher shifts quickly between different denominations of a line and addresses the two possibilities of either label the vertices with capital letters or the side with the corresponding length, respectively (line 152). In the second instance, the teacher first repeats the wrong term 'base' that the student offered and substitutes it with "*line segment*" (line 164) while also changing the grammar from past tense to imperative mood. He does not provide an explanation, why he changes the technical term, but justifies the use of "*line segment*" en passant ("*because it is a line segment from one point to the other*" (line 164)).

Adjusting students' wordings might be motivated by the requirement "*to be mathematical precise in verbalizing and describing what we did*" (line 159-160). However, he does not make this explicit to the students.

DISCUSSION

The analysis of the episode reveals that the teacher implements important genre features of geometric construction texts. The main theme of the lesson is to solve the geometric construction problem on the level of the drawing. The geometric construction text is introduced as a supplement to the drawing. Therefore, geometric construction texts are introduced from the retrospective perspective after a construction step has been carried out on the level of the drawing. The mode that is enacted in the learning situation induces the use of the past tense when verbalizing the action. This is coherent with the teachers own use of language in the classroom communication. Accordingly, the realized type of act (mode) is that of reporting (the problem solution) or of verbalizing (actions). Thus, the teacher enacts one particular modeas a feature of the type of act of geometric construction texts in the classroom discourse.

Furthermore, the teacher introduces the sequence model and the use of the imperative mood as features of geometric construction texts. These text type features match the features of geometric construction texts as procedures, which are passed on by the textbook. He explicitly refers to these text type features in the classroom discourse and realizes them in the developing written text product. The imperative mood, however, is appropriate for a different type of act (mode), namely a comprehensible (verbal) description of single steps in geometric constructions. Consequently, the implemented text type features do not coincide with the explicated type of act features. Furthermore, they would have required a different structure of the lesson. An interplay between the type of act and text type features is neither explicitly addressed nor is it inherent in the classroom discourse.

In summary, the teacher borrows properties from different contexts of situation, namely didactics of mathematics and school mathematics education (textbooks) and blends them in instruction. Therefore, his implementation is characterized by inconsistencies between the type of act (function) and the text type (form) of the genre. As a result, two different genres of schooling play a part in the episode: procedure and procedural recount. While the student's use of the present perfect is in accordance with the whole classroom discourse and refers to the genre of procedural recount the teacher changes the perspective from retrospective to prospective and offers the imperative mood, which is a typical feature of the genre procedure. Although we cannot infer any substantial effects on students' learning of geometric construction texts it is questionable if this blending does contribute to the development of language proficiency in mathematics. Even if the students are able to write geometric construction texts after instruction, it is likely that the students do not understand the reasons for the language features. From our point of view, this should be a goal for integrated subject matter and language learning.

Furthermore, we cannot infer from the limited data why the teacher introduced geometric construction texts in the way he did. We can see that he generally seems to be motivated to communicate features of geometric construction texts to the students. Therefore, we hypothesize that he is probably not aware of other features, and especially not the interrelations between type of act and text type. Otherwise, he would have probably communicated these to his students.

CONSEQUENCES FOR TEACHING SUBJECT-SPECIFIC GENRES

Our analysis of geometric constructions texts reveals that different contexts of situation inform the features of this subject-specific genre. As a consequence, the features of this genre vary according to the context of situation and a blended genre emerges. Our analysis of the episode revealed how these varying and sometimes conflicting features were blended in the teaching of the genre in the classroom. While a geometric construction text with similar features as in the used textbook was developed in the classroom, the teacher's contradictory explications of genre features are likely to impede the students' understanding of the relationship between the genre's function (type of act) and form (text type).

Subject-specific genres play an integral role with regard to integrated subject matter and language teaching. While their type of act relates to subject specific learning goals, their text

type features enact their subject specific function with the appropriate genre-specific linguistic means. Therefore, we see it as an important aspect of successful integrated subject matter and language teaching that type of act and text type features of a genre are enacted in a consistent, mutually related manner. Therefore, we ask how to empower teachers to teach subject-specific genres in this way. From our point of view, the answer to this question and to our fourth question (What are the consequences for teaching geometric construction texts in mathematics classrooms?) lies in what Devitt (2009) has termed "the teacher's genre awareness", i.e."the teacher being conscious of the genre decisions he or she makes and what those decisions will teach students" (p. 339). We argue that "genre awareness" is an important aspect of teacher knowledge in order to contribute to a successful subject matter and language integrated teaching. Since Devitts definition of the teachers' genre awareness remains vague, our aim in this section is to detail the notion of genre awareness based on our analysis of geometric construction texts as a subject-specific genre and its implementation in the classroom.

In our analysis of the episode we concluded that the teacher was probably not aware of the multiple and inconsistent features of geometric construction texts. Accordingly, we argue that an important prerequisite for an integrated teaching of mathematics and language is to know about the features of subject-specific genres and possible genre-variations. In order to contribute to subject-specific learning goals, it seems vitally important to know about the epistemologically grounded subject-specific functions of the genre.

However, mere knowledge about the genre features and possible variations does not seem to suffice. On the one hand, our analysis of geometric construction texts has shown that type of act and related text type features of a genre might even be ambiguous or contradictory due to influences from different contexts of situation. On the other hand, our case study of the implementation of geometric construction texts in a mathematics classroom revealed how the language of the teacher induces features of the genre, which conflict his explicit teaching of genre features. The latter aspect is also inherent in the whole classroom discourse: Introducing geometric construction texts as a retrospective of what was done and at the same time seeking for the imperative mood as the appropriate language for geometric construction texts implements an inconsistent genre, which is likely to yield confusion or misunderstanding. Besides knowing about genre features and possible variations it is also important for teachers to know about the mutual dependencies and possible incompatibilities of genre features. This knowledge is a prerequisite for designing appropriate learning arrangements. As suggested in genre-pedagogy, an 'appropriate' learning arrangement starts with setting the context, i.e. "revealing genre purposes and the settings in which it is commonly used" (Hyland, 2007, p. 159). In the case of geometric construction texts this could be achieved by focusing on

a) the epistemological function as an existence proof;
b) the didactical functions (report of the solution of the problem, verbal description of single steps in geometric constructions, the verbalization of actions and the communication in the classroom).

The introduction of further genre features has to relate to and has to be consistent with the genre purposes, which are inherent in the learning arrangement. Finally, the language of the teacher has to adjust to the implemented genre purposes and related genre features.

In summary, genre awareness comprises

a) knowledge about genre features, their interrelations and possible variations
b) to design learning arrangements according to genre purposes,
c) to use a language that is consistent with the enacted genre purposes and to implement genre features that are consistent with the learning arrangement and the enacted genre purposes.

CONCLUSIONS

Referring to genre pedagogy and the question of the relevant context of a genre we argued that focusing subject-specific genres is a promising approach for integrated subject matter and language teaching, because

a) the type of act of a subject-specific genre is grounded in disciplinary and in subject-specific didactical purposes,
b) the text type features are means to achieve these purposes.

Our analysis of geometric construction texts revealed that genre features vary according to the context of situation under consideration. Genre features of different contexts of situation influence the classroom discourse of subject-specific genres. The challenge in integrated subject matter and language teaching is to implement subject-specific genres in the classroom with consistent interrelations between function and form.

We suggested "genre awareness" in subject matter teaching as an important aspect of teacher knowledge in order to address this problem. We have argued that genre awareness comprises

a) knowledge about genre features, their interrelations and possible variations
b) to design learning arrangements, according to genre purposes,
c) to use a language that is consistent with the enacted genre purposes and to implement genre features that are consistent with the learning arrangement and the enacted genre purposes.

Although grounded in the empirical analysis of a case, genre awareness is a theoretical and normative concept. The question remains, if teachers' genre awareness will actually improve students' understanding and writing of geometric construction texts. This will be an aspect of further investigation.

So far, it is not common to apply genre theory in order to analyze the implementation of genre features into the classroom. Our methodological approach to analyze classroom discourse based on our model of genre features proved to be fruitful in order to better understand the implementation of genre features into the classroom. Therefore, our methodology contributes to the methodological repertoire of research into genre-pedagogy.

NOTES

1. With the term „didactics of mathematics“ we refer to the scientific discipline that investigates the teaching and learning of mathematics and is in charge of mathematics teacher education. „School mathematics education“ refers to the teaching and learning of mathematics in school.

2. In German speaking countries the common tools used for geometric construction are compass and ‚Geodreieck'. A ‚Geodreieck' is a special set square or triangle combining a 90-45-45 triangle with a protractor and a ruler into a single tool made of clear plastic.

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Formation of Language Identities in a Bilingual Teaching Intervention on Fractions

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ABSTRACT

Students' identities are connected to their productive participation in the mathematics classroom. A bilingual Turkish-German teaching intervention intended to foster the students' conceptual understanding of fractions has to account for the students' identities, since the students' identities within the intervention influence how the students utilize the learning opportunities. To account for the dynamic and interactive nature of identities, positioning theory was applied to reconstruct the students' identities as multilingual mathematics learners in four teaching intervention groups with four different teachers. In each group, students developed different identities as multilingual mathematics learners, ranging from "student in need of help" to "student responsible for mathematics." These identities were differently affected by the Turkish language. The analysis indicates that Turkish becomes a resource in the mathematical conversations when the students collaborate towards a consensual solution and are made responsible for each other's understanding. As a consequence, for developing teaching interventions aiming at building on students' multilingual resources for participating in mathematical discourses, the ways in which students can develop identities must be taken into account in order to enhance productive engagement.

Keywords: multilingual learning, identity, fractions, language, bilingual teaching intervention

INTRODUCTION

While bilingual Turkish-German students use Turkish and German language in their everyday life, Turkish is usually excluded from German mathematics classrooms by most schools' language policies. Establishing the official language as the exclusive language of instruction by not allowing other languages in the classroom has often been criticized (Planas & Setati, 2009), as it contributes to establishing a "language of power" associated with academic success (similar to the English language in South Africa; Setati, 2008).

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State of the literature

- Students' identities influence how they participate in mathematics classrooms and thus their opportunities to learn mathematics.
- There is a need for understanding the specific conditions that support multilingual students in activating their multilingual resources for mathematics learning in a mainly monolingual school system.
- It is an open question as to how the inclusion of home languages in teaching interventions might affect the students' identities as mathematics learners in general and their identity as multilingual mathematics learners specifically.

Contribution of this paper to the literature

- The empirical analysis extrapolates different forms of identities as multilingual mathematics learners. These identities can be shown to be interactively established, with teachers' substantial impact on the process.
- Making students accountable for arriving at a consensual solution seems to enhance the establishment of identities as mathematics learners with responsibility for the mathematics. These patterns occur often in situations where students use both languages from their multilingual repertoire.
- Future multilingual teaching interventions need to address issues of language identities in order to facilitate the activation of multilingual resources.

As a result, many multilingual students in Germany identify themselves as German-speaking mathematics learners. However, this is problematic: First, it is against the official recommendation of the Council of Europe to include students' home languages in subject matter courses (Baecco et al., 2010). Second, empirical studies have shown that when multilingual students feel that their home language "is good enough for learning mathematics" (Noren, 2008, p. 45), their interest in mathematics can increase (Noren, 2008). In particular, when multiple languages are allowed for negotiating the meaning of concepts, student participation has been shown to be promoted (Noren, 2015). Thus, there may be benefits for multilingual Turkish-German students to identify themselves as Turkish speakers in the mathematics classroom.

The use of language shapes the ways in which students identify themselves: "We use language to get recognized as taking on a certain identity or role, that is, to build an identity here and now" (Gee, 1999, p. 18). While in everyday life mixing and code-switching between Turkish and German is the normal way of speaking for many second- and third-generation Turkish-German bilinguals (Auer, 2011), in the mathematics classroom this identification is not possible, as the activation of multilingual resources is usually not allowed (Meyer, Prediger, César, & Norén, 2016). Thus, students have to cope with contrasting language contexts whether they are in class at school or outside of the school in their normal societal environment. This may lead them to developing distinct identities depending on these contexts.

From a sociocultural perspective, student identity has proven a useful construct to capture participation patterns of underprivileged students with a special social status in mathematics classrooms, such as low-performing students (Lange, 2016), language learners, or multilingual students (e.g., Planas, 2011; for an overview on the identity construct, see Bishop, 2012).

This study investigates how a bilingual Turkish-German teaching intervention, intended for fostering students' conceptual understanding of fractions and implemented in 11 groups by four teachers, might support the students' identification with being Turkish mathematics learners.

In particular, the article will

- argue that opportunities to learn depend at least in part on how students identify themselves as multilingual mathematics learners in the ongoing and evolving conversations of the teaching intervention (Sections 2 and 3);
- present a teaching intervention intended to support students' learning of fractions by activating the students' Turkish language resources (Section 4); and,
- by qualitatively analyzing four different groups, show that students' identities develop differently, where different storylines evoke and allow for different personal stories by which students' find their place in the ongoing conversations (Section 5).

IDENTITY IN THE MATHEMATICS CLASSROOM

Identity manifests itself in stories about individuals, either told or held true by individuals themselves or by others (Sfard & Prusak, 2005). It encompasses the individuals' identification with the activities in the mathematics classroom (Cobb, Gresalfi, & Hodge, 2009) in the form of stories that individuals tell about themselves, here regarded as personal stories. At the same time, it encompasses stories told by others about individuals to identify them in certain ways, for example, a teacher identifying a student as multilingual (Reeves, 2009), here more generally referred to as stories. In this study, of particular interest is the students' identity as multilingual mathematics learners.

The different ways of being identified and of identifying oneself in the classroom affect students' opportunities to participate. Students who have been identified as having special needs tend to refrain from participating as they do not want to interfere with the regular classroom, or they see themselves as having nothing to contribute to the mathematics at hand (Civil & Planas, 2004). Therefore, supporting students' identity as "problem solvers, claim makers, and solution reporters" (Empson, 2003, p. 337) in a study on fostering conceptual understanding is one factor that can foster low-achieving students' mathematical success.

Students' identities are interactively established and can be subject to change in ongoing conversations in the mathematics classroom, as the stories that are told about them continually develop in these conversations. Accordingly, both the teacher and peers may influence the student's identity as a multilingual mathematics learner. Teachers who perceive multilingual students as underprivileged immigrants may attribute to them limited

mathematical capacities, which may result in the assignment of different tasks (Planas & Gorgorio, 2004). Perceiving English language-learning students as no different from their peers hinders teachers' ability to see these students' specific additional linguistic resources, resulting in fewer ways to utilize such resources (Reeves, 2009). In contrast, identifying English-learning students as problem solvers and mathematical thinkers has been shown to help students to develop their identity as capable mathematics learners (Turner, Dominguez, Maldonado, & Empson, 2013). Peers can have an equally high impact. In peer-to-peer interactions, students influence how they identify each other in terms of competence, which establishes unequal opportunities to participate. As a result, students may not work cooperatively on cooperative tasks (Bishop, 2012).

For this article, a teaching intervention was investigated in which 11 different groups were taught by four different teachers. The empirical analysis extrapolates quite different dynamics and opportunities for students to develop their mathematical identity in four of these 11 groups. It was an open question how the inclusion of the students' home languages in this teaching intervention might impact the students' identities as mathematics learners in general and their identity as multilingual mathematics learners specifically.

IDENTITY AS INTERACTIVE AND REFLEXIVE POSITIONING

Students—and teachers—identify themselves in the conversations in the classroom in the form of telling personal stories about themselves in regard to mathematics and using multiple languages. Positioning theory can account for the dynamics of identifying others and oneself in a teaching intervention group based on the actions of the individuals in the conversation, in this case teachers and students. Conversations unfold along storylines, where storylines can be understood as "mutually agreed upon contexts" (van Langenhove & Harré, 1999, p. 9) that establish culturally shared patterns of how a conversation develops. A conversation can revolve around multiple storylines, as the individuals can make reference to moral dilemmas, prototypical characters (the good, the evil, the multilingual), or cultural stereotypes (teacher/student, nurse/patient). The individuals involved understand these storylines differently since they will be based on their own individual previous experiences (see Davies & Harrè, 1990). In line with the perceived storyline(s), individuals position themselves and others in the unfolding conversations.

In an ongoing conversation, the participants try to be certain kinds of people (Bucholtz & Hall, 2005). The participating individuals will continually position themselves and others based on the perceived storyline of the conversation in which they are involved (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015) and on the personal stories the individuals tell about who they are in this conversation. "Positioning . . . is the discursive process whereby selves are located in conversations as observably and subjectively coherent participants in jointly produced story lines" (Davies & Harré, 1990, p. 48).

For example, a teacher can take a position (P) of helping a student understand (P1), so the student is positioned as in need of help (P2). These positions allow the teacher to make remarks on the correctness of the student's thinking, while it might relegate the student to ask comprehension questions; these rights and constraints characterize positions P1 and P2.

The teacher's and student's contributions might establish a storyline of tutoring in the eyes of both teacher and student and result in the conversation unfolding in line with this storyline (see van Langenhove & Harré, 1999, p. 17f). There is, however, no pre-determined way to take a position. The teacher might position a student as being in need of help, but the student might resist this positioning and take a different position – individuals can resist a teacher's positioning by means of personal stories that they tell themselves. However, being positioned in a certain way by the teacher or their peers might cause students to actualize or change their personal stories about themselves, for instance, by identifying themselves with the mathematics in a teaching intervention in new ways (Moghaddam, 1999, p. 75). At the same time, the teacher has a certain illocutionary force that makes resisting difficult for students, as teachers have a culturally acknowledged strong position in teacher/student-related storylines (see Davies & Harré, 1990).

The ongoing actualization of personal stories, a student's identity, is based on the dynamics of being continually positioned and of positioning oneself. The former is interactive positioning, here understood as the constraints for action that are interactionally placed upon the student, the expectations that are interactionally established, and the space for actions in which the students are free to act. For example, when a teacher encourages the students to speak in Turkish, the students might change their personal stories because they had previously been forbidden to speak Turkish in the regular classroom. The latter is reflexive positioning, composed of the constraints that students see for themselves, by the expectations they fulfill, and the individual possibilities to act that they see for themselves based on their individual personal stories (Moghaddam, 1999).

By distinguishing between reflexive and interactive positioning, my study reconstructed the ways that individuals' identities—their personal stories—aligned with the affordances of the teaching intervention—the storylines of the unfolding conversations and the general stories held true by the teachers about the students. In the eyes of the students, the teaching intervention might revolve around familiar storylines of teacher-student interactions from regular classrooms, this way suggesting traditional personal stories. However, I assumed that the teachers—with their coercive power to shape the conversation (see Reeves, 2009, for ELLs; also Yoon, 2008)—could act against such traditional storylines and positively influence the students' identities as multilingual mathematics learners in the teaching intervention groups. In summary, there is a complex dynamic of how personal stories develop, and there might even be cases where the personal stories of students have no room in the teaching intervention due to peers and the teacher holding true different stories about an individual.

The theoretical perspective presented here only allows reconstruction of identity as a highly contextualized phenomenon that is dependent upon the specifics of the teaching intervention as well as the notions and activities of the teacher and students.

Research Questions

This study addresses the following research questions:

- Q1. What storylines underlie the interactive positionings, and what personal stories are suggested by the reflexive positionings in the different teaching interventions?
- Q2. What is the spectrum of possible identities that are available to the students in the different teaching interventions, where each teacher might differently contribute to the students' identities?
- Q3. How are these related to the use of students' Turkish home language in the intervention groups?

RESEARCH CONTEXT AND RESEARCH DESIGN

This section introduces the operationalization of individual and normative identities, the research context as given by the teaching intervention of the larger project MuM-Multi (funded by the German ministry BMBF, grant 01JM1403A, held by Prediger, Redder, and Rehbein), its underlying design principles, and the methods of study for case selection and data analysis.

Operationalization: Individual and Normative Identity

The identification of the students' reflexive positionings provides insights into the students' ways of identifying themselves with both the mathematics and the Turkish language. In order to do that, it is important to identify the possibilities "to act" that students see for themselves and the expectations and constraints the need to fulfill. More specifically:

- The possibilities that a student sees for actions in the ongoing conversation are operationalized with the category *initiative*. I assume that students' initiative to contribute to the conversation is equivalent to them intentionally taking a position (van Langenhove & Harré, 1999, p. 22f) and is thus indicative of the personal stories that guide the students' actions. Categories for initiative in educational contexts have been empirically reconstructed by Waring (2011).
- The expectations and constraints that students associate with their positioning are operationalized with the category *participation*. It is operationalized by the length of a contribution, which is dependent on the reflexive positioning of the student. Longer contributions that span two or more sentences are assumed to be instances where students have positioned themselves in line with personal stories that revolve around having an active part in the mathematics in the intervention. Short utterances (a sentence or less) indicate personal stories that have a more passive part in the teacher intervention. In a teaching intervention group where students are given only a few opportunities to participate, short utterances indicate that there are fewer opportunities for the students to act.

The reconstruction of interactive positionings provides insight into the stories that the teachers hold true about the students. Due to the teachers' coercive power, these stories frame the potential for the multilingual students to identify themselves with their multilinguality and the mathematics. Here, the focus is on how the students are positioned by the teacher in regard to the use of language, as they indicate the stories that are held true:

- The positionings that constrain and facilitate the use of multiple languages are operationalized with the category *language tasks and how they are accomplished*. This category encompasses on the one hand the assignment of a language task by the teacher and on the other hand the resulting ways in which the teacher takes up how the students accomplish this task.
- The established possibilities for language use in the conversation are operationalized with the category *language support and regulation*. It encompasses three facets: First, the help and support that is given by the teacher in regard to language; second, the praising of utterances; and third, the rejection of utterances in the conversation (see Table 1).

To address the issue of language-specific positionings, that is, whether the positionings are specifically associated with Turkish or German, the above categories were expanded to include the use of Turkish and German: Each attribution of a category was coded with the underlying language use, either Turkish (T), German (G), or, in cases of code-switching and -mixing, both (B). For example, if a language task is assigned in Turkish, then it is coded as T; this might indicate that the teacher acts in line with a storyline where students are continually expected to use Turkish for working on the assigned tasks, as they are interactively positioned as Turkish mathematics learners.

Design Principles for the Teaching Intervention

This study was embedded in a short-term German-Turkish bilingual teaching intervention that is part of the larger project MuM-Multi. The intervention aimed at fostering seventh grade multilingual students' conceptual understanding of fractions in regard to the part-of-

Table 1. Interpretation scheme and its operationalization

Establishing identities in a teaching intervention group	
Indicators for stories behind interactive positionings	**Indicators for personal stories behind reflexive positionings**
Language tasks and how they are accomplished: - Nature of language tasks - Use of language for accomplishing the task	**Initiative:** - Rephrasing the teacher - Offering the unfitted - Piggybacking - Activating source - Stepping in - Initiating action - Self-selecting for taking turns (Warwick, 2011)
Language support and regulation: - Support for language - What contributions are valued and by whom? - What contributions are rejected and by whom?	**Participation:** - Utterance spanning more than one sentence - Utterance spanning one sentence or less

whole concept, equivalence, and order of fractions. The intervention ran for five 90-minute sessions. In the project, a bilingual Turkish-German intervention was compared with a parallel monolingual intervention and a control group in a mixed methods design with a randomized control trial. The bilingual intervention was designed to foster multilingual students' conceptual understanding by activating their home language, Turkish. Forty-one multilingual students participated in 11 small groups. This study focused on four groups in the bilingual teaching intervention.

The bilingual teaching intervention is an adaption of a German monolingual teaching intervention for fostering students' conceptual understanding of fractions (Prediger & Wessel, 2013). Three main design principles guided the bilingual adaption of the monolingual teaching intervention (see Schüler-Meyer, Prediger, Kuzu, Wessel, & Redder, submitted):

1. Creating opportunities for bilingual communication and Turkish language production: Due to institutionally limited experience in speaking Turkish in schools (Grosjean, 2001), the Turkish language production is fostered systematically by material and teacher (Meyer & Prediger, 2011).
2. Applying the design principles of macro-scaffolding (Gibbons, 2002) and developing the Turkish formal registers: The learning trajectory was sequenced in line with scaffolding mechanisms and by specifically establishing everyday contexts that connect to the students' multilingual out-of-school experiences (Dominguez, 2011). Furthermore, we provided meaning-related words and phrases in those instances where they might be needed for conceptual understanding. For example, the words *Anteil* (German) and *düsen pay* (Turkish), meaning of "part of a whole," were introduced to express fractions (Kuzu, 2014).
3. Relating registers and languages within the relating registers approach: Moving continually upwards and downwards between everyday and formal registers provides learners with possibilities to construct meaning for mathematical language (Prediger, Clarkson, & Bose, 2016). Beyond that, the German and Turkish languages were continually related in the material (so that Turkish becomes a transparent resource; Setati, Molefe, & Langa, 2008) and code-switching was encouraged (Auer, 2011).

Methods for Data Gathering

Within MuM-Multi, students with a low achievement in a pre-test on fractions and a low proficiency in the German language (measured with a C-Test; Grotjahn, Klein-Braley, & Raatz, 2002) were chosen to participate, as these students are especially at risk and might profit most from a language-integrated teaching intervention on fractions. The students' varying proficiencies in Turkish were measured by a Turkish C-Test. Having few previous experiences in Turkish mathematics, the students' Turkish academic or technical language was less developed than their German academic and technical language. All students participated voluntarily in the teaching intervention.

Each session in each of the 11 teaching intervention groups was videotaped (11 groups x 5 sessions). The camera focused on a group of 2 to 3 students, while another 2 to 3 students participated in the same intervention group but were not videotaped. The video material was transcribed and translated by Turkish-German bilingual university students in ways that preserved the meaning of the Turkish utterances as much as possible.

Four teachers implemented the teaching interventions in one to four of the 11 groups. In most groups, the teachers stayed with their group over the course of the intervention. The teachers were trained in a preparation course to implement the teaching intervention in line with the presented principles.

Case Selection for Data Analysis

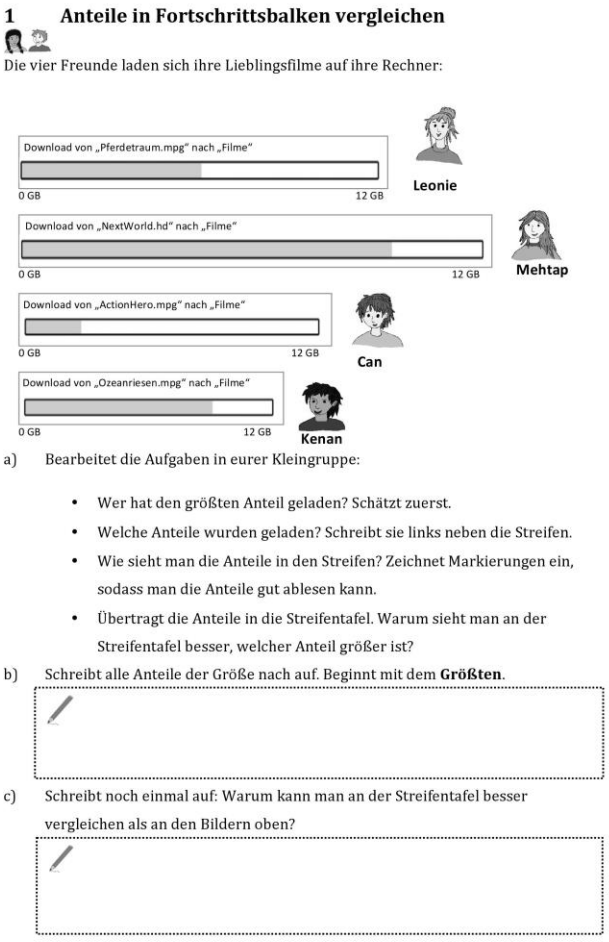
As discussed above, the teachers had important roles in the students' identity development in the teaching intervention. In order to capture and contrast diverse ways in which identities can be established in a bilingual teaching intervention, the group with the most vivid communication from each teacher was selected. This resulted in four focus intervention groups.

The analysis reported here focuses on the conversations within the first task of the third teaching intervention session. This focus task was based on the context of downloading movies: Four children downloaded movies; each download was presented with its own download bar (see Figure 1). The students were asked to reflect on the idea of the need for a standardized medium for comparing the downloads, namely, a fraction bar with the same length. The task was given in Turkish and German.

The task was chosen as a focus task for this study for two reasons:

- It is located at a central point of the intended learning trajectory: In Sessions 1 and 2, students had a chance to understand the use of the fraction bar and relevant keywords associated with it. In Session 3, it was intended that the students would internalize the nature of the whole in the part-of-a-whole relationship, for example, that the size of a fraction does not depend on the length of the fraction bar.

- The selected task implements the three design principles: In line with the first design principle, it connects to the students' everyday experiences, in this case the context of downloading movies. This task is also exploratory and encourages collaborative work. It provides room for the students to use both Turkish and German languages while collaboratively working on it. In line with the third principle, the task allows students to work with different representations, in this case the fraction bar and the symbolic representation.



Four friends are downloading movies on their PCs.

a) Work on the tasks in pairs:
- Who has downloaded the biggest share? Estimate first.
- What shares were downloaded? Write them next to the fraction bars.
- How do you see the shares in the fraction bar? Place marks so that you see them better.
- Transfer the shares into the fraction bar board. Why is the fraction bar board better for comparing the shares?

b) Write down the shares according to their size. Start with the biggest.

c) Write again: Why is the fraction bar board better for comparing the shares?

Figure 1. Download task: The first task in teaching intervention Session 3 (given to students in German and Turkish versions)

Data Analysis

The conversations in the focus tasks and the four focus intervention groups were analyzed with regard to interactive and reflexive positionings (see Table 1 for an overview), employing content analysis with the above described categories (Mayring, 2015). Within a category, stories are identified based on frequently occurring phenomena in the material for each group that are then condensed ("reducing procedures," p. 373) and explicated.

A comparison of stories in each teaching intervention group can indicate the conditions in the teaching intervention groups for students to develop their personal stories, that is, their

identities as multilingual learners of mathematics. Furthermore, it can capture how the students' actual personal stories develop within the conversation, that is, the actual development of the students' identity as multilingual mathematics learners. This gives insight into research question Q2. Comparisons across the groups indicate how the opportunities for developing an identity as a multilingual mathematics learner differ between the groups and can provide insights into storylines in line with research question Q1. The analysis of the use of language within interactive and reflexive positionings will indicate how the students' identity development is related to the use of Turkish and German, addressing research question Q3.

RESULTS

In the following, I compare and contrast two cases. These cases can be read as "extreme cases" with respect to very different opportunities for students to develop their identity. Based on these two cases, hypotheses for mechanisms underlying the construction of a positive identity towards multilingual mathematics learning were generated (Section 5.3). An overview of the results of the analysis for all four analyzed teaching intervention groups is given in Table 2.

Identities in Group E (Teacher: Mr. Flid)

Interactive positionings and related storyline of the conversation

In the following episode from teaching intervention Group E, whose teacher is Mr. Flid, Atiye presents a solution. In reaction to her, Mediha presents a competing solution (Turns 3218-3229). The teacher intervenes in the conversation and asks Atiye to explain how she arrived at her solution. In this way, he positions her as being knowledgeable of the mathematics at hand.

In Turn 3217, Atiye uses the notion of common denominator from the download context—namely that every fictional student downloads 12 gigabytes—to explain her answer, $\frac{7}{12}$. She argues that the grey area has a length of 7 in relation to the length of the fraction bar of 12. Atiye's utterance is followed by the teacher asking Mediha to also present her solution, which is "*Ikide üç?*" ("12 therein 3?") (Turn 3223). After that, the teacher asks Mediha to explain her solution, but she is not able to give an explanation that is understood by the others, so he asks Atiye to explain her solution to Mediha (Turn 3230).

In this episode, the teacher asks all participating students to present their solutions and to verbalize them. The teacher positions the students as being responsible for arriving at a shared understanding, since he asks Atiye to explain her thinking so that Mediha will understand the correct solution. But he also positions the students by requiring that they listen to each other (Turn 3232). While the teacher continually speaks Turkish, he allows the students to answer in German (Turns 3217, 3231), which suggests that his positionings of the students are language independent.

Transcript E1

Turn Person	**Original** (Turkish in black, German in grey)	**English Translation** (from Turkish in red, from German in orange)
3217 Atiye	Wenn man- burası şimdi on iki olsa, dann wird hier ungefähr Sieben.	If one, if this over here would be twelve *[points at the Kenan's download bar on the worksheet, see Figure 2]*, then this would be seven or so.
3218 Flid	Atiye, sence? . . .	*[to Mediha, gets the names wrong]* Atiye, in your opinion?
3223 Mediha	Ikide üç? . . .	Twelve, therein three? *[in Turkish, fractions are expressed "denominator therein enumerator"]*
3230 Flid	Tamam tamam. Ehm o zaman Atiye Mediha'ya açıklar mısın nasıl on ikide yediği bulduğunu.	OK, OK, um, Atiye, can you then explain to Mediha, how you arrived at the twelve therein seven7?
3231 Atiye	Ehm zum Beispiel du hast ja hier Zwölf, ne?	Ehm, for example here you have twelve, haven't you? *[points at Kenan's fraction bar]*
3232 Flid	Okay, sen de dinle!	*[to Okay]* Okay, do also listen!

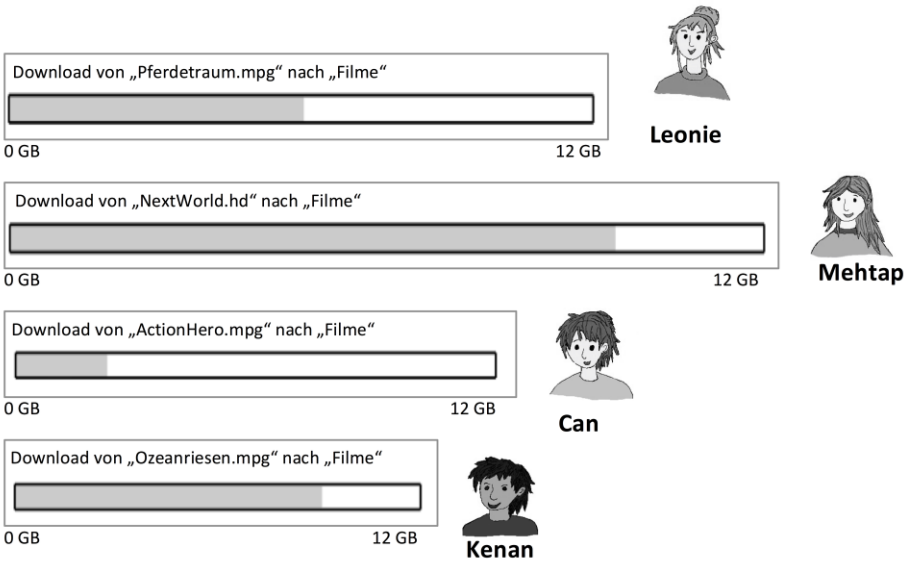


Figure 2. Fraction bars on the worksheet

Table 2.1 *Overview of the storylines in the four teaching intervention groups*

	Group E	Group D	Group J	Group L
Language tasks and how they are accomplished:	-Teacher positions himself as guide of conversation: Focused, goal-oriented discourse established by repeating task formulations or by adaptively bringing students questions as tasks into the discourse.	-Teacher positions himself as tutor and guide of conversation: Teacher takes responsibility for explanations and observations and for building on the students' utterances	-Teacher positions himself as traditional teacher who allocates speaking rights: -Mathematical correctness of solutions is not relevant; rather, presenting it to the others is more relevant. -Solutions are not negotiated but presented sequentially.	-Teacher positions himself as supporter of mathematical utterances (not conversations): Students are accountable for working in the fraction bar and locating and drawing the fractions, but not for negotiations or building on each other's utterances -No accountability for mathematical correctness
Language support and regulation:	-Teacher continually speaks Turkish. -Students positioned as responsible for task solution: German everyday language is accepted, also for conceptual work; Support in working with the fraction bar (in Turkish).	-Teacher positions himself as tutor and guide of conversation: Expected answers are enforced and unexpected answers are ignored or rephrased so that they fit the expectations -Students positioned as learners in need of help: Short answers allowed, lots of support	-Task solutions have to be presented in Turkish. -Words are treated as vocabulary: not their meanings are negotiated, but their translation Students position themselves as Turkish language learners: When working collaboratively without teacher support, students speak German.	-Teacher positions himself as supporter of mathematical utterances (not conversations): -Verbalizations (e.g., comparing solutions) are supported by the teacher (everybody has to share their solution) Support in using Turkish language.
Storylines that underlie the teacher's positionings of the students	*Storyline of quest for a consensual solution, where conversation is guided by the teacher; students positioned as responsible for mathematics*	*Storyline of working on tasks under guidance of teacher, students positioned to find acceptable answers*	*Storyline of Turkish language learning in mathematics: Students positioned to give solutions in Turkish (German is language for mathematical thinking among peers)*	*Storyline of quest for individual solution, where use of representations is supported by the teacher, but students not positioned as responsible for correct mathematics*

Table 2.2 *Overview of the personal stories and identities in the four teaching intervention groups*

	Group E	Group D	Group J	Group L
Initiative:	-Students position themselves as responsible for understanding: Students initiate turns for helping each other or to collaboratively arrive at a solution.	-Students position themselves as learners in IRE pattern or as in need of help: Students initiate conversations with the teacher about expectations in the task or to inquire about the "expected" solution	-Students position themselves interchangeably as Turkish language learners or mathematics learners: -students initiate conversations to arrive at a solution when working in pairs; -Students initiate the translation of words when they need a word in writing down a solution.	-Students position themselves in need of language support: Students initiate turns in order to establish clarity (e.g., how to express gigabyte in Turkish); -Students initiate discourses that are "off-topic"
Participation	-Students position themselves as responsible for solution: -Students are conjecturing, explaining, observing and discursively building on each other's utterances, -fraction bar as an aid for this. Students participate more in translanguaging mode	-Students position themselves within closed conversation pattern and orient their actions accordingly: naming fractions, locating fractions in the fraction bar, in reaction to the closed discourse pattern (both Halim and Hakan)	-Students position themselves as teacher-guided learners: Students participate to question the teacher for explicating the expectations for the solution	-Students position themselves as supervised learners in search of correct solution: Students participate to verbalize solutions with the help of the fraction bar (in German and again in Turkish) to each other and the teacher
Personal stories/identities that the guide the students' actions through reflexive positionings	*Stories of collaboratively working towards understanding and solving the task. Students identify themselves as contributors to this endeavor, where their Turkish language is a means to explicate one's understanding*	*Stories of eager mathematics learner (Halim) or receptive participant (Hakan). Students identify themselves with their part in a traditional conversation pattern of teacher/student interactions in school*	*Stories of bilingual mathematics learners:* *Students identify themselves as Turkish language learners while working independently on the tasks*	*Stories of supervised, material-centered mathematics learning: Students identify with a role of giving a linguistically acceptable task solution (to the teacher) through independent work on the fraction bars*

Reflexive positionings and students' personal stories

In the following episode, Mediha shows initiative and steps in to give an explanation as to why $\frac{3}{12}$ is an incorrect solution.

Transcript E2

Turn Person	**Original** (Turkish in black, German in grey)	**English Translation** (from Turkish in red, from German in orange)
3144 Flid	Sence?	*[addressing the student Okay]* In your opinion?
3145 Mediha	Drei Zwölftel geht doch nicht.	Three twelves does not work after all.
3146 Okay	Ist falsch glaube ich.	Is wrong, I believe.
3147 Flid	Mhm mhm.	Mhm mhm *[agreeing]*.
3148 Mediha	Çünkü mesela sen buraya yapsan, dann kannst du ja nicht mehr weiter machen.	Because, if you do it like that *[points at Leonie's download bar on the worksheet, see Figure 2]* then you can't do it any further.

Mediha steps in to give an explanation in Turns 3145 and 3148, where the teacher asks Okay for her opinion. With this, she gives an opportunity to the student Okay, who had mistakenly suggested $\frac{3}{12}$, to explain her false reasoning (Turn 3144). Mediha and Okay agree that the previously suggested solution $\frac{3}{12}$ is not correct (Turns 3145, 3146). The teacher accepts Mediha's initiative.

The students take up multiple positionings in which they stand in for the other and explain their solutions to each other. Here, Mediha stands in for Okay by answering for him (Turn 3148). Furthermore, the students seem to accept each other's positions and cooperatively arrive at a shared understanding. The students take up positionings in the conversation accordingly. Thus, the students identify themselves with the aim that everyone has to understand.

The next episode takes place shortly after the previous one and illustrates how Atiye conjectures about how to determine which download is the largest share.

Atiye observes that the different lengths of the fraction bars in the task do not allow direct comparisons of the grey areas, but that the ratio of the grey area matters. She suggests that one has to shorten all fraction bars to the same length, so that one can compare the grey areas. Her conjecture is that the fraction bars need to have the same length in order to be able to compare the fractions.

Transcript E3

Turn Person	**Original** (Turkish in black, German in grey)	**English Translation** (from Turkish in red, from German in orange)
3161-3163 Atiye	Ich glaub Balkenler gleich değil ya. Deswegen kann man das nicht so— çünkü bu kurz olsa dann wäre das ja ungefähr bis da	I believe the bars are not identical. *[Flid: # Mhm mhm]* That is why one can't like— because if that were short *[points at Mehtap's download bar on the worksheet, Figure 2]* then that would be till around here.

Atiye engages in a conversation in which the students explain their thinking and question their current, unfinished explanations. Atiye builds on the previous observations of her peers of the fraction bars. It seems that Atiye is positioning herself to be responsible for each of the student's understandings, which is part of the collaborative endeavor to arrive at a shared understanding. Interestingly, this goes hand in hand with translanguaging, that is, with mixing German and Turkish (García, 2009). Hence, the students seemed to identify themselves as multilingual "doers" of mathematics in mathematical activities such as conjecturing.

Relation between interactive and reflexive positionings and the development of the students' identity in intervention Group E

The students' personal stories in intervention Group E developed hand in hand with the teacher-enforced storylines that guided the conversation. Positioning the students as being responsible for the correct solution in a storyline where every student has to understand the others' solutions and explanations is coherent with Atiye and Mediha's reflexive positionings, in which they make themselves responsible for cooperatively arriving at a solution and for the other's understanding. This suggests that the students are able to develop identities as multilingual mathematics learners and that they identify themselves as doers of mathematics across both languages.

Identities in Teaching Intervention GROUP D (Teacher: Mr. Flek)

Interactive positionings and related storyline of the Conversation

The following episode from intervention Group D illustrates how the teacher, Mr. Flek, assigns language tasks and how he evaluates and takes up the students' answers to his tasks. Two students, Halim and Hakan, work together; the teacher is also in charge of three students who work at a separate table.

Halim is working with fraction bars that have 12 as the denominator (Figure 2). In the beginning, when the teacher assigns a task, Halim answers in Turkish by naming the correct

fraction. The teacher positively evaluates Halim's answer by revoicing it in the same wording.

Transcript D1

Turn Person	Original (Turkish in black, German in grey)	English Translation (from Turkish in red, from German in orange)
3091 Flek	. . . senin bu sıralamana göre Mehtap kaç ne kadar indirdi? Düşen payı ne- ne kadar Mehtap'in?	. . . how much has Mehtap downloaded, according to your ordering? How, how big is Mehtap's share?
3092 Halim	Ehm on ikide on.	Ehm twelve therein ten.
3093 Flek	On ikide on demi? Kenan'ın ne kadar?	Twelve therein ten, isn't it? How much is Kenan?

The teacher positions himself as responsible for evaluating and building on the students' utterances when working with Halim and Hakan. In this episode, the teacher takes up Halim's answer and builds on it by asking a follow-up question (Turn 3093). The interaction positions students as having to answer to the teacher, where short answers will be accepted to "fulfill" these positionings. These positionings might indicate that the teacher and students are together establishing a storyline of tutor and learner in need of support using the Turkish language.

The following episode gives deeper insight into this storyline. The teacher asks Halim to explain the reasoning behind his solution, in which Halim has observed that he has to account for the different lengths of the fraction bars when determining the ratio:

Transcript D2

Turn Person	Original (Turkish in black, German in grey)	English Translation (from Turkish in red, from German in orange)
3089 Flek	Yani, sen ne diyorsun tam olarak?	So *[points at Halim]*, what do you mean, exactly?
3090 Halim	Ja, dass dass die hier größer. Weil das unterschiedliche Balken sind. Also das und dann das. Hier, weil aber obwohl dieser Balken kleiner ist. Aber der hat mehr- mehr- runter- runter- runtergeladen. Das ist das Wichtigste, wer mehr- mehr- runtergeladen hat.	Yes, that that they here bigger *[points at Mehtap's and Can's download bars, Figure 2]*. Because they are different bars. So that *[points at Mehtap's download bar]* and then that *[points at Kenan's download bar]*. Here, because, but in spite of this bar being smaller *[points at Kenan's bar]*. But he has down- downloaded more, more. That is the most important, who downloaded more, more. So #

	Transcript D2 contiuned	
	Also #	
3091 Flek	# Sen öyle diyorsun demi? Şimdi sen diyorsun ki ehm Mehtap ve Kenan'ın düşen payı aynı. Du siehst da einen Unterschied. Das ist ja auch völlig ok.	#you mean it like that, don't you? You are saying now that Mehtap and Kenan have the same share. You are seeing a difference there. That is entirely okay, yes.

Halim explains his thinking in everyday language and by using the fraction bar deictically. He suggests that while the fraction bars are different in length, this does not matter (Turn 3090). Instead, he focuses what has been downloaded in each fraction bar. The teacher frames Halim's utterance by asking for an explanation (Turn 3089) and then by evaluating it (Turn 3091). The teacher takes the position of being responsible for evaluating Halim's answer and interprets its meaning by rephrasing it (Turn 3091). The teacher then evaluates Halim's answer based on this rephrased answer, which mirrors his understanding of what Halim tried to express. As a result, by directing the conversation away from Halim's answer after Turn 3091, the teacher positions Halim as not contributing to the current conversation. As a consequence, Halim might identify himself as not having understood the task and/or the embedded mathematics correctly. It is an open question if this positioning is also connected to Halim's use of German. This interaction indicates, together with the above episode, a storyline of "tutor and learner in need of support" in which the teacher is in a position to frame and evaluate the students' utterances.

Reflexive positionings and students' personal stories

In the following episode Halim and Hakan engage in the mathematical conversation by initiating questions directed to the teacher.

Transcript D3

Turn Person	**Original** (Turkish in black, German in grey)	**English Translation** (from Turkish in red, from German in orange)
3062 Flek	çubuğa aktardığınız düşen payları ehm büyüklüğüne göre en büyüğü ile başlayarak şuraya yazın.	*[points at the symbolic fractions on Halim's worksheet, Figure 3]* these shares that you transferred to the bar according to their size *[points at task 1b]* beginning with the biggest, write them down there.
3063 Hakan	Die beiden sind gleich.	These two are equal *[points at two download bars]*
3064 Flek	Tamam. Güzel.	Ok. Nice.

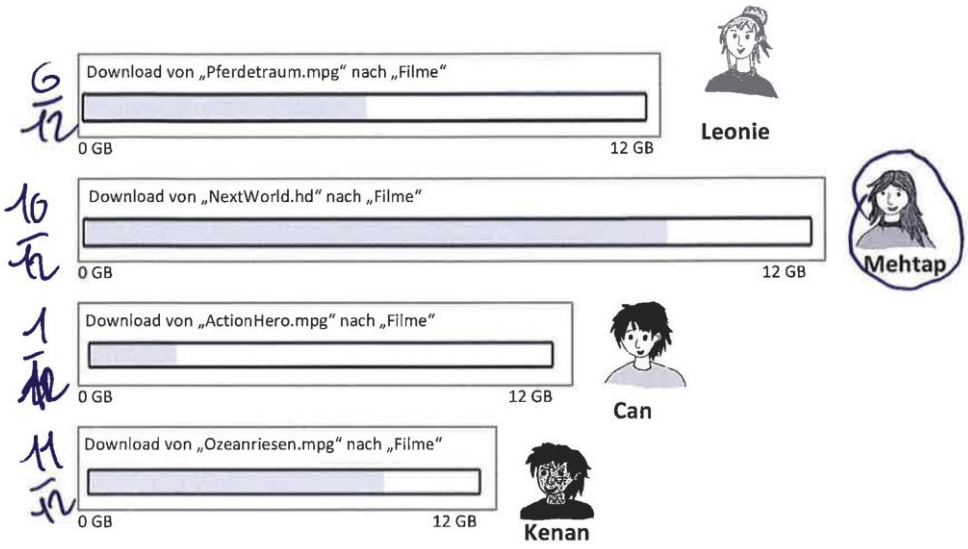


Figure 3. Halim's worksheet

The teacher assigns a task to Halim and Hakan (Turn 3062). The nature of the task involving writing something down suggests that the teacher intends the students to work alone on this task, without assistance. However, Hakan steps in and gives an answer where it is not expected (Turn 3063). The teacher values the utterance from Hakan, which breaks up the individual work and reestablishes the assistance given by the teacher (Turn 3064 and following).

Hakan's stepping in with an answer or a clarifying question after the teacher has assigned a task is a usual pattern in this group. Usually it leads the teacher to give more explanations, in this instance by fragmenting the larger task into smaller tasks. By stepping in in this way, the students might position themselves as being in need of help, and they usually receive help after stepping in. Hence, Hakan's actions seem to connect to personal stories where he sees himself as a receptive participant in the conversation and positions himself as being in need of help; in the course of the conversation, Hakan participates in Turkish, which suggests that he has actualized his personal stories into the Turkish language.

The following episode D4 illustrates how Halim tends to participate in the conversation.

Previous to this episode, the teacher assigned tasks from the worksheet to the students. This confuses Halim (Turn 3019) and the teacher comes to Halim (Turn 3022). Then, Halim engages in the conversation with longer than usual utterances to express what irritates him (Turn 3023). In the next Turn (3024), this exchange is ended by an explanation from the teacher that results in the assignment of a more specific task.

Transcript D4

Turn Person	Original (Turkish in black, German in grey)	English Translation (from Turkish in red, from German in orange)
3019 Halim	Hää? Hat wieviel hat? Wie? Ehm, ich verstehe das nicht.	Heee? Has how much has? How? Ehm *[looks at the teacher]* I do not understand this. . . .
3022 Flek	Mhm mhm! Tam olarak neyi anlamiyorsun?	Mhm mhm! What exactly do you not understand?
3023 Halim	Ja! Das. So also das hier, das hier Also aber ich weiß nicht. Das sind doch Anteile. Das sollen doch also hier ist es doch sowas wie Anteil	*[points at his worksheet]* Yes. That. So, thus that one, that one *[points at the download bars]* But I don't know. These are parts of a whole. That here should be something like part of a whole *[points at the fraction bar board]*

Halim engages in the conversation to clarify the assigned task. This way, Halim might position himself as—most likely—a German mathematics learner like in regular classrooms, which usually involves working on assigned tasks in order to learn mathematics. At the same time, he positions the teacher as being responsible for how the task is meant to be solved. This suggests that Halim is acting in line with personal stories of being an eager mathematics learner who works thoroughly on assigned tasks in order to learn under the guidance of the teacher. These stories might have been transferred from the regular mathematics classroom and are thus told in German. This seems to be the usual way for Halim to participate in the conversation in this task.

Relation between interactive and reflexive positionings and the development of the students' identity in intervention Group D

In teaching intervention Group D, the teacher establishes a storyline of "tutor guiding students who are in need of help" while working on the task. Halim and Hakan position themselves differently in this storyline: Hakan as learner who is in need of assistance and Halim as learner who works thoroughly on the mathematical tasks in order to learn under the guidance of the teacher. Accordingly, the teacher and both students contribute to perpetuating this storyline in which the teacher is the guide/helper.

Halim and Hakan's identities might develop on different pathways: Hakan might develop an identity as multilingual learner who specifically needs assistance—coherent with being placed in a teaching intervention intended to foster his understanding—while Halim might develop an identity similar to his mathematical identity in the regular classroom, where he

perceives himself as a mathematics learner who needs to thoroughly work on tasks under the guidance of the teacher in order to learn.

Comparison of the Four Cases

The four cases presented in Table 2, from which the two in-depth cases presented above have been taken, differ in the overarching storylines that are established in each. These differences are a product of the teachers trying to implement the Turkish language into the intervention group in line with the design principles of this study. For example, the Turkish language can become a medium for mathematical talk, such as conjecturing and observing as in Group E, or it can become an aspect of the content to learn, as in Group J. In the former, the Turkish language possibly enriched the mathematical conversations, but in the latter, the Turkish language probably did not directly contribute to the mathematical conversation. This suggests that the extent to which the Turkish language contributed to the mathematical conversation depended to a large degree on the established storyline within the teaching intervention group. Here, the storylines were usually teacher centered because the teacher's illocutionary force allowed him to guide the conversations and the intervention was designed to be teacher centered.

The storylines in the teaching intervention groups provide the stage for the students to develop their personal stories. Hence, in each group there are different opportunities for the students to develop the stories they tell about themselves, that is, their identities. As shown above, in intervention Group E, Atiye's and Mediha's personal stories of being responsible for the solution and each other's understanding connect to "collective" positionings: Atiye and Mediha positionings are characterized more by being part of the group than by being an individual in the conversation. This is quite different from Group D, where the students for some reason work individually despite sitting at the same table. Halim and Hakan position themselves individually, and this results in different opportunities to engage in the conversation. Hence, not only do the storylines differ between the teaching intervention groups, the students also position themselves differently within the teaching intervention groups. As a result, there is a large spectrum of possibilities for identity development in the bilingual teaching intervention in the larger project MuM-Multi. At the same time, however, this spectrum might be limited by the students' previous identities from the regular classrooms that the students import into the teaching intervention. For example, Hakan's personal stories might connect to the regular German-dominated classroom, where he also might continually seek assistance from the teacher.

SUMMARY AND DISCUSSION

With respect to research question Q1, we observe that in the four analyzed teaching intervention groups, a specific storyline was established in each that guided the conversation. This storyline was teacher centered in these groups and relatively stable over

the course of the analyzed focus task presented here. These storylines opened a stage for the personal stories to develop, resulting in personal stories that were clearly connected to the storyline of the conversation.

In regard to research question Q2, we see that each of the four groups developed unique storylines that set the stage for unique ways for students to develop their personal stories. At the same time, the storylines revolved in some way around relating mathematics and the Turkish language, and thus were relatively similar. Within an intervention group, the spectrum of identities was limited by the room the established storyline provided for students to develop their personal stories.

The students' personal stories—their identities as multilingual mathematics learners within the teaching intervention—were connected to the use of Turkish in the four analyzed intervention groups, just as the storylines in the intervention were a product of the multilingual nature of the teaching intervention (Q3). Accordingly, students were able to include Turkish as a part of their mathematical identity when they engaged in Turkish in activities such as conjecturing and explaining (Group E). The Turkish language also became the language to ask for and to receive help and could in this way connect to an identity of "needing assistance in mathematics" (Hakan in Group D). Turkish also became the language of correct solutions, resulting in identities of being Turkish language learners of Turkish mathematical language (Group J).

This study has focused on the comparison of students' identities in four different teaching intervention groups led by four different teachers and how they were influenced by the storyline of the conversations generated by one task that occurred during the third session of a five-session teaching intervention. It has not, however, given insights into the development of the students' identities over the course of the five sessions. Furthermore, only four out of the 11 groups were analyzed. Nevertheless, assuming that the teachers acted similarly in all five sessions, the students may have developed relatively stable identities. A cursory examination of the other seven groups suggests that in other groups, the same teacher may have established very different storylines; thus, in these groups there were different opportunities for students to develop their identities.

In this study, I have used the identity construct to investigate identities of individuals. At the same time, I have used the construct of storylines to characterize the conversations in the different intervention groups in order to assign a form of normative identity to each group. This, however, can be problematic in conversations where multiple storylines are enacted and guide the individuals in developing their personal stories (see Herbel-Eisenmann et al., 2015).

Elsewhere it has been shown that teaching interventions that segregate students from their regular mathematics classrooms can affect their participation in mathematics, mediated by their identification with having specific needs (Civil & Planas, 2004). In this study, the initial positioning of the students was similar, as the students were asked to participate in a teaching intervention in addition to their regular mathematics classroom. However, this study shows that students can profit from a teaching intervention in regard to their identity as long as an adequate storyline is established that guides the conversations in the

intervention. For example, the students in Group E developed an identity that allowed them to be comfortable using Turkish to explain, conjecture about, or observe mathematical phenomena. For this to be successful, a "quest for a consensual solution" storyline might be necessary in which students are positioned as responsible for the mathematics at hand. Such an accountability for consensual understanding (Greeno, 2006) led the students to use Turkish while exercising authorship of mathematical ideas and agency ("taking up room"; Hand, 2012).

This teaching intervention was built on research on multilingual mathematics learning, which has suggested that, under certain conditions, multilingualism is a resource for mathematics learning. These studies focus on the mathematical side of such interventions: mathematics that connects to the students' everyday experiences, which allows them to initiate a multilingual, everyday mathematical discourse (e.g., Dominguez, 2011; Moschkovich, 2015). However, the results presented here suggest that it might not be sufficient that the material and conversation are multilingual. Instead, it seems that students need to identify themselves as multilingual in storylines where they are made responsible for the mathematics at hand: In teaching intervention group D, students likely fell back to storylines from their regular classroom and thus to monolingual use of language, whereas in Group E, Turkish became a resource. The results presented here suggest that a teaching intervention aiming to build on students' multilingual resources for participating in mathematical discourses has to carefully consider the ways in which students can develop identities as multilingual mathematics learners in ways that students do not fall back on monolingual identities from regular classrooms.

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How to Integrate Content and Language Learning Effectively for English Language Learners

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ABSTRACT
This paper describes the challenges and successes of developing and scaling up a research-based instructional intervention known as the SIOP (Sheltered Instruction Observation Protocol) Model. The SIOP Model is an approach used widely in the United States for teaching subjects like mathematics and science to students learning through English, a new language. Teachers integrate techniques that make the concepts accessible with techniques that develop the students' skills in the academic language of the specific subjects. This article describes a program of research that developed the SIOP Model in one study and then tested its efficacy and refined its professional development design in subsequent studies in a number of different contexts over 15 years. Results revealed that students with teachers who were trained in the SIOP Model and implemented it with fidelity performed better on assessments of academic language than students with teachers who were not trained in the model.

Keywords: sheltered instruction, content-based language learning, English as a second language, academic language development, SIOP Model

INTRODUCTION

This article describes a program of research that developed an instructional model for students in U.S. elementary and secondary schools who have to learn English as a new language at the same time they have to study mathematics, science, and other subjects that are taught through English. The model was created in one study and then tested in subsequent studies in a number of different contexts over 15 years to demonstrate its effectiveness. The model which will be described here is known as the SIOP (Sheltered Instruction Observation Protocol) Model. In the United States it is widely used in all subject areas and at all grade levels.

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State of the literature

- There are research-based studies of instructional techniques, such as reciprocal teaching (a reading technique) and information gap activities (for oral interaction) but very few that have examined a combination of techniques that could be used consistently to plan lessons that integrate content and language in any subject area.
- Few empirical studies of sheltered instruction exist that look at the effects on language development of students in content area classrooms.

Contribution of this paper to the literature

- This paper discusses how subject area teachers can learn about the academic language of their subject area and how to teach it using a research-based approach.
- The research and development designs that led to the SIOP Model may be applied to other interventions that are being developed and refined over time in response to an pressing educational problem.

This article will highlight some of the SIOP Model's implementation with math and science teachers. The goal of SIOP instruction is for teachers to develop the learners' academic English skills while using specialized techniques to teach and have students engage with the subject area topics in a comprehensible manner (Echevarria, Vogt & Short, 2017). The article is offered as a design framework for other researchers who may have to develop an intervention for a pressing educational problem, identify the promising practices, determine how best to provide professional development to teachers on the intervention, and refine the process of implementation over time.

Historical Context

In the United States, the 1990s were a pivotal time for the education of English language learners for two reasons. The population of school-aged English language learners (ELLs) grew much more rapidly than the general school population and major educational reforms were implemented at the national level.

From 1995 to 2000, the percentage of ELLs grew 39% but the population of all students (including ELLs) decreased by 1%. In the 1990s, most English language learners were placed in English as a second language (ESL) programs for one to three years and the focus of their classes was learning to read, write, speak, and listen in English. Few states offered bilingual education. There was no national ESL curriculum nor state-level frameworks and so ESL instruction was uneven and varied from district to district and from state to state (Sheppard, 1995). Teaching these learners grade-level content subjects was often delayed until they developed some proficiency in English.

The educational reform movement in elementary and secondary schools led numerous professional teaching organizations to write standards for their subjects (e.g., mathematics, science, history, language arts) that delineated the depth and breadth of what students should

learn. As states transformed standards into curriculum frameworks, the instruction and expectations for student learning became more rigorous.

The focus on standards served as a catalyst to some educators of ELLs. In order to make the ESL classes more relevant to school, ESL teachers developed lessons around subject area themes (e.g., the solar system) (Crandall, 1993). This approach was known as content-based language instruction. In addition, some general education teachers began to use an approach called sheltered instruction. They integrated ESL techniques in their lessons, using visuals, gestures, and modeling to make content topics comprehensible. They did not however pay attention to English language development, except to teach subject-specific vocabulary. Both types of classrooms were quite diverse in their instructional practices and no research studies had yet identified which techniques were effective for student learning (Sheppard, 1995).

The Problem

As schools implemented more and more standards-based curricula in mathematics, science, and other subjects, educators noticed that ELLs who exited the ESL and bilingual programs were not successful in their general education classes where English was the medium of instruction. Given the increasing numbers of ELLs in schools, more general education teachers than ever were instructing these students but many had had no training to work with learners who did not speak English (Batalova, Fix, & Murray, 2007; National Center for Education Statistics, 2002). The teacher training programs in universities lagged behind the needs of the schools (Ballantyne, Sanderman, & Levy, 2008). In the mid 1990s, only California and Florida required specific coursework for all preservice teachers on topics like ESL methods and second language acquisition. Even 10 years later the number of states with such requirements had only risen to six of the 50 (National Comprehensive Center on Teacher Quality, 2009).

As the standards movement strengthened, schools were held accountable for student performance. Starting in 2002, the federal government required states to assess students in mathematics and reading based on the new standards. (Science was added later.) Even English language learners who were not proficient in the language of the tests were assessed. Schools were penalized and labeled "low performing" or "needs improvement" if their ELLs did not attain testing achievement targets set for native English speakers on tests that had not been designed or normed for English language learners (Abedi, 2002).

This situation created some need for change. ELLs lagged significantly behind their English-speaking peers on state standardized tests given in English (California Department of Education, 2004; Kindler, 2002). Performance on the sole national test in the U.S.—the National Assessment of Educational Progress (NAEP)—consistently showed wide achievement gaps between English language learners and non-English language learners in mathematics and reading at all grade levels tested (4th, 8th and 12th) (Braswell, Dion, Daane, & Jin, 2005; Grigg, Daane, Jin & Campbell, 2003). ELLs also had higher drop-out rates from high school than English-speaking students (Ruiz-de-Velasco & Fix, 2000). It became more and more evident that current educational practices were insufficient to meet the academic and language needs of the ELLs.

The problem then was the following: How could all teachers could help English language learners develop academic language skills and subject area knowledge concurrently so they would achieve in school? For over 15 years, we have conducted research to grapple with this persistent problem. Three major studies are described here along with a discussion of what we have learned, implications for math and science teachers, and future directions.

Developing the Sheltered Instruction Observation Protocol (SIOP) Model: The CREDE Study (1996-2002)

In 1996, we began a design research study funded by the U.S. Department of Education through the National Center for Research on Education, Diversity & Excellence (CREDE). Our research questions were 1) What are the characteristics of a model of sheltered instruction that result in ELL achievement gains? and 2) What are the characteristics of an effective professional development program for implementing quality sheltered instruction to a high degree? We hoped to design and test a model of sheltered instruction that could be used with consistency across subject areas and across grade levels.

THEORETICAL BACKGROUND

To develop the model, we drew from theories of second language acquisition and the sociocultural view of teaching and learning. Acknowledging the distinction between conversational and academic English (Cummins 1981), we recognized that although students develop a moderate command of spoken English in social settings in 1 – 2 years, they need a longer time frame (i.e., 4 – 7 years) and more support to comprehend and use academic English successfully in school. They must master semantic and syntactic knowledge and functional language use in different academic subjects. For example, ELLs must be able to read and understand expository prose found in math textbooks; pose hypotheses before science experimentation; and justify solutions to word problems. They must also learn how to complete instructional tasks, such as writing a geometric proof or interpreting charts and graphs.

Research on second language acquisition provided direction to the types of supports that teachers can employ to help students learn the subject area topics and develop appropriate language skills. The theoretical underpinning is that language acquisition is enhanced through meaningful use and interaction. Comprehensible input (Krashen, 1985; Gass, 2013) is crucial when students are not proficient in the language of instruction. Teachers therefore use visuals, gestures, less complex speech, modeling, and other techniques to present key information. Comprehensible output (Swain, 1985; Gass, 2013) is also important so students can articulate their ideas, practice academic language, develop automaticity, and get feedback. Techniques, such as using sentence stems and structured conversations, can guide student output. Explicit instruction on language forms, academic vocabulary, and language learning strategies, along with building literacy from classroom talk, also contribute to language development and content comprehension (Ellis, 1999; Gibbons, 2003; Norris & Ortega, 2000).

Research on the sociocultural perspective gave insight into dynamics of the classroom. Student learning is promoted through social interaction and contextualized communication (Tharp &

Gallimore, 1988; Vygotsky, 1978), guided by "more capable others." Teachers can scaffold instruction so that students can construct meaning and understand complex concepts (Bruner, 1983) and they organize partner or group work so English learners have peer support to grapple with complex ideas. Teachers also assist learning by beginning instruction at a student's level of understanding and, with appropriate support, incrementally advance their knowledge and language skills. Teacher scaffolds might include preteaching key vocabulary before a reading assignment, adjusting speech by paraphrasing or elaborating on a student response, and asking questions that elicit detailed responses from the students rather than one-word answers.

Sociocultural research also revealed that students from immigrant families benefit from explicit socialization to the implicit cultural expectations of the classroom, such as turn-taking and participation rules (Cook-Gumperz, 2006; Moschkovich, 2007). Teachers can make explicit their assumptions for classroom behavior and interactional styles, such as encouraging students to ask questions and take roles in cooperative learning groups. They can also engage in culturally responsive teaching that recognizes and builds upon culturally different ways of learning, behaving, and using language (Nieto & Bode, 2008).

SIOP Model Development

To address the first research question, researchers at California State University, Long Beach and at the Center for Applied Linguistics in Washington, DC collaborated with a small group of middle school teachers (n =11) from core subject areas (math, science, social studies, and language arts) in three districts in the United States. We identified best practices for teaching content and language to ELLs from the professional literature and tested combinations of these techniques to build a model of sheltered instruction. At first, we organized these techniques into an observation protocol, but the collaborating teachers suggested that the SIOP Model be used for lesson planning and delivery as well. So we reframed it as an instructional approach that shows subject area teachers how to integrate academic language development into content instruction and how to use ESL techniques to make the concepts comprehensible. This was a departure from typical instruction and we anticipated that it would require considerable professional development because most math, science and social studies teachers had no background in linguistics, second language acquisition, or ESL methodology.

The most significant change concerned what became a hallmark of the SIOP Model—that all lessons include a language objective. Teachers would continue to plan lessons around a content objective (e.g., Students will represent translations, reflections, and rotations of an object in a coordinate plane.) but would now add a language objective too (e.g., Students will orally describe the position of the resulting image compared to the original position of the object.).

The increased focus on academic English was disconcerting for some. The collaborating teachers were willing to try planning with two objectives, but when we included eight more teachers from their schools in subsequent years, there was some resistance and anxiety to the notion of being responsible for language development in math or science class.

Over four years, we piloted and refined the model. To allay some teachers' concerns, we

worked with them to develop lesson plans and identify techniques and activities to include. We met in study groups to identify the critical features for instruction that would support both content and language learning. The teachers tried out various groupings of these features until in 2000, we finalized the SIOP Model with 30 features of instruction organized in eight components—Lesson Preparation, Building Background, Comprehensible Input, Strategies, Interaction, Practice & Application, Lesson Delivery, and Review & Assessment (Echevarria, Vogt & Short, 2000). Figure 1 provides brief descriptions of each component.

Once the SIOP Model was finalized, it was operationalized in the observation protocol, which has a 5-point scale for each of the 30 features. The tool allows observers to rate teachers' lessons for the degree of fidelity to the model and to provide explicit feedback to help teachers implement the model more consistently. A separate study established the validity and reliability of this protocol (Guarino, et al., 2001). (See the appendix for the protocol.)

Student Achievement

Beyond developing the model, we also needed to determine if SIOP implementation improved the language performance of English language learners, given the growing importance of testing and accountability in U.S. schools. We investigated the model's effects using a quasi-experimental design. Two groups of ELLs in sheltered classes participated: students whose teachers were trained in the SIOP Model (the intervention [aka SIOP] group) and a similar group of ELLs in the same district programs whose teachers had no exposure to the SIOP Model (the comparison group). Students in both groups were in Grades 6–8, represented a comparable range of English proficiency levels, and spoke a variety of native languages.

The writing assessment from the Illinois Measurement of Annual Growth in English (IMAGE) test was used as an outcome measure of academic literacy. A standardized test of reading and writing, the IMAGE was used by Illinois districts at that time to measure the annual growth of these skills for ELLs in Grades 3 and above. The test was valid and reliable and had correlational and predictive value for achievement scores on the standardized state achievement tests in reading and mathematics (Illinois State Board of Education, Assessment Division, 2004). It provided scores for five subtests on aspects of writing—language production, focus, support/elaboration, organization, and mechanics—as well as a total score for each student.

We administered the IMAGE as a pretest in the fall and a posttest in the spring. Scores were analyzed for the students who were present for both administrations (n = 241 for students in SIOP classes, n = 77 for students in comparison classes). Because there were differences between the two groups in their pretest scores, analyses of co-variances (ANCOVA) were conducted. Comparisons between SIOP and comparison groups on their total scores found the students whose teachers were trained in the SIOP Model made significantly better gains than the comparison group in writing ($F(1,312) = 10.79$; $p<.05$). Follow-up analyses on student performance on the various subtests of the writing assessment found that the SIOP group performed at a significantly higher level in language production ($F(1,314) = 5.00$; $p<.05$), organization ($F(1,315) = 5.65$; $p<.05$), and mechanics ($F(1,315) = 4.10$; $p<.05$) than the comparison group. The SIOP group also made gains over the comparison group in the focus

and support/elaboration subtests, but not to a statistically significant level (Echevarria, Short & Powers, 2006).

Lesson Preparation: Each SIOP lesson has separate language and content objectives that are linked to the curriculum & standards and taught systematically. Teachers plan their lessons carefully, with appropriate content concepts, the use of supplementary materials, adaptation of content as needed, and meaningful activities that integrate concepts with language practice.

Building Background: Teachers make explicit links between new concepts and past learning and between concepts and students' personal experiences. These connections help students organize new information as part of their cognitive processing. Teachers must directly teach and emphasize the key academic vocabulary and provide opportunities for ELLs to use this vocabulary in meaningful ways.

Comprehensible Input: Teachers modulate their rate of speech, word choice, and sentence structure complexity according to the proficiency level of ELLs. They explain academic tasks clearly, both orally and in writing, and provide models and examples. Lessons incorporate a variety of techniques to make instruction accessible, including the use of visuals, hands-on activities, demonstrations, gestures, and body language.

Strategies: Lessons provide students with instruction in and practice with a variety of learning strategies. Teachers scaffold the delivery of new information as they guide students to a higher level of understanding and independent practice. They also promote higher-order thinking through a variety of question types and tasks.

Interaction: Lessons are designed with frequent opportunities for interaction and extended discussion among students and with the teacher so students practice important skills like elaborating, negotiating meaning, persuading, disagreeing, and evaluating. Teachers group students to support the content and language objectives, provide sufficient wait time for student responses, and clarify concepts in the student's first language, if possible and as needed.

Practice & Application: Lessons include hands-on materials, manipulatives, and/or physical movement to practice new content. Teachers plan activities for students to apply their content and language knowledge through all language skills (reading, writing, listening, and speaking).

Lesson Delivery: Teachers implement lessons that clearly support content and language objectives with appropriate pacing, while students are engaged 90% to 100% of the instructional period. All students must have opportunities to practice language skills within the context of the academic tasks.

Review & Assessment: Teachers provide a comprehensive review of key vocabulary and concepts, regularly give specific, academic feedback to students, and conduct assessment of student comprehension and learning throughout the lesson. Teachers should offer multiple ways for students to demonstrate their understanding of the content.

Adapted from Echevarria, Vogt & Short (2017).

Figure 1. Description of the SIOP Model Components

We also calculated the Cohen's *d* effect size of the intervention which was .833. This effect size is considered large by most indices (Cohen, 1998), suggesting that the SIOP intervention led to significant gains over time in students' overall writing performance.

Professional Development for the SIOP Model

From 2000-2002, we began to address the second research question but did not answer it fully. We had learned that help with lesson planning and a teacher study group were helpful. We developed some professional learning materials, namely two videos of exemplary SIOP instruction and a teacher training manual. The materials were designed for teachers at all grade levels in all core subject areas and ESL, because the need was spread across these diverse areas. We also started to provide workshops in other school districts and to collect feedback on what teachers understood about the SIOP and what they had questions about. However, the full scope of an effective professional development program wasn't realized until our next large-scale research project which involved teachers who had not been part of the design study.

Scaling Up SIOP Research: The New Jersey SIOP Study (2004-2006)

In 2004 we expanded the research to a new study with more teachers and students, funded by the Carnegie Corporation of New York. It was also quasi-experimental in design and took place in two matched districts (one SIOP, one comparison) in northern New Jersey, each with two middle schools and one high school. The research questions for this study were 1) Do teachers reach high levels of implementation of the SIOP Model during a sustained professional development program after 1 year or 2 years? and 2) Does implementation of the SIOP Model in subject area classrooms result in increased student achievement after 1 year or 2 years? We had a representative sample of teachers in both districts who taught in Grades 6-12. Teachers in the SIOP district taught mathematics, science, history, language arts, ESL, special education, and technology. Approximately 35 teachers participated for two years (Cohort 1) and an additional 23 during the second year (Cohort 2). The comparison district did not have cohorts so the same 19 teachers participated both years. The comparison teachers taught mathematics, science, history, and ESL.

The SIOP Professional Development Program

Before we could measure the teachers' level of implementation, we had to provide professional development in the SIOP district. These middle and high school teachers were not building the model with us, as was the case with the CREDE study, so we needed to craft a complete professional learning program that would support the teachers. We began with the basic question: What did teachers need to know about the academic language of their subject? We concluded they needed to know how to identify the types of language in their standards, textbooks, and curriculum frameworks so they could generate language objectives. They then needed to know how to teach these aspects of language and which techniques they could incorporate in lessons to let student practice and apply the language while studying the content topics.

We therefore designed a series of workshop around the SIOP components that highlighted

language learning. The first ones introduced the teachers to categories of academic language that should be considered when writing language objectives, such as vocabulary, language skills and functions, and language structures. Academic vocabulary was the "low hanging fruit." Teachers were comfortable teaching key content words, but we had to broaden their instruction. First, they needed to realize that ELLs required more than the subject-specific terms like *logarithm* and *exponent*. They needed to learn general academic terms too that would let them talk, read, and write about a concept or topic: a) process verbs like *determine*, *solve*, and *represent*; b) cross-curricular nouns such as *result*, *effect*, and *condition*; c) conjunctions and logical connectors such as *given that*, *however*, and *in sum*; and d) polysemous terms that might cause confusion like *power* and *division*. Second, the teachers had to instruct in ways that facilitated ELLs' meaning-making—not a quick orally stated definition or a glossary entry, but a technique that involved students in learning the words, such as using a concept definition map or drawing pictures of the terms. Third, the teachers needed to plan activities in the lessons where students would practice using the words, such as reading them in mathematics text, using them in a writing task, or talking about them when problem solving.

In similar ways, we explored the other categories with the teachers, asking them to identify the language skills students use in class (e.g., Are students reading to find a scientific claim or to follow directions? Are they taking notes or writing a summary of what they learned?) and the language functions they want students to produce (e.g., Will students be asked to justify their solution to a problem? Will they hypothesize?). To identify language structures, we had teachers work with authentic texts that students would read and samples of student written work. We drew attention to examples of passive voice, nominalizations, conditional sentences, pronoun referents, and the like, and discussed how these aspects of language might be difficult for English language learners.

Once teachers began to understand what kind of language students were expected to use in class, we showed them ways to teach these language targets explicitly. Figure 2, for example, illustrates sample language objectives for algebra and how they could be addressed in several lessons. This type of instruction was more challenging for teachers at the secondary level than for those in elementary schools who commonly taught language skills. We had to reassure the secondary teachers that they weren't expected to become teachers of grammar but that they should call attention to language structures that appeared frequently in their materials (e.g., use of the imperative in lab directions).

We wanted teachers to build student academic talk through interaction. So when teachers asked students to "discuss the solution to a problem" in a small group or "turn and talk" to a partner, we encouraged them to also give guidance regarding the type of language they wanted students to use. For instance, if students were asked to discuss comparisons in an elementary math classroom, they could be explicitly taught to use frames like:

"___ is smaller than ... "
"___ and ___ are equivalent because ...."

If they were in a secondary math class and had to interpret and analyze graphs, they might practice with frames like:

"Based on the graph, we conclude that ..."
"The intersection of the lines shows that ..."

In all of these classes, a word bank with key terms could also be provided to highlight targeted vocabulary that the ELLs were expected to use. These language frames, sentence starters, and word banks were scaffolds for students to become fluent in academic language in classroom contexts (see Donnelly & Roe, 2010; Zwiers & Crawford, 2009, Short & Echevarria, 2016 for more discussion).

Type of Language Objective	**Algebra Example**
Academic Vocabulary: key terms needed to discuss, read, or write about the lesson's topic (subject-specific, general academic, or word parts) ↓	Students will define and give examples of positive and negative slope.
What it means instructionally	Teacher uses a concept definition map with class to define slope, call attention to related words (e.g., increase, decrease, vertical, horizontal), and elicit real-life examples of slope.
Language Skills and Functions: skills students will use in the lesson (e.g., read for main idea) or the specific purpose for using language (e.g., to compare, to persuade) ↓	Students will orally justify the slope of a line between two points.
What it means instructionally	Teacher demonstrates how to find slope using a geoboard and offers language frames to justify the determination, such as "The slope is positive/negative __ because ...".
Language Structures: grammar or language structures in the written or spoken discourse of the lesson ↓	Students will use if-then statements to describe what happens to a line when the slope changes.
What it means instructionally	Teacher teaches (or reviews) how to form two types of if-then sentences: 1) when the if clause comes first and 2) when it comes in the second half of the sentence. Teacher points out use of present tense in the *if* clause and future tense in the *then* clause.

Adapted from Echevarria, Vogt & Short (2017).

Figure 2. Categories and Examples of Language Objectives

In other workshops, we demonstrated techniques that the teachers could add to their lessons to help students practice and apply the new language and concepts they were learning. Instead of telling students to read a chapter in the textbook on solving problems with improper fractions and do exercises, we suggested a teacher write a problem's solution on paper as a series of steps, make numerous copies, cut up the steps into strips, scramble them, and have student pairs put them in order. Students would discuss their ideas, look for logical connections between one step and the next. Or, as another activity, student groups could create their own fraction problem, write it on an index card, and send it to another group to solve. That group would discuss, solve, and send it back to the original group to assess.

We also proposed that teachers assign tasks that would require higher-order thinking and communication among students. For example, suppose student groups each had to make a poster about a different human organ system (e.g., respiratory system, circulatory system). The assignment might be "List the top five facts a person should know about this system and draw a diagram". The students would have to discuss facts about the system, negotiate some consensus on the top five, create the poster, and illustrate the system. They might have to read and extract information from a textbook or Internet resources.

To further academic language practice, the teacher might use the "Walkers and Talkers" technique. Each group's poster is placed on a wall and one student remains at the poster as the talker to explain it to others who come by. The rest walk around and listen to the explanations of the other systems. These walkers record the information they learn in a notebook. (The teacher might model an effective oral presentation of a sample poster, in advance.) When time is up, the groups reconvene at their posters and the walkers explain what they learned from the other posters to the talker who takes notes in his or her notebook. Through this project, students practice all of the language skills.

The final SIOP workshops focused more on lesson planning, after the teachers had become familiar with a variety of language development techniques. We used a series of progressive activities. First, teachers were given a SIOP math or science lesson plan which had a content objective and a language objective. They were asked to indicate where in the lesson the teacher explicitly instructs on the language objective and how students practice it. This activity raises teacher awareness of language development within instruction. Next, teachers were given a fairly traditional math or science lesson and asked to write a language objective for the lesson and add some language practice opportunities. Finally, teachers were tasked with writing their own SIOP lesson plan and received feedback from the teacher trainer.

Teacher Implementation

To address the first research question in this NJ SIOP study, we delivered the SIOP professional development program to the teacher cohorts in the treatment district. In Year 1, this consisted of seven workshops spread over the course of one year. By teaching components of the SIOP Model over time, we gave teachers a chance to practice in their classes and build on their knowledge. To help teachers incorporate the model into their teaching, we organized the workshops in a participatory manner for the teachers with hands-on activities, cooperative mini-projects, analysis of videotaped instruction, and integration of research and theory. We

also recruited and trained three local coaches at the school sites (part-time teachers) to observe in classes and give feedback, and we offered technical assistance via electronic media.

In addition to the workshops, the coaches and sometimes the researchers observed and gave feedback to teachers to assist with implementation. Because the coaches were on-site, some teachers also sought their advice in lesson planning. We created a project website and posted sample lesson plans and step-by-step explanations of instructional techniques. Teachers could use the closed group electronic list to share information, challenges, and successes.

The PD workshops in the second year for the Cohort 2 teachers were very similar to Year 1. We also offered three additional workshops on lesson and unit planning to Cohort 1 teachers to help them implement SIOP better. We added more teaching materials to the project website and hosted online chats on topics like math language techniques. With district support, we increased the number of coaches from three to five and they were given more time during the day to devote to coaching. This was an important change because the number of teachers involved rose to almost 60 in this second year.

Comparison teachers did not receive SIOP Model professional development, but all teachers had a one-hour, district-sponsored workshop on student diversity and accommodating ELLs in the classroom. The ESL teachers had workshops on topics such as designing thematic units and using the new content-based ESL textbooks.

We conducted classroom observations, took field notes, and rated lessons of each SIOP and comparison teacher on the SIOP protocol twice per year, in the fall and spring. In this way, teacher fidelity to the intervention was measured. Comparison teachers were also observed because it was anticipated that they might incorporate some characteristics of sheltered instruction in their lessons. We recorded scores on individual items and the overall percentage score in a teacher database. We used the scores to determine which teachers were high, medium, or low implementers based on the following guidelines: high implementers scored 75% or higher; medium implementers scored between 50% and 75%; and low implementers scored 50% or below on the protocol's scale.

Teacher implementation data revealed that in the SIOP district, after one year of professional development, 56% of Cohort 1 (in Year 1) and 74% of Cohort 2 (in Year 2) were high implementers of the SIOP Model. After two years, 71% of Cohort 1 reached a high level. As a subset of the teachers, the Cohort 1 math teachers (n=10) reached the second highest level of implementation in Year 2, after the language arts teachers, averaging a score on the SIOP protocol of 83.3%. The Cohort 1 science teachers (n=5) implemented the SIOP Model less well, reaching an average score of 69.2%.

At the comparison site, only 5% of the teachers reached a high level in Year 1; 17% in Year 2. The features of the SIOP Model were thus much better implemented in the SIOP district (Short, Fidelman, & Louguit, 2012).

Student Achievement

To address the second research question, we examined student performance on tests required by the state to assess student knowledge of the standards. The outcome measure of academic

literacy was the IDEA Language Proficiency Tests (IPT), the standardized assessment of English language proficiency in New Jersey at that time. The IPT provided a total score for each student and subtest scores for oral language, reading, and writing. We also examined student performance on several state subject achievement tests. The subjects of the data collection were students in the ESL programs in Grades 6-12 in both districts. SIOP students (n = 387) spoke more than 15 different native languages and were from 35 countries of origin. Comparison students (n =193) spoke eight different native languages with 25 countries of origin.

English language proficiency. We collected the IPT scores for ELLs in both districts. We first gathered baseline IPT data on all ELLs from the Spring 2004 administration. In 2005 and 2006, we collected the IPT scores of ELLs with at least one SIOP teacher or at least one comparison teacher. It is important to note that in these district ESL programs, new students enter and others exit annually. As a result we had a cross-section of students that was not matched across the years and so we examined the average mean scores of the groups. Because the districts had a high level of student mobility, only a small number participated in all three IPT administrations, so no longitudinal analyses were undertaken.

We compared IPT mean proficiency level scores for SIOP and comparison groups each year. Then, using 2006 data, we employed analysis of variance (ANOVA) measures to determine if the teachers' SIOP training influenced the students' English language achievement.

For oral language proficiency, the average mean scores were at about the same level in both districts in the baseline year, but SIOP students performed better than comparison students in 2005 and continued to outperform them in 2006 at which time the average mean score in the SIOP district was statistically significantly higher than in the comparison district ($F(1, 434) = 8.49$, $p < .004$). Reading had a similar trend except that SIOP students performed better than comparison students only in 2006 and the differences in average mean scores did not reach statistical significance ($F(1, 434) = 2.49$, $p = .12$). In writing, comparison ELLs had slightly higher performance in baseline year; however, in 2005 SIOP students had higher mean scores. By 2006, this difference was statistically significantly ($F(1, 433) = 9.74$, $p < .002$).

Total English proficiency level scores showed the same trends as the oral language and writing data results but were moderated by the reading results. Nonetheless, we found that although comparison students had better total proficiency scores than SIOP students in the baseline year, SIOP students surpassed them in 2005 and showed a statistically significant difference in mean scores by 2006 ($F(1, 433) = 5.36$, $p < .02$).

The ANOVA results provided some evidence of SIOP as a predictor of achievement in oral language, writing, and total English proficiency. We calculated Cohen's d effect size (Cohen, 1988) and found that the SIOP scores were more than one fourth of a standard deviation higher than those of the comparison group for oral language (0.29), almost one third of a standard deviation higher for writing (0.31), and close to one fourth for total English (0.23). These were considered small to moderate effects (Short, Fidelman & Louguit, 2012).

Subject area performance. We wanted to examine the SIOP's effects on student achievement in the subject areas too but the data collection and analyses of the state tests were

problematic for several reasons: a) the number of student subjects was very small for most tests (n<30 in one or both groups) so the results were not generalizable, b) tests were only administered in Grades 6, 7, 8, and 11, at the end of the school year, c) the students took these tests only once, while in that particular grade, and d) New Jersey changed tests in 2006. Nonetheless, with these limitations in mind, the results showed a significant difference ($p < .05$) in mean scores in favor of SIOP students on five state subject area tests: reading and language arts for Grade 6 in 2005, language arts for Grades 6 and 7 in 2006, and mathematics for Grade 11 in 2006. There was a significant difference ($p < .05$) in mean scores in favor of students in the comparison district on one state content test: social studies for Grade 7 in 2005. There were no significant differences between groups on the other 19 content tests (Short, Echevarria & Richards-Tutor, 2011).

We also tried to compare the results of students who had been in the study for two years on the tests that they took in the second year (2005-06). We wanted to see if participation in SIOP instruction over two years influenced student performance. The number of students in both the SIOP and comparison groups exceeded 30 for only two tests however—the mathematics and language arts tests given in Grade 11. For these assessments, we found a significant difference in favor of SIOP for mathematics ($p < .05$), but no significant difference for language arts.

Focusing on One Subject Area: The CREATE SIOP Science Study (2005 – 2007)

As a result of the New Jersey SIOP study, we felt that we needed to refine the professional development program and try to increase teacher fidelity to enhance student outcomes. We decided to concentrate our efforts on one subject area, science. With funding from the U.S. Department of Education through the National Center for Research on the Educational Achievement and Teaching of English Language Learners (CREATE), we created an experimental-control study for middle school science and we included native English speakers and former ELLs in the analyses in addition to English language learners. Science was selected because of its importance in schooling and because it was a recent addition to federal testing mandates. The research question was the following: What are the effects of the SIOP Model on the acquisition of science language among English language learners in middle school science classrooms?

In 2005–06, with teacher consultants, we developed four Grade 7 SIOP life science units (Cell Structure and Function, Photosynthesis and Respiration, Cell Division, and Genetics) based on the district curriculum and state standards to support the fidelity of implementation. With these units, teachers would have lessons already embedded with SIOP features to teach while they were learning the model. We also designed and field-tested curriculum-based assessments for each unit to measure life science concepts, scientific vocabulary, reading comprehension skills, and writing skills. These assessments included multiple choice and essay items.

In 2006–07, we randomly assigned ten middle schools in southern California to SIOP or control conditions. Two control schools dropped out however as the research began, leaving a total of

12 teachers in the study. Because of California's teaching certification requirements, the science teachers in both conditions had had some university preparation for teaching ELLs.

SIOP Professional Development

Life Science was taught at Grade 7 for only one semester in this district. This was a limitation that condensed the time for professional development, data collection, and potential impact. Teachers in the five SIOP schools received only 3 days of training in the SIOP Model. As part of the professional development, participants explored the eight components of the SIOP Model by watching videos illustrating effective classroom implementation of each component's features, rating the video lessons using the SIOP protocol, and practicing SIOP techniques to deepen their understanding. Participants were given binders of the SIOP Life Science units with supplementary materials (e.g., graphic organizers for students to take notes from text readings, activity sheets to be used for lab experiments and vocabulary development). They reviewed the SIOP lesson plans in these units and were able to ask questions and suggest changes.

From September through December, SIOP teachers taught the four science units. Biweekly coaching support for all SIOP teachers was provided by the project staff. The teacher and coach reviewed the lesson plan beforehand, the coach observed and recorded notes using the SIOP protocol, and then the coach met with the teacher to provide detailed feedback. We viewed this coaching process as critically important given the reduced number of workshops we could provide.

In the three control schools, no SIOP training or coaching was provided. Teachers taught the same curricular topics but with their own unit lessons. Both groups of teachers had the same textbooks available for use.

Student Achievement

Students in both conditions were given the CREATE science language assessments as a pretest at the beginning of each unit and as a posttest at the end to measure growth in acquisition of science language. The essays were scored using the IMAGE writing rubric. In the data analysis, we compared the assessment results of students in the SIOP classes (n = 649) to those of control students (n = 372). The sample included students who were native English speakers, former ELLs who had been redesignated, and English language learners.

We used hierarchical linear modeling (HLM) to determine if SIOP instruction had an impact on students' science language and concept development. The individual students were nested within teaching sections, sections within teacher, and teachers within schools. In this way we could analyze individual student results on the assessments in conjunction with teacher and school variables. Because student and teacher level fixed effect variables may influence student outcomes, we examined the students' pretest scores and their language classification (e.g., native speaker, ELL) as student variables, and the level of SIOP implementation (high, medium, low) and the condition (SIOP or control) as teacher variables. We aggregated the scores of the four posttest assessments but analyzed the composite scores for the essay and multiple choice components separately as outcome variables.

Results from the conditional ANCOVA model of HLM indicated that students in the treatment condition for all language proficiency classifications—outperformed, on average, those in the control, although not to a statistically significant degree. There was an approximate 0.9 point advantage (ϒ = 0.9, s.e. = 2.1, t =.429, p =.67) for students in SIOP schools on the multiple choice component of the posttest and a larger 5.5 point advantage (ϒ = 5.5, s.e. = 6.8, t =.809, p =.418) on the essay component.

We also calculated the Hedges' g effect sizes (Hedges, 2007). The effect of SIOP instruction on the multiple choice component of the posttest was associated with Hedges' g = .103, whereas the effect on the essay component of the posttest was g =.197. These results indicated small positive effects (Echevarria, Richards-Tutor, Canges, & Francis, 2011).

Teacher Implementation

The results were disappointing but not too surprising, given our classroom observations and the limited time we had for professional development. SIOP and control teachers were observed five times and their lessons were rated using the SIOP protocol. The scores were averaged into an overall score and the teachers were categorized as high, medium, and low implementers (as in the NJ SIOP study). We found that teachers in both groups scored across these ranges. Some SIOP teachers were low implementers and some control teachers were high (possibly because of their university preparation).

We decided next to examine whether the level of teacher implementation played a role in student achievement. We compared teacher results with their students' average scores across the four assessments. The analyses indicated a positive relationship between teacher implementation level and average student gains. In other words, students whose teachers implemented the SIOP Model to a high degree performed significantly better on the assessments than students whose teachers were low implementers (R^2=.22, p<.05), emphasizing the importance of fidelity to the model. This result held true for English language learners, former English language learners, and native English speakers (Echevarria, Richards-Tutor, Chinn, & Ratleff, 2011).

DISCUSSION

While the research we have conducted to date has had positive results, the outcomes were not as strong as desired. In the SIOP design study, the students of SIOP-trained teachers significantly outperformed the comparison students on the writing assessment. In the NJ study, the significant differences in the average mean scores in favor of the SIOP student group on oral language, writing, and total English proficiency indicated that the SIOP professional development had a positive impact on the development of English among the ELLs in classes with SIOP-trained teachers. There was a small impact on achievement in some subject areas but results were not generalizable, given the limitations of the testing process. In the CREATE study, which had a shortened time frame for professional development and implementation, the science language achievement results were less robust.

One important finding from the work is that sustained, high quality professional development is critical if teachers are to implement interventions with fidelity. Providing workshops and lesson plans alone, as in the CREATE study, may not lead to high levels of implementation, particularly for a comprehensive approach like the SIOP Model. Site-based coaching appears to be an essential element of overall professional development, offering teachers the job-embedded support required to sustain changes in their practice.

In the NJ study we found that only 56% of the treatment teachers in Cohort 1 became high implementers of SIOP after one year whereas 74% of the Cohort 2 teachers reached the high implementation level in that time frame. We argue that the context of the SIOP Model initiative played a role in this difference. SIOP was a new initiative in 2005 when Cohort 1 teachers participated in the professional development. The coaching support was more limited then, the notion of focusing on language development in content courses was new, and a culture of working in a cross-disciplinary way was lacking. In contrast, Cohort 2 teachers entered an existing SIOP culture in 2006 and joined a team of teachers and coaches who had already experienced success. The SIOP Model was also viewed favorably at that point by the administration which devoted more staff time to coaching, affording teachers more support.

Another finding was that fidelity to the model led to higher student achievement. In the CREATE study this was true for all types of learners—ELLs, former ELLs, and native English speakers. SIOP instruction did not just benefit students learning English as a new language, it benefited all students in the classes. But fidelity does not occur naturally. Teachers need time to get good at the SIOP Model and we have to contextualize the PD activities in their particular subject areas and classrooms to facilitate their mastery of the teaching practices.

IMPLICATIONS FOR MATH AND SCIENCE TEACHERS

Because U.S. educational reforms put pressure on English language learners (and their teachers and schools) to reach the average performance levels of native English speakers on high-stakes tests in reading, math, and science before they were proficient in English, our early focus had been on serving as many teachers as possible through professional learning opportunities. However, in recent years we have had opportunities to offer subject-specific PD on the SIOP Model (Echevarria, Vogt, & Short, 2010; Short, Vogt, & Echevarria, 2011) and we think this is a sensible evolution of our research program.

Our work with math and science teachers showed us that they did not typically think about developing academic language among students when they planned their lessons. They were capable of communicating concepts, explaining procedures, exploring theories, modeling problems, analyzing patterns, and making real-world connections through a variety of methods. Yet although they used academic discourse themselves, they were not prepared to teach it explicitly to their English language learners. They readily applied SIOP techniques that helped make the content concepts comprehensible (e.g., pre-teaching key terms, using videos to illustrate a concept, using manipulatives and other hands-on materials) but were not comfortable at first with the pedagogical practices we

recommended to elicit mathematical or scientific language from students who were still learning English. They understood that ELLs should interact with the content concepts by talking, reading, and writing about them, but they needed support to make such communication happen. Therefore, we needed to enhance the instructional repertoire of the mathematics and science teachers so they could more easily integrate language and content instruction.

In order to transform math and science teaching practices, job-embedded professional development is needed. This involves a range of workshop activities geared to the specific curricula: demonstrations of math and science language techniques, discourse analysis of the language in math and science textbooks, examination of student work from a language perspective, video clips of meaningful lessons that simultaneously teach content and develop English language skills, practice opportunities to write language objectives for lessons, collaboration with language teachers to prepare language frames, and more. Workshops alone will not lead to high levels of implementation, so in-class coaching and lesson planning assistance are needed as well to promote teacher uptake of the new practices and to sustain the professional learning over time.

DIRECTIONS FOR FUTURE RESEARCH

Although we have conducted research on the SIOP Model for 15 years, there is more to learn. The following suggestions could be applied to a single content area, like mathematics or science, or could be used with mixed content areas.

In the SIOP research studies conducted to date, the student achievement data has been collected concurrently with the teacher professional development. Consequently, medium and high levels of implementation are not reached by all teachers before student assessments begin. Future research might consider investigating the effects on student achievement after SIOP professional development is completed and teachers implement the model with fidelity. Such a study would offer a more valid picture of SIOP's impact on student performance.

Another gap in the research is a longitudinal analysis of SIOP implementation. It would be worthwhile to examine the effects on student performance in language development and subject matter knowledge after they have had continuous exposure to SIOP instruction. For example, an experimental study could look at the effects of SIOP instruction on the same cohort of students over three to five years' time and compare their achievement to that of a control group. Teachers would need to be trained in advance (e.g., one year prior). The study would also need to carefully plan for appropriate pre- and posttest measures to capture growth over time.

A third area is for researchers to examine the specific genres and academic language of subject areas like mathematics and science. This can be accomplished with discourse analysis of written and spoken discourse and case studies of teachers and students (see, for example, Moschkovich, 2010 and Unsworth, 2000). We know that it is not enough to make content teachers familiar with the techniques of integrated language and content instruction, we also have to strengthen their understanding of how academic language is

used in their subject area and how they can build student competence with it. We have begun some work in this area but more is needed.

Finally, most of our professional development work has been with practicing teachers. Our field would benefit from some design research at the undergraduate level in teacher education. The course load for teacher candidates is already full, but it is not preparing them for today's students. How can pedagogy courses incorporate more attention to strategies and techniques that are effective in teaching content and academic language to English language learners? What practicum opportunities can not only expose teachers-to-be to culturally and linguistically diverse classrooms but allow them to observe high quality teaching? What action research might they undertake to learn about the students' real-life experiences with second language acquisition?

CONCLUSION

The pressing need to improve instructional practices and academic language development for English language learners in the 1990s in the United States led to the development of the SIOP Model. Drawing from the professional literature and teacher input, we built an instructional approach and tested it over time. In an iterative process, we refined the professional development program based on classroom observations, teacher feedback, and student performance on assessments. The pressure has continued into the 21st century and the overarching goal of the SIOP Model, namely to make language an integral part of lesson design and delivery, is still relevant. It is one approach that has merit and a research base which shows that high levels of implementation lead to student achievement.

The SIOP Model can be applied to any schooling situation where students are learning content through a new language. It can used with any state standards, in any grade and in any subject (see for example, Watkins & Lindahl, 2010 and Whittier & Robinson 2007.). However as with most interventions, lasting effects require structures at the school level, such as onsite coaching, to sustain the teacher development necessary for delivering effective sheltered instruction (Batt, 2010; Friend, Most & McCrary, 2009; McIntyre et al., 2010).

Currently, the movement of peoples across borders and into lands of cultural and linguistic diversity is widespread. Many countries are educating immigrants and refugees who do not speak the language used in school. The SIOP Model offers a framework for instruction that accommodates their varied levels of proficiency in the new language yet allows them access to the subject matter at the same time. In the United States, the SIOP has been used in dual language classrooms where some non-native speakers, of Spanish for instance, are studying the grade-level curriculum through Spanish (Howard, Sugarman & Coburn, 2006). In the Netherlands, SIOP instruction has been used in Dutch math and science classrooms with immigrant learners (Hajer & Meestringa, 2009). In Korea, the SIOP has been used to train English as a foreign language teachers (Song, 2016).

With SIOP instruction, teachers use techniques to make academic topics accessible to students and practice the academic language as it is used in specific subject areas. Key features for the academic success of English language learners include language objectives

in every lesson, the development of background knowledge, the acquisition of academic vocabulary, cooperative group activities, and the emphasis on subject-specific genres for reading, writing, listening, and speaking tasks. When teachers reach high levels of implementation, with coaching support and time, English language learners develop academic language and content knowledge.

APPENDIX

The Sheltered Instruction Observation Protocol (SIOP)
(Echevarria, Vogt, & Short, 2017)

Observer: _____
Date:_____
Grade:___ _____
Class:_____

Teacher:_____
School:__ _____
ESL level:_____
Lesson: Multi-day Single-day (circle one)

Cite under "Comments" specific examples of the behaviors observed

Total Score: _____ % Score _____

Lesson Preparation	**4**	**3**	**2**	**1**	**0**	**NA**
1. Clearly defined content objectives for students	☐	☐	☐	☐	☐	
2. Clearly defined language objectives for students	☐	☐	☐	☐	☐	
3. Content concepts appropriate for age and educational background level of students	☐	☐	☐	☐	☐	
4. Supplementary materials used to a high degree, making the lesson clear and meaningful (graphs, models, visuals)	☐	☐	☐	☐	☐	
5. Adaptation of content (e.g., text, assignment) to all levels of student proficiency	☐	☐	☐	☐	☐	☐
6. Meaningful activities that integrate lesson concepts (e.g., surveys, letter writing, simulations, constructing models) with language practice opportunities for reading, writing, listening, and/or speaking *Comments:*	☐	☐	☐	☐	☐	

Building Background	**4**	**3**	**2**	**1**	**0**	**NA**
7. Concepts explicitly linked to students' background experiences	☐	☐	☐	☐	☐	☐
8. Links explicitly made between past learning and new concepts	☐	☐	☐	☐	☐	
9. Key vocabulary emphasized (e.g., introduced, written, repeated and highlighted for students to see) *Comments:*	☐	☐	☐	☐	☐	

Comprehensible Input	**4**	**3**	**2**	**1**	**0**
10. Speech appropriate for students' proficiency level (e.g., slower rate, enunciation and simple sentence structure for beginners)	☐	☐	☐	☐	☐
11. Explanation of academic tasks clear	☐	☐	☐	☐	☐
12. A variety of techniques used to make content concepts clear (e.g., modeling, visuals, hands-on activities, demonstrations, gestures, body language) *Comments:*	☐	☐	☐	☐	☐

Strategies	**4**	**3**	**2**	**1**	**0**	
13. Ample opportunities provided for student to use learning strategies	☐	☐	☐	☐	☐	
14. Consistent use of scaffolding techniques throughout lesson, assisting and supporting student understanding such as think-alouds	☐	☐	☐	☐	☐	
15. A variety of question types used throughout the lesson including those that promote higher-order thinking skills	☐	☐	☐	☐	☐	
Comments:						
Interaction	**4**	**3**	**2**	**1**	**0**	**NA**
16. Frequent opportunities for interactions and discussion between teacher/student and among students, which encourage elaborated responses about lesson concepts		☐	☐	☐	☐	
17. Grouping configurations support language and content objectives of the lesson	☐	☐	☐	☐	☐	
18. Sufficient wait time for student response consistently provided	☐	☐	☐	☐	☐	
19. Ample opportunities for students to clarify key concepts in L1 as needed	☐	☐	☐	☐	☐	☐
Comments:						
Practice & Application	**4**	**3**	**2**	**1**	**0**	**NA**
20. Hands-on materials and/or manipulatives provided for students to practice using new content knowledge	☐	☐	☐	☐	☐	☐
21. Activities provided for students to apply content and language knowledge in the classroom	☐	☐	☐	☐	☐	☐
22. Activities integrate all language skills (i.e., reading, writing, listening, and speaking)	☐	☐	☐	☐	☐	
Comments:						
Lesson Delivery	**4**	**3**	**2**	**1**	**0**	
23. Content objectives clearly supported by lesson delivery	☐	☐	☐	☐	☐	
24. Language objectives clearly supported by lesson delivery	☐	☐	☐	☐	☐	
25. Students engaged approximately 90-100% of the period	☐	☐	☐	☐	☐	
26. Pacing of the lesson appropriate to the students' ability level	☐	☐	☐	☐	☐	
Comments:						
Review & Assessment	4	3	2	1	0	
27. Comprehensive review of key vocabulary	☐	☐	☐	☐	☐	
28. Comprehensive review of key content concepts	☐	☐	☐	☐	☐	
29. Regular feedback provided to students on their output (e.g., language, content, work)	☐	☐	☐	☐	☐	
30. Assessment of student comprehension and learning of all lesson objectives (e.g., spot checking, group response) throughout the lesson	☐	☐	☐	☐	☐	

Comments:

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